Fractal Decoded Image Quality Prediction Based on Accumulated Collage Error Coefficient

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Abstract

To predict the fractal decoded image quality more efficiently, an accumulated collage error coefficient (ACEC) based method was proposed in this study. Firstly, the definition of ACEC was introduced to describe the relationship among the upper bound, lower bound, and actual value of the accumulated collage error of all range blocks. Moreover, the definition and monotonicity of the relative error of ACEC were also defined and discussed. While the relative error of ACEC reaches a relatively small value, the average collage error (ACER) can be estimated approximately, and then the encoding process can be terminated to directly predict the peak signal-to-noise ratio (PSNR) quality of decoded images. Experimental results show that compared with the previous method, the proposed method can predict the decoded image quality more accurately with fewer computations.

Full Text

Preamble

Fractal Decoded Image Quality Prediction Based on Accumulated Collage Error Coefficient

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Abstract: To predict fractal decoded image quality more efficiently, this study proposes a method based on the accumulated collage error coefficient (ACEC). First, we introduce the definition of ACEC to describe the relationship among the upper bound, lower bound, and actual value of the accumulated collage error across all range blocks. Additionally, we define and analyze the monotonicity properties of the relative error of ACEC. When the relative error of ACEC reaches a sufficiently small value, the average collage error (ACER) can be approximated, allowing the encoding process to terminate early while directly predicting the peak signal-to-noise ratio (PSNR) quality of decoded images. Experimental results demonstrate that compared with previous methods, the proposed approach achieves more accurate prediction of decoded image quality with fewer computational requirements.

Keywords: Fractal image coding; Decoded image quality; Accumulated collage error coefficient

1. Introduction

Images play a crucial role in human perception, and vast quantities of images are stored, processed, and transmitted daily, creating significant pressure on storage and transmission resources. Efficient image compression technology provides an effective solution for reducing data volume. Unlike conventional techniques such as discrete cosine transform and discrete wavelet transform, fractal image coding achieves compression by establishing an iterated function system (IFS) whose fixed point can closely approximate the original image.

The core concept of fractal image coding was first introduced by Barnsley, with Jacquin subsequently proposing the first block-based fractal coding method [?, ?]. Although fractal image coding offers several advantages—including a novel theoretical foundation, fast decoding, potential for high compression ratios, and resolution independence [?, ?]—its primary drawback has been high computational complexity during the encoding process. Consequently, research on fast fractal encoding has become a key direction in this field. In recent years, numerous fast encoding methods have been proposed by converting global block matching into local block matching [?]. For instance, Chaurasia proposed composite statistical features, while Gupta extracted block features in the DCT domain. Both approaches locate nearest neighbors in feature space as candidate domain blocks, restricting block-matching operations to these candidates. Rather than seeking the best-matched domain block, Zheng specified update times for fractal codes to obtain acceptable domain blocks, thereby effectively shortening the encoding process. Furthermore, no-search fractal image coding methods directly assign a single domain block as the best match for each range block without any search operations, enabling real-time encoding and higher compression ratios at the cost of poor decoded image quality [?]. After years of development, fractal image coding has been gradually applied to various image processing tasks, including image denoising [?], image retrieval [?], image super-resolution [?], watermarking [?], image hashing [?, ?], and head pose estimation [?, ?].

The fractal decoded image quality prediction method is a recently proposed technique that can predict decoded image quality using only a partial encoding process [?]. Previous methods utilized the average percentage of accumulated collage error for prediction but did not consider the proportional relationship among the upper bound, lower bound, and actual value of the accumulated collage error. To address this limitation, we propose an accumulated collage error coefficient (ACEC) based method for more efficient quality prediction. By examining the upper and lower bounds of the accumulated collage error across all range blocks, we define ACEC to characterize the proportional relationship among these bounds and the actual accumulated collage error value. We further define and analyze the monotonicity properties of the relative error of ACEC. During encoding, when the relative error of ACEC becomes sufficiently small, the process can terminate early, allowing us to approximate the average collage error (ACER) and directly predict the peak signal-to-noise ratio (PSNR) quality of decoded images. Experimental results show that compared with previous methods, our approach predicts PSNR quality more accurately while requiring fewer computations.

The main contributions of this study are summarized as follows:

1. Based on the upper and lower bounds of accumulated collage error, we propose the definition of accumulated collage error coefficient (ACEC) to describe the relationship among the upper bound, lower bound, and actual value of the accumulated collage error for all range blocks.
2. We introduce and discuss methods for estimating ACEC, along with the definition and monotonicity of ACEC' s relative error. This enables our ACEC-based prediction method to achieve higher accuracy with fewer computations than previous approaches.
3. The proposed method can be integrated with existing fractal encoding methods to predict decoded image quality using only a partial encoding process.

This paper is organized as follows: Section 2 reviews conventional fractal image coding. Section 3 describes the principle and detailed procedures of the proposed method. Sections 4 and 5 present and analyze experimental results, respectively. Section 6 concludes the paper.

2. Conventional Fractal Image Coding

Fractal image coding aims to establish an IFS whose fixed point can approximate the original image. The first block-based fractal encoding method was proposed by Jacquin, with detailed procedures described in [**?**]:

Step 1: Divide the $M \times N$ input image f uniformly into non-overlapping $B \times B$ range blocks R_i, where $i = 1, 2, 3, \dots, \text{NumR}$ and NumR denotes the total number of range blocks.

Step 2: Slide a $2B \times 2B$ window over the original image horizontally and vertically to obtain domain blocks D_j, where $j = 1, 2, 3, \dots, \text{NumD}$ and NumD denotes the total number of domain blocks.

Step 3: Establish a domain block pool (DBP) by contracting the domain blocks to the same size as range blocks. Then obtain an extended domain block pool (EDBP) by performing eight isometric transformations on each domain block within the DBP.

Step 4: Find the best-matched domain block for each range block by searching within the EDBP and minimizing the following function:

$$\min_{\substack{j=1,2,3,\ldots,\text{NumD} \\ 8 \text{ transformations}}} \text{CE}(R_i) = \|R_i - (s_i \cdot D_j + o_i \cdot I)\|^2, \quad i = 1, 2, 3, \ldots, \text{NumR}$$

where $\text{CE}(R_i)$ denotes the collage error of R_i, I denotes a $B \times B$ matrix whose components are all ones, s_i and o_i denote the scaling and offset coefficients of the affine transformation, respectively, and D_j is the best-matched domain block of R_i. The coefficients s_i and o_i can be computed using the least squares method as:

$$s_i = \frac{\langle R_i - \bar{R}_i \cdot I, D_j - \bar{D}_j \cdot I \rangle}{\|D_j - \bar{D}_j \cdot I\|^2}, \quad o_i = \bar{R}_i - s_i \cdot \bar{D}_j$$

where $\langle \cdot \rangle$ denotes the inner product, and $\bar{R}_i$ and $\bar{D}_j$ denote the averages of R_i and D_j, respectively.

In the fractal decoding process, an arbitrary $M \times N$ image can be selected as the initial image. The same transformations established during encoding are then performed to reconstruct all range blocks until one iteration is completed. After approximately 10 iterations, the decoded image can be obtained. Figure 1 Figure 1: see original paper illustrates the rotated 256×256 Bird image used as the initial image, while Figures 2 Figure 2: see original paper-(f) show the first five iteration images during the decoding process when the Bird image is encoded. The figure caption "Initial and the first five iteration images in decoding process" accompanies these images.

3. Prediction of Decoded Image Quality with the Accumulated Collage Error Coefficient

3.1 Definition of the Accumulated Collage Error Coefficient

Substituting Eq.(2) back into Eq.(1) yields:

$$\text{CE}(R_i) = \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \text{LCC}_i^2)$$

where LCC_i denotes the linear correlation coefficient between R_i and D_j. Since $0 \le \text{LCC}_i \le 1$, we have:

$$0 \le \text{CE}(R_i) \le \|R_i - \bar{R}_i \cdot I\|^2$$

From Eq.(4), we observe that for any range block, its variance provides the upper bound for the associated collage error, and only range blocks with large variances may produce large collage errors. Therefore, all range blocks are sorted by their variances from largest to smallest and encoded sequentially. During this process, the accumulated collage error (ACE) of all range blocks can be expressed as:

$$\mathrm{ACE} = \sum_{i=1}^{\mathrm{NumR}} \mathrm{CE}(R_i) = \sum_{i=1}^{\mathrm{NumR}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \mathrm{LCC}_i^2)$$

If we use $\mathrm{LCC}_{\mathrm{min}}$ and $\mathrm{LCC}_{\mathrm{max}}$ to represent the minimum and maximum of all LCCs, the upper and lower bounds of ACE can be described as:

$$\mathrm{ACE}_{\mathrm{lower}} = \sum_{i=1}^{\mathrm{NumR}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \mathrm{LCC}_{\mathrm{max}}^2)$$

$$\mathrm{ACE}_{\mathrm{upper}} = \sum_{i=1}^{\mathrm{NumR}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \mathrm{LCC}_{\mathrm{min}}^2)$$

During encoding, range blocks can be divided into two categories: coded range blocks and uncoded range blocks. The ACE of coded range blocks can be represented as:

$$\mathrm{ACE}_{\mathrm{coded}} = \sum_{i=1}^{\mathrm{NumR}_{\mathrm{coded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \mathrm{LCC}_i^2)$$

where $\mathrm{NumR}_{\mathrm{coded}}$ denotes the number of coded range blocks. Similarly, the ACE of uncoded range blocks can be represented as:

$$\mathrm{ACE}_{\mathrm{uncoded}} = \sum_{i=1}^{\mathrm{NumR}_{\mathrm{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \mathrm{LCC}_i^2)$$

where $\mathrm{NumR}_{\mathrm{uncoded}}$ denotes the number of uncoded range blocks.

If we use $\mathrm{LCC}_{\mathrm{uncoded}}^{\mathrm{min}}$ and $\mathrm{LCC}_{\mathrm{uncoded}}^{\mathrm{max}}$ to represent the maximum and minimum of the LCCs of uncoded range blocks, the upper and lower bounds of the ACE of uncoded range blocks, $\mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{upper}}$ and $\mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{lower}}$, can be expressed as:

$$\mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{lower}} = \sum_{i=1}^{\mathrm{NumR}_{\mathrm{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - (\mathrm{LCC}_{\mathrm{uncoded}}^{\mathrm{max}})^2)$$

$$\mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{upper}} = \sum_{i=1}^{\mathrm{NumR}_{\mathrm{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - (\mathrm{LCC}_{\mathrm{uncoded}}^{\mathrm{min}})^2)$$

We then have the inequality:

$$\mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{lower}} < \mathrm{ACE}_{\mathrm{uncoded}} < \mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{upper}}$$

Moreover, because the uncoded range blocks are a subset of all range blocks, $\text{LCC}_{\text{uncoded}}^{\text{min}}$ and $\text{LCC}_{\text{uncoded}}^{\text{max}}$ can be approximated by LCC_{min} and LCC_{max}, respectively. Eq.(9) can then be rewritten by replacing $\text{LCC}_{\text{uncoded}}^{\text{min}}$ and $\text{LCC}_{\text{uncoded}}^{\text{max}}$ with LCC_{min} and LCC_{max} as:

$$\text{ACE}_{\text{uncoded}}^{\text{lower}} = \sum_{i=1}^{\text{NumR}_{\text{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \text{LCC}_{\text{max}}^2)$$

$$\text{ACE}_{\text{uncoded}}^{\text{upper}} = \sum_{i=1}^{\text{NumR}_{\text{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - \text{LCC}_{\text{min}}^2)$$

For all range blocks, to describe the proportional relationship among ACE, $\text{ACE}_{\text{lower}}$, and $\text{ACE}_{\text{upper}}$, we define the accumulated collage error coefficient (ACEC) as the ratio of the ratio of ACE to $\text{ACE}_{\text{lower}}$ to the ratio of $\text{ACE}_{\text{upper}}$ to $\text{ACE}_{\text{lower}}$, which can be calculated by:

$$\text{ACEC} = \frac{\text{ACE}/\text{ACE}_{\text{lower}}}{\text{ACE}_{\text{upper}}/\text{ACE}_{\text{lower}}} = \frac{\text{ACE}}{\text{ACE}_{\text{upper}}}$$

Further, Eq.(12) can be expanded as:

$$\text{ACE} = \text{ACE}_{\text{coded}} + \text{ACE}_{\text{uncoded}}$$

$$\text{ACE}_{\text{upper}} = \text{ACE}_{\text{coded}}^{\text{upper}} + \text{ACE}_{\text{uncoded}}^{\text{upper}}$$

Consequently, ACEC can be expressed as:

$$\text{ACEC} = \frac{\text{ACE}_{\text{coded}} + \text{ACE}_{\text{uncoded}}}{\text{ACE}_{\text{coded}}^{\text{upper}} + \text{ACE}_{\text{uncoded}}^{\text{upper}}}$$

Furthermore, by Eq.(10), the upper and lower bounds of ACEC, denoted as $\text{ACEC}_{\text{upper}}$ and $\text{ACEC}_{\text{lower}}$, can be represented respectively as:

$$\text{ACEC}_{\text{upper}} = \frac{\text{ACE}_{\text{coded}} + \text{ACE}_{\text{uncoded}}^{\text{upper}}}{\text{ACE}_{\text{coded}}^{\text{upper}} + \text{ACE}_{\text{uncoded}}^{\text{upper}}}$$

$$\text{ACEC}_{\text{lower}} = \frac{\text{ACE}_{\text{coded}} + \text{ACE}_{\text{uncoded}}^{\text{lower}}}{\text{ACE}_{\text{coded}}^{\text{upper}} + \text{ACE}_{\text{uncoded}}^{\text{upper}}}$$

While $\text{ACE}_{\text{uncoded}}^{\text{upper}}$ can approach $\text{ACE}_{\text{uncoded}}^{\text{lower}}$ sufficiently, we can estimate ACEC as:

$$\text{ACEC} \approx \frac{\text{ACEC}_{\text{upper}} + \text{ACEC}_{\text{lower}}}{2}$$

3.2 Relative Error of ACEC and Predicting PSNR Quality

Based on the dynamic range of ACEC, i.e., the deviation between $\mathrm{ACEC}_{\mathrm{upper}}$ and $\mathrm{ACEC}_{\mathrm{lower}}$, we define the relative error of ACEC, denoted as ε, as the ratio of half the deviation between $\mathrm{ACEC}_{\mathrm{upper}}$ and $\mathrm{ACEC}_{\mathrm{lower}}$ to the estimated ACEC from Eq.(15):

$$\varepsilon = \frac{(\mathrm{ACEC}_{\mathrm{upper}} - \mathrm{ACEC}_{\mathrm{lower}})/2}{\mathrm{ACEC}}$$

By Eqs.(15), (16), and (14), ε can be expanded as:

$$\varepsilon = \frac{\mathrm{ACE}^{\mathrm{upper}}_{\mathrm{uncoded}} - \mathrm{ACE}^{\mathrm{lower}}_{\mathrm{uncoded}}}{2(\mathrm{ACE}_{\mathrm{coded}} + \mathrm{ACE}_{\mathrm{uncoded}})}$$

Because a sufficiently large number of domain blocks within the EDBP can guarantee that $\mathrm{LCC}^{\min}_{\mathrm{uncoded}} \approx 0$, Eq.(17) can be rewritten as:

$$\varepsilon \approx \frac{\mathrm{ACE}^{\mathrm{upper}}_{\mathrm{uncoded}}}{2(\mathrm{ACE}_{\mathrm{coded}} + \mathrm{ACE}_{\mathrm{uncoded}})} = \frac{\sum_{i=1}^{\mathrm{NumR}_{\mathrm{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2}{2 \cdot \mathrm{ACE}}$$

By setting the derivatives of ε with respect to $\mathrm{NumR}_{\mathrm{coded}}$ and $\mathrm{NumR}_{\mathrm{uncoded}}$ to zero respectively, we obtain:

$$\frac{\partial \varepsilon}{\partial \mathrm{NumR}_{\mathrm{coded}}} = -\frac{\|R_{\mathrm{coded}} - \bar{R}_{\mathrm{coded}} \cdot I\|^2}{2 \cdot \mathrm{ACE}} + \frac{\mathrm{ACE}^{\mathrm{upper}}_{\mathrm{uncoded}} \cdot \|R_{\mathrm{coded}} - \bar{R}_{\mathrm{coded}} \cdot I\|^2 \cdot (1 - \mathrm{LCC}^2_{\mathrm{coded}})}{2 \cdot \mathrm{ACE}^2} = 0$$

$$\frac{\partial \varepsilon}{\partial \mathrm{NumR}_{\mathrm{uncoded}}} = \frac{\|R_{\mathrm{uncoded}} - \bar{R}_{\mathrm{uncoded}} \cdot I\|^2}{2 \cdot \mathrm{ACE}} - \frac{\mathrm{ACE}^{\mathrm{upper}}_{\mathrm{uncoded}} \cdot \|R_{\mathrm{uncoded}} - \bar{R}_{\mathrm{uncoded}} \cdot I\|^2 \cdot (1 - \mathrm{LCC}^2_{\mathrm{uncoded}})}{2 \cdot \mathrm{ACE}^2} = 0$$

At the early stage of encoding, $\mathrm{NumR}_{\mathrm{coded}}$ is much smaller than $\mathrm{NumR}_{\mathrm{uncoded}}$. From Eq.(18), we know that as encoding proceeds, ε decreases monotonically while $\mathrm{NumR}_{\mathrm{coded}}$ increases and $\mathrm{NumR}_{\mathrm{uncoded}}$ decreases. At the final stage of encoding, $\mathrm{NumR}_{\mathrm{coded}}$ is much larger than $\mathrm{NumR}_{\mathrm{uncoded}}$. By Eq.(18), we know that ε approaches zero. Thus, setting ε at a relatively small value can ensure that a sufficiently large number of range blocks have been encoded, allowing the maximum and minimum LCCs of coded range blocks, $\mathrm{LCC}^{\max}_{\mathrm{coded}}$ and $\mathrm{LCC}^{\min}_{\mathrm{coded}}$, to approximate those of all range blocks, $\mathrm{LCC}_{\max}$ and $\mathrm{LCC}_{\min}$, respectively:

$$\mathrm{LCC}^{\max}_{\mathrm{coded}} \approx \mathrm{LCC}_{\max}, \quad \mathrm{LCC}^{\min}_{\mathrm{coded}} \approx \mathrm{LCC}_{\min}$$

By Eq.(20), Eq.(6) can be rewritten by replacing $\mathrm{LCC}_{\max}$ and $\mathrm{LCC}_{\min}$ with $\mathrm{LCC}^{\max}_{\mathrm{coded}}$ and $\mathrm{LCC}^{\min}_{\mathrm{coded}}$, respectively, as:

$$\mathrm{ACE}_{\mathrm{lower}} = \sum_{i=1}^{\mathrm{NumR}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - (\mathrm{LCC}^{\max}_{\mathrm{coded}})^2)$$

$$\text{ACE}_{\text{upper}} = \sum_{i=1}^{\text{NumR}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - (\text{LCC}_{\text{coded}}^{\min})^2)$$

Similarly, Eq.(11) can be rewritten as:

$$\text{ACE}_{\text{uncoded}}^{\text{lower}} = \sum_{i=1}^{\text{NumR}_{\text{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - (\text{LCC}_{\text{coded}}^{\max})^2)$$

$$\text{ACE}_{\text{uncoded}}^{\text{upper}} = \sum_{i=1}^{\text{NumR}_{\text{uncoded}}} \|R_i - \bar{R}_i \cdot I\|^2 \cdot (1 - (\text{LCC}_{\text{coded}}^{\min})^2)$$

Thus, $\text{ACE}_{\text{upper}}$ in Eq.(14) can be obtained. Moreover, by Eq.(16), a small ε can also guarantee that ACEC is relatively close to the true value. Then ACE can be estimated by Eq.(15), and $\text{ACE}_{\text{upper}}$ can be computed by Eqs.(12) and (21) as:

$$\text{ACE}_{\text{upper}} = \text{ACE}_{\text{coded}}^{\text{upper}} + \text{ACE}_{\text{uncoded}}^{\text{upper}}$$

Further, the average collage error (ACER) of all range blocks can be computed as:

$$\text{ACER} = \frac{\text{ACE}}{\text{NumR}}$$

Additionally, the peak signal-to-noise ratio (PSNR) is used to measure decoded image quality as:

$$\text{PSNR} = 10 \log_{10} \frac{255^2}{\frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} (f(x, y) - \hat{f}(x, y))^2}$$

where f and $\hat{f}$ represent the input and decoded images, respectively. Moreover, a logarithmic relationship exists between ACER and PSNR:

$$\text{PSNR} = \alpha + \beta \log_{10} \text{ACER}$$

where α and β are constant values. Thus, by Eqs.(23) and (24), we can directly predict the PSNR quality of the decoded image. The detailed procedures of the proposed method are as follows:

Step 1: Divide the input image into range and domain blocks, and sort the range blocks by their variances from largest to smallest.

Step 2: Take one uncoded range block and encode it. Based on $\text{LCC}_{\text{coded}}^{\max}$ and $\text{LCC}_{\text{coded}}^{\min}$, compute $\text{ACE}_{\text{upper}}$ and $\text{ACE}_{\text{lower}}$ using Eqs.(14), (21), and (22). Further, compute ε using Eq.(16). If ε is smaller than 0.15, proceed to Step 3; otherwise, return to Step 2.

Step 3: Calculate ACER using Eqs.(24), (23), (21), (15), and (14), then predict the decoded image quality using Eq.(26).

4. Experiments

Five 256×256 test images (Bird, Owl, Trees, City, and Cat) shown in Figure 2 [Figure 2: see original paper] are used in our experiments. Three state-of-the-art fractal coding methods along with Jacquin' s method are employed to evaluate the proposed method' s performance. The scaling and offset coefficients, s and o, are quantized using 5 and 7 bits, respectively. We compare the previous prediction method with our proposed approach in terms of prediction accuracy and computational requirements.

Tables 1 , 2, 3, and 4 list the experimental results for Jacquin' s, Chaurasia' s, Zheng' s, and Gupta' s methods, respectively. In Table 1, when the range block size is 4×4, the second row shows the PSNR values of decoded images for Jacquin' s method. The third row indicates that 4096 range blocks need to be encoded for all test images, with the percentages of computations required (PCRs) considered as 100%. For the previous method, rows four through six list the predicted PSNRs, deviations between predicted and actual PSNRs, and PCRs. Rows seven through nine show the corresponding values for the proposed method. From Table 1, we observe that for 4×4 range blocks, the average deviation decreases from 0.03 dB to 0.02 dB, while the average PCR decreases from 62.72% to 56.20%. For 8×8 range blocks, the average deviation decreases from 0.15 dB to 0.12 dB, and the average PCR decreases from 62.81% to 54.47%. Thus, for Jacquin' s method, the proposed approach provides higher prediction accuracy with fewer computations than the previous method.

Tables 2 , 3, and 4 present the experimental results for Chaurasia' s, Zheng' s, and Gupta' s methods, respectively. In Table 2, for range block sizes of 4×4 and 8×8, the average PCRs decrease from 62.98% to 56.46% and from 62.91% to 54.65%, respectively, while the average deviations decrease from 0.09 dB to 0.07 dB and from 0.18 dB to 0.09 dB, respectively. In Table 3 , for range block sizes of 4×4 and 8×8, the average PCRs decrease from 70.97% to 64.64% and from 73.49% to 64.17%, respectively, with average deviations remaining at 0.05 dB and decreasing from 0.18 dB to 0.10 dB, respectively. In Table 4 , for range block sizes of 4×4 and 8×8, the average PCRs decrease from 63.54% to 56.97% and from 63.11% to 54.86%, respectively, while the average deviations decrease from 0.08 dB to 0.06 dB and from 0.17 dB to 0.11 dB, respectively. In summary, the proposed method predicts the PSNR quality of decoded images more accurately with fewer computations than the previous method.

5. Discussion

When the threshold ε is set to 0.15, Eq.(18) yields:

$$\varepsilon = \frac{\mathrm{ACE}_{\mathrm{uncoded}}^{\mathrm{upper}}}{2(\mathrm{ACE}_{\mathrm{coded}} + \mathrm{ACE}_{\mathrm{uncoded}})} = 0.15$$

From this, we derive:

$$\frac{\text{ACE}_{\text{coded}}}{\text{ACE}_{\text{uncoded}}} = 6.13$$

We define the actual percentage of accumulated collage error (APACE) as:

$$\text{APACE} = \frac{\text{ACE}_{\text{coded}}}{\text{ACE}_{\text{coded}} + \text{ACE}_{\text{uncoded}}}$$

The minimum APACE, $\text{APACE}_{\text{min}}$, can be defined and calculated using Eqs.(29) and (28) as:

$$\text{APACE}_{\text{min}} = \frac{\text{ACE}_{\text{coded}}}{\text{ACE}_{\text{coded}} + \text{ACE}^{\text{upper}}_{\text{uncoded}}}$$

In previous work, the encoding process terminated when APACE reached 90%. Thus, setting ε to 0.15 requires fewer computations than setting APACE to 90%. Specifically, when ε reaches 0.15, APACE is only about 86%, which is smaller than 90%. This implies that the previous method must encode an additional 4% of range blocks to achieve APACE = 90%. Consequently, the proposed method saves corresponding computations. For example, when range block size is 4×4, the green and red blocks in Figure 3 [Figure 3: see original paper] represent range blocks encoded in the previous method, while the green blocks alone represent those encoded in the proposed method. For the Bird image, we have 4096 total range blocks. In the previous method, when APACE reaches 90%, we must encode 2826 range blocks (represented by red and green blocks in Figure 3(a)), comprising 68.99% of total range blocks as shown in Table 1. In the proposed method, when ε reaches 0.15, we only need to encode 2489 range blocks (represented by green blocks in Figure 3(a)), comprising 60.77% of total range blocks in Table 1. This saves computations for 337 range blocks (represented by red blocks). When range block size is 8×8, similar results are illustrated in Figure 4 [Figure 4: see original paper]. For the Bird image, we have 1024 total range blocks. In the previous method, when APACE reaches 90%, we must encode 689 range blocks (represented by red and green blocks in Figure 4(a)), comprising 67.29% of total range blocks in Table 1. In the proposed method, when ε reaches 0.15, we only need to encode 568 range blocks (represented by green blocks in Figure 4(a)), comprising 55.47% of total range blocks in Table 1. This saves computations for 121 range blocks (represented by red blocks). Furthermore, ACEC in the proposed method captures the proportional relationship among ACE, $\text{ACE}_{\text{lower}}$, and $\text{ACE}_{\text{upper}}$, which was not considered in the previous method. Therefore, the proposed method provides more accurate prediction results. In summary, our approach predicts decoded image quality more accurately with fewer computations than the previous method.

6. Conclusion

This study proposed an accumulated collage error coefficient-based fractal prediction method. We first introduced the definition of ACEC to describe the proportional relationship among the upper bound, lower bound, and actual value of the accumulated collage error. We then defined and analyzed the monotonicity properties of ACEC' s relative error. Finally, when the relative error of ACEC reaches a small value, the accumulated collage error of all range blocks can be estimated to directly predict decoded image quality. Experimental results verify that the proposed method achieves higher prediction accuracy with fewer computations than previous methods. Future work will focus on further improving the performance of fractal prediction methods in terms of both prediction accuracy and computational efficiency.

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Note: Figure translations are in progress. See original paper for figures.