Recent Progress on Halo Nuclei in Relativistic Density Functional Theory

Authors: Kaiyuan Zhang, Cong Pan, Siyu Chen, Qingjin Luo, Keyan Wu, Yifeng Xiang, Cong Pan

Date: 2024-02-26T00:00:00+00:00

Abstract

Since the discovery of the halo nucleus 11Li in 1985, halo phenomena in exotic nuclei have always been an important frontier in nuclear physics research. The relativistic density functional theory has achieved great success in the study of halo nuclei, e.g., the self-consistent description of the halo nucleus 11Li and the microscopic prediction of deformed halo nuclei. This paper introduces some recent progress, including the investigation of the halo nucleus 37Mg and the prediction of the N = 28 shell collapse and a deformed halo in the new isotope 39Na based on the deformed relativistic Hartree-Bogoliubov theory in the continuum (DRHBc), as well as the exploration of triaxially deformed halo nuclei by the newly developed triaxial relativistic Hartree-Bogoliubov theory in the continuum (TRHBc).

Full Text

Preamble

Nuclear Physics Review, Vol. , No. , Feb. 2024
Article ID: 1007-4627(2024)000000-10

Recent Progress on Halo Nuclei in Relativistic Density Functional Theory

Kaiyuan Zhang[1], Cong Pan[2], Siyu Chen[1], Qingjin Luo[1], Keyan Wu[1], Yifeng Xiang[1]
(1. Institute of Nuclear Physics and Chemistry, China Academy of Engineering Physics, Mianyang, Sichuan 621900, China;
2. Department of Physics, Anhui Normal University, Wuhu, Anhui 241000, China)

Abstract: Since the discovery of the halo nucleus ^{11}Li in 1985, halo phenomena in exotic nuclei have remained an important frontier in nuclear physics research. Relativistic density functional theory has achieved great success in the study of halo nuclei, including the self-consistent description of the halo nucleus ^{11}Li and the microscopic prediction of deformed halo nuclei. This paper introduces some recent progress, including the investigation of the halo nucleus ^{37}Mg and the prediction of $N = 28$ shell collapse and a deformed halo in the new isotope ^{39}Na based on the deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc), as well as the exploration of triaxially deformed halo nuclei using the newly developed triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc).

Key words: halo phenomena; relativistic density functional theory; ^{37}Mg; ^{39}Na; ^{42}Al
Document code: A
CLC number: O571.21
DOI: 10.11804/NuclPhysRev.00.01.00

Received date: 26 Feb. 2024; Revised date: 26 Feb. 2024
Foundation item: Natural Science Foundation of China (12305125, U2230207, U2030209); Natural Science Foundation of Sichuan Province (124NSFSC5910); National Key R&D Program of China (2020YFA0406001, 2020YFA0406002)
Biography: Kaiyuan Zhang (1995-), working on nuclear physics; E-mail: zhangky@caep.cn
Corresponding author: E-mail: cpan@ahnu.edu.cn

Introduction

The discovery of the first halo nucleus [?] in 1985 has attracted great interest. The radius of a halo nucleus significantly deviates from the empirical formula $r_{0A}^{1/3}$ (where $r_0 \approx 1.2$ fm and A is the mass number), shaking the simple assumption that the nucleus is incompressible. Therefore, the description of halo phenomena has challenged traditional models for nuclear structure. The experimental exploration of halo nuclei has also stimulated the development of new generations of radioactive ion beam facilities worldwide.

During the past decades, about 20 halo nuclei or candidates have been identified or suggested in experiments, as shown in Fig. 1 [Figure 1: see original paper]. On the theoretical side, nuclear models studying halo phenomena include the few-body model [?, ?], shell model [?, ?], antisymmetrized molecular dynamics [?, ?], halo effective field theory [?, ?], nonrelativistic [?, ?] and relativistic density functional theories [?, ?], among others.

Relativistic density functional theory is designed for nuclear many-body problems based on quantum field theory and density functional theory [?]. Due to many appealing advantages [?, ?], such as the automatic inclusion of spin-orbit coupling [?, ?] and reasonable description of its isospin dependence [?], the competition between scalar and vector densities giving rise to a new saturation mechanism [?], natural explanation of pseudospin symmetry in the nucleon spectrum [?, ?, ?] and spin symmetry in the antinucleon spectrum [?, ?], and self-consistent treatment of nuclear magnetism [?, ?], relativistic density functional theory has become one of the most popular nuclear theories [?, ?, ?, ?, ?, ?, ?, ?, ?, ?].

In the study of halo nuclei, great progress has been achieved based on relativistic density functional theory. The relativistic continuum Hartree-Bogoliubov (RCHB) theory [?, ?], which solves the relativistic Hartree-Bogoliubov (RHB) equation in coordinate space by assuming spherical symmetry, is suitable for describing spherical exotic nuclei. Based on the RCHB theory, the halo phenomenon in ^{11}Li was described in a microscopic and self-consistent way [?], the giant halo formed by more than two neutrons was predicted [?], and the neutron halo in hypernuclei was explored [?].

The deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc) [?, ?, ?, ?], which considers axial deformation and solves the RHB equation in a Dirac Woods-Saxon (DWS) basis [?], can provide an adequate description for axially deformed exotic nuclei. The DRHBc theory has predicted the deformed halo nuclei ^{42,44}Mg and the shape decoupling between the core and the halo [?, ?], which is considered one of the interesting new phenomena near the drip line [?]. While the heaviest magnesium isotope synthesized in the laboratory so far is ^{40}Mg [?] and the DRHBc prediction for ^{42,44}Mg remains to be verified, many known halo nuclei have been successfully described using the DRHBc theory, including ^{17,19}B [?, ?], ^{15,19,22}C [?, ?], ^{31}Ne [?], and ^{37}Mg [?, ?]. A deformed two-neutron halo in ^{39}Na has also been predicted by the DRHBc theory [?].

In addition, a DRHBc mass table including both deformation and continuum effects is under construction [?, ?, ?, ?]. During this construction, many interesting phenomena have been investigated or predicted using the DRHBc theory [?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. The DRHBc theory has also been extended with angular momentum projection, enabling the exploration of rotational excitations of deformed halo nuclei [?, ?].

It should also be mentioned that there are many studies on deformed halos using other models, such as the single-particle model [?], antisymmetrized molecular dynamics [?], Hartree-Fock theory [?, ?], Hartree-Fock-Bogoliubov theory [?, ?], relativistic Hartree-Fock-Bogoliubov theory [?, ?], and ab initio theory [?]. Most of these studies have assumed axial symmetry for nuclear deformation.

The triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc) [?], which considers triaxial deformation and solves the RHB equation in the DWS basis, has been developed very recently to explore triaxially deformed halo nuclei. The observed heaviest odd-odd aluminum isotope, ^{42}Al [?], has been predicted as a triaxial halo nucleus with a novel shape decoupling between the core and the halo at the triaxial level [?].

This paper is structured as follows. The frameworks of the DRHBc and TRHBc theories are briefly presented in Section 1. Several of the above-mentioned studies on halo nuclei are introduced in Section 2, including ^{37}Mg, ^{39}Na, and ^{42}Al. A summary is given in Section 3.

1 The DRHBc and TRHBc Theories

In the following, we briefly introduce the main formalism for the DRHBc and TRHBc theories. Starting from a meson-exchange or point-coupling Lagrangian density, the RHB equation can be derived by treating the nuclear mean field and pairing correlations on the same footing [**?**]. In Eq. (1), λ is the Fermi energy, and E_k and $(U_k, V_k)^T$ are respectively the quasiparticle energy and the quasiparticle wave function. h_D denotes the Dirac Hamiltonian,

$$h_D(\mathbf{r}) = \alpha \cdot \mathbf{p} + V(\mathbf{r}) + \beta[M + S(\mathbf{r})],$$

with the scalar potential $S(\mathbf{r})$ and the vector potential $V(\mathbf{r})$. Δ represents the pairing potential,

$$\Delta(\mathbf{r}_1, \mathbf{r}_2) = V_{pp}(\mathbf{r}_1, \mathbf{r}_2)\kappa(\mathbf{r}_1, \mathbf{r}_2),$$

where V_{pp} is the pairing interaction and κ is the pairing tensor.

The present DRHBc and TRHBc theories adopt a density-dependent zero-range pairing interaction,

$$V_{pp}(\mathbf{r}_1, \mathbf{r}_2) = V_0(1 - P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2)\left[1 - \frac{\rho(\mathbf{r}_1)}{\rho_{\text{sat}}}\right],$$

in which V_0 is the pairing strength, $(1 - P_\sigma)$ is the projection operator for the spin-zero component, and ρ_{sat} is the saturation density of nuclear matter.

In the DRHBc theory for axially deformed nuclei, the densities and potentials are expanded as follows,

$$f(\mathbf{r}) = \sum_l f_l(r) P_l(\cos\theta),$$

where P_l is the l-order Legendre polynomial, and the assumed spatial reflection symmetry limits l to even numbers. For light nuclei, it is sufficient to truncate the Legendre expansion at $l_{\max} = 6$ [**?**].

In the TRHBc theory for triaxially deformed nuclei, the densities and potentials are expanded in terms of spherical harmonics,

$$f(\mathbf{r}) = \sum_{lm} f_{lm}(r) Y_{lm}(\theta, \varphi).$$

The spatial reflection symmetry and the mirror symmetries with respect to xy, xz, and yz planes lead to the limitations that l and m are even numbers and $f_{lm}(r) = f_{l-m}(r)$ [**?**].

The RHB equations are solved in the DWS basis [**?**, **?**], whose wave function has more appropriate asymptotic behavior compared with the harmonic oscillator basis and is thus suitable for expanding the wave functions of weakly bound nuclei. In practical calculations, an energy cutoff of $E_{\mathrm{cut}} = 300$ MeV and an angular momentum cutoff of $J_{\mathrm{max}} = 23/2\,\hbar$ for the DWS basis can guarantee the convergence of numerical results [**?**]. In such a basis space, the pairing strength $V_0 = -325$ MeV·fm^3, the saturation density $\rho_{\mathrm{sat}} = 0.152$ fm^{-3}, and a pairing window of 100 MeV can reproduce the odd-even mass differences of calcium and lead isotopes [**?**]. The blocking effects in odd-mass and odd-odd nuclei are taken into account via the equal filling approximation [**?**, **?**, **?**].

2 Recent Progress

2.1 The Halo Nucleus ^{37}Mg

The deformed *p*-wave neutron halo nucleus ^{37}Mg, confirmed experimentally in 2014 [**?**, **?**], is the heaviest nuclear halo system to date. Although ^{37}Mg had been synthesized and identified in 1996 [**?**], theoretical studies over the subsequent two decades, including phenomenological calculations based on a Woods-Saxon potential [**?**] and microscopic ones based on nonrelativistic and relativistic density functional theories [**?**, **?**, **?**], did not predict its neutron halo. Even after 2014, a fully microscopic description of the neutron halo in ^{37}Mg remained a challenge. For instance, a large quadrupole deformation ($\beta_2 \gtrsim 0.5$) must be introduced in calculations based on Woods-Saxon potentials to reproduce the *p*-wave occupation of the valence neutron in ^{37}Mg [**?**, **?**, **?**, **?**]. Calculations based on antisymmetrized molecular dynamics reproduced the matter radii of $^{24-36}$Mg but significantly underestimated the matter radius of ^{37}Mg [**?**, **?**]. Results from nonrelativistic Hartree-Fock-Bogoliubov calculations were found to be density-functional dependent [**?**, **?**, **?**], with only the M3Y-P6 functional capable of providing the *p*-wave components of the valence neutron in ^{37}Mg [**?**].

The deformed *p*-wave neutron halo in ^{37}Mg has been described in a microscopic and self-consistent way using the DRHBc theory [**?**]. Figure 2 [Figure 2: see original paper] illustrates the two-dimensional neutron density distributions for $^{35-37}$Mg in the DRHBc calculations with the density functional PC-F1 [**?**]. The neutron density distribution of ^{37}Mg is remarkably more diffuse, consistent with the picture of nuclear halos. The root-mean-square (rms) radii for $^{35-37}$Mg calculated from nuclear densities are respectively 3.44, 3.49, and 3.57 fm, in satisfactory agreement with the empirical radii of 3.44(03), 3.49(01), and 3.62(03) fm deduced from reaction cross sections [**?**]. Furthermore, the densities and wave function of the valence neutron from the DRHBc theory can be fed as microscopic inputs into the Glauber model to study reaction observables, which proved successful in describing the halo nucleus ^{31}Ne [**?**]. Such work for magnesium isotopes has also been performed recently, and the reaction cross section of ^{37}Mg bombarding a carbon target and the longitudinal momentum distribution of ^{36}Mg fragments after its breakup are reproduced very well [**?**].

To further understand the halo phenomenon in ^{37}Mg, Fig. 3 [Figure 3: see original paper] presents the single-neutron orbitals around the Fermi energy and their rms radii. The rms radius of the weakly bound orbital 5 is approximately 5.6 fm, significantly larger than others. Orbital 5 is occupied by the valence neutron in ^{37}Mg, with 38% $2p_{1/2}$ and 19% $2p_{3/2}$ components, comparable to the $\approx$ 40% p-wave halo components suggested in experiments [?]. The low centrifugal barrier for p waves allows considerable tunneling of the wave function into the classically forbidden region, which, together with the weak binding of orbital 5, results in its largest rms radius. While certain s- and p-wave components are found for orbitals 1 and 2, respectively, the diffuseness of their wave functions is suppressed by relatively deep binding. For orbitals 3 and 4 dominated respectively by d and f waves and orbitals 6 and 7 with occupation probabilities $\lesssim 0.1$, their contribution to the neutron halo in ^{37}Mg is marginal. Therefore, it is natural to decouple the neutron density of ^{37}Mg into a core and a halo, with the Fermi energy as a threshold. As seen in Fig. 2, the core density closely resembles that of ^{36}Mg, while the halo density predominantly contributes to the total neutron density far from the center.

Results from DRHBc calculations based on other density functionals, such as PC-PK1 [?], NL3* [?], and PK1 [?], are quantitatively consistent with the above results. Therefore, the microscopic self-consistent description of the DRHBc theory for the halo nucleus ^{37}Mg is essentially density-functional independent. It is worth noting that the DRHBc theory was only applied to even-even nuclei in magnesium isotopes before 2014, missing the prediction of the neutron halo in ^{37}Mg.

2.2 The New Isotope ^{39}Na

The exploration of boundaries in the nuclear chart, i.e., drip-line locations, has always been a major goal of nuclear physics [?]. Although the proton drip line has been experimentally delineated up to neptunium with $Z = 93$ [?], the neutron drip line is only known up to neon with $Z = 10$ [?]. In 2022, further experimental exploration discovered the currently most neutron-rich isotope of sodium with $Z = 11$, ^{39}Na, but the neutron drip line has not yet been determined [?]. The neutron number of ^{39}Na is $N = 28$, a traditional magic number. The disappearance of traditional magic numbers and emergence of new ones are novel phenomena in exotic nuclei near drip lines, e.g., the disappearance of $N = 28$ magicity in the isotones of ^{39}Na, ^{40}Mg, and ^{42}Si [?, ?]. On the other hand, as shown in Fig. 1, clues to halo nuclei or candidates are found in every isotopic chain from He to P, except for Na and Si. Therefore, it is necessary to study the shell structure of ^{39}Na and explore possible halo phenomena in neutron-rich Na isotopes, which could provide references for future experiments.

Figure 4 [Figure 4: see original paper] shows the evolution of single-neutron levels around the Fermi energy with quadrupole deformation obtained from DRHBc calculations with PC-PK1. In the spherical limit, the orbitals $2p_{3/2}$ and

$1f_{7/2}$ are nearly degenerate and close to $2p_{1/2}$. In the traditional shell model, there is a sizable energy gap between $1f_{7/2}$ and $2p_{3/2}$, forming the $N = 28$ shell closure and making the spherical shape energetically favored. In ^{39}Na, the lowering of $2p$ orbitals results in the collapse of the $N = 28$ shell closure, and down-sloping levels split from the $1f_{7/2}$ orbital lead to a large ground-state deformation of $\beta_2 > 0.4$. In addition, as shown in Fig. 4, pairing correlations lead to partial occupation of the weakly bound or continuum $1/2^-$ and $3/2^-$ orbitals by valence neutrons in ^{39}Na. Due to deformation effects, pf components are mixed in the wave functions of these orbitals, and p-wave components in valence neutrons favor the formation of a neutron halo. Therefore, the halo phenomenon may exist in ^{39}Na.

Figure 5 [Figure 5: see original paper] shows neutron density distributions along and perpendicular to the symmetry axis for odd-mass Na isotopes with $N \geq 20$. The ground-state deformation of ^{31}Na is spherical due to the $N = 20$ shell closure, while other more neutron-rich Na isotopes are well deformed with quadrupole deformation $\beta_2 > 0.35$. Therefore, the significant increase in density distribution along the symmetry axis from ^{31}Na to ^{33}Na in Fig. 5(a) can be understood from deformation effects. From ^{33}Na to ^{41}Na, the neutron density distribution along the symmetry axis gradually becomes more diffuse with increasing neutron number. In Fig. 5(b), similar gradual growth is seen in the neutron density distribution perpendicular to the symmetry axis from ^{31}Na to ^{37}Na. However, far from the center, the density distributions of ^{39}Na and ^{41}Na are very diffuse, even though they are prolate deformed. This suggests oblate neutron halos in ^{39}Na and ^{41}Na, with prolate-oblate shape decoupling between the core and halo.

Further analysis reveals that the $1/2^-$ and $3/2^-$ orbitals embedded in the continuum contribute predominantly to the halos in ^{39,41}Na. These two orbitals have considerable p-wave components. According to angular momentum coupling, both spherical harmonic functions $|Y_{10}(\theta, \varphi)|^2$ and $|Y_{1\pm1}(\theta, \varphi)|^2$ contribute to the $1/2^-$ orbital, with the latter dominating. For the $3/2^-$ orbital, only $|Y_{1\pm1}(\theta, \varphi)|^2$ contributes. The angular distribution is prolate for $|Y_{10}(\theta, \varphi)|^2 \propto \cos^2\theta$ and oblate for $|Y_{1\pm1}(\theta, \varphi)|^2 \propto \sin^2\theta$. The shape of the halo is thus oblate.

Therefore, based on DRHBc calculations with density functional PC-PK1, the disappearance of the traditional magic number $N = 28$ and the appearance of a deformed neutron halo in the new isotope ^{39}Na are predicted. For results from other density functionals for Na isotopes, discussion on the neutron dripline location, and further exploration of the microscopic mechanism for halo formation, see Ref. [?].

2.3 The Triaxial Halo Nucleus ^{42}Al

Non-axial deformation, namely triaxial deformation, is one of the basic deformation degrees of freedom in atomic nuclei. Triaxiality is closely related to rich nuclear phenomena, e.g., chiral rotation [?], wobbling motion [?], and fission

[?]. It has recently been found that information on nuclear deformation can be extracted from relativistic heavy-ion collision experiments [?], and evidence for triaxial structure in ^{129}Xe has been reported by analyzing data from the CERN Large Hadron Collider [?], which further stimulates interest in triaxial nuclei.

It would be interesting to explore the possible existence of triaxial halo nuclei. In 2021, a calculation based on the Woods-Saxon potential suggested that the region of halo nuclei could be extended by triaxial deformation that allows the appearance of s- or p-wave components in some weakly bound orbitals [?]. However, the triaxial deformation, depth, and width of the potential are adjustable parameters, and crucial pairing and continuum effects are not included in such phenomenological models. In contrast, the TRHBc theory takes into account triaxial deformation, pairing correlations, and continuum effects in a microscopic and self-consistent way, making it capable of properly describing triaxial halo nuclei. The TRHBc theory predicts ^{42}Al as a triaxial halo nucleus by examining neutron separation energies, rms radii, density distributions, and single-neutron orbitals around the Fermi energy for aluminum isotopes [?].

Figure 6 [Figure 6: see original paper] exhibits density distributions in the xy, xz, and yz planes for the core and halo of ^{42}Al, calculated from the TRHBc theory with PC-PK1. The halo density is significantly more diffuse than the core density, especially in the yz plane. Quantitatively, the rms radius is 3.85 fm for the core and 5.26 fm for the halo. The deformation parameters (β, γ) are $(0.38, 50^\circ)$ and $(0.79, -23^\circ)$ for the core and halo, respectively. The negative γ value means that y is the intermediate axis while x is the short one, which is just the reverse of the case for the core. With the corresponding rms radius and deformation parameters, schematic pictures for the core and halo are also displayed in Fig. 6, where the long, intermediate, and short axes can be distinguished clearly. Therefore, ^{42}Al is a triaxial halo nucleus with a novel triaxial shape decoupling between the core and halo. For detailed results and discussion on the triaxial halo characteristics of ^{42}Al, see Ref. [?].

3 Summary

Halo phenomena in exotic nuclei have long been an important frontier in nuclear physics research. Relativistic density functional theory has made great progress in the study of halo nuclei. Recently, a microscopic and self-consistent description of ^{37}Mg, including its small one-neutron separation energy, large rms radius, diffuse density distribution, and p-wave components for the halo neutron, has been obtained using the DRHBc theory [?]. Based on structure input purely from the DRHBc theory, the reaction cross section of ^{37}Mg on a carbon target and the momentum distribution of ^{36}Mg fragments after its breakup are reproduced very well by the Glauber model [?]. Therefore, a unified description of the halo characteristics of ^{37}Mg from nuclear structure to reaction dynamics has been achieved.

The recently discovered isotope ^{39}Na has been investigated with the DRHBc theory [?]. It is revealed that the lowering of $2p$ orbitals in the spherical limit results in the disappearance of $N = 28$ magicity in ^{39}Na. Pairing correlations and pf component mixing driven by deformation lead to partial occupation of weakly bound and continuum orbitals with certain p-wave components, giving rise to the formation of an oblate neutron halo around the prolate core in ^{39}Na.

To explore triaxial halo nuclei, the TRHBc theory has been developed, which self-consistently includes triaxiality, pairing correlations, and continuum effects [?]. The observed heaviest odd-odd aluminum isotope, ^{42}Al, is predicted as a triaxially deformed halo nucleus, in which a novel triaxial shape decoupling—i.e., the exchange of intermediate and short axes between the triaxial core and triaxial halo—is found.

Acknowledgments

Helpful discussions with members of the DRHBc Mass Table Collaboration, the collaboration with J. L. An, Y. Kim, J. Meng, M.-H. Mun, P. Papakonstantinou, X.-X. Sun, S. Q. Zhang, and S. S. Zhang, and the support from H. Yan are gratefully appreciated.

References

[1] TANIHATA I, HAMAGAKI H, HASHIMOTO O, et al. Phys Rev Lett, 1985, 55: 2676. DOI: 10.1103/PhysRevLett.55.2676.
[2] ZHUKOV M, DANILIN B, FEDOROV D, et al. Phys Rep, 1993, 231(4): 151. DOI: 10.1016/0370-1573(93)90141-Y.
[3] HANSEN P G, JENSEN A S, JONSON B. Annu Rev Nuc Part Sci, 1995, 45(1): 591. DOI: 10.1146/annurev.ns.45.120195.003111.
[4] OTSUKA T, FUKUNISHI N, SAGAWA H. Phys Rev Lett, 1993, 70: 1385. https://link.aps.org/doi/10.1103/PhysRevLett.70.1385.
[5] KUO T T S, KRMPOTIĆ F, TZENG Y. Phys Rev Lett, 1997, 78: 2708. https://link.aps.org/doi/10.1103/PhysRevLett.78.2708.
[6] HORIUCHI H, KANADA-EN' YO Y, ONO A. Z Phys A, 1994, 349(3): 279. https://doi.org/10.1007/BF01288975.
[7] ITAGAKI N, AOYAMA S. Phys Rev C, 1999, 61: 024303. https://link.aps.org/doi/10.1103/PhysRevC.61.02
[8] RYBERG E, FORSSÉN C, HAMMER H W, et al. Phys Rev C, 2014, 89: 014325. https://link.aps.org/doi/10.1103/PhysRevC.89.014325.
[9] JI C, ELSTER C, PHILLIPS D R. Phys Rev C, 2014, 90: 044004. https://link.aps.org/doi/10.1103/PhysRevC.90.044004.
[10] TERASAKI J, HEENEN P H, FLOCARD H, et al. Nucl Phys A, 1996, 600(3): 371. https://www.sciencedirect.com/science/article/pii/037594749600036X. DOI: 10.1016/0375-9474(96)00036-X.
[11] SCHUNCK N, EGIDO J L. Phys Rev C, 2008, 78: 064305. https://link.aps.org/doi/10.1103/PhysRevC.78.
[12] MENG J, RING P. Phys Rev Lett, 1996, 77: 3963. DOI: 10.1103/PhysRevLett.77.3963.
[13] PÖSCHL W, VRETENAR D, LALAZISSIS G A, et al. Phys Rev Lett,

1997, 79: 3841. https://link.aps.org/doi/10.1103/PhysRevLett.79.3841.

[14] ZHANG K Y, PAPAKONSTANTINOU P, MUN M H, et al. Phys Rev C, 2023, 107: L041303. https://link.aps.org/doi/10.1103/PhysRevC.107.L041303.

[15] MENG J. International review of nuclear physics: volume 10 relativistic density functional for nuclear structure[M]. World Scientific, Singapore, 2016.

[16] RING P. Phys Scr, 2012, T150: 014035. https://doi.org/10.1088%2F0031-8949%2F2012%2Ft150%2F014035. DOI: 10.1088/0031-8949/2012/t150/014035.

[17] MENG J, ZHANG K Y. Physics, 2021, 50: 789 (in Chinese).

[18] KOEPF W, RING P. Z Phys A, 1991, 339(1): 81.

[19] REN Z X, ZHAO P W. Phys Rev C, 2020, 102: 021301(R). https://link.aps.org/doi/10.1103/PhysRevC.102

[20] SHARMA M M, LALAZISSIS G, KÖNIG J, et al. Phys Rev Lett, 1995, 74: 3744. DOI: 10.1103/PhysRevLett.74.3744.

[21] WALECKA J. Ann Phys, 1974, 83(2): 491. https://www.sciencedirect.com/science/article/pii/0003491674 DOI: https://doi.org/10.1016/0003-4916(74)90208-5.

[22] GINOCCHIO J N. Phys Rev Lett, 1997, 78: 436. https://link.aps.org/doi/10.1103/PhysRevLett.78.436.

[23] MENG J, SUGAWARA-TANABE K, YAMAJI S, et al. Phys Rev C, 1998, 58: R628. DOI: 10.1103/PhysRevC.58.R628.

[24] LIANG H, MENG J, ZHOU S G. Phys Rep, 2015, 570: 1. DOI: 10.1016/j.physrep.2014.12.005.

[25] ZHOU S G, MENG J, RING P. Phys Rev Lett, 2003, 91: 262501. DOI: 10.1103/PhysRevLett.91.262501.

[26] HE X T, ZHOU S G, MENG J, et al. Eur Phys J A, 2006, 28: 265. DOI: 10.1140/epja/i2006-10066-0.

[27] KOEPF W, RING P. Nucl Phys A, 1989, 493(1): 61. DOI: 10.1016/0375-9474(89)90532-0.

[28] KOEPF W, RING P. Nucl Phys A, 1990, 511(2): 279. https://www.sciencedirect.com/science/article/pii/0 DOI: https://doi.org/10.1016/0375-9474(90)90160-N.

[29] RING P. Prog Part Nucl Phys, 1996, 37: 193. DOI: 10.1016/0146-6410(96)00054-3.

[30] VRETENAR D, AFANASJEV A V, LALAZISSIS G A, et al. Phys Rep, 2005, 409(3): 101. DOI: 10.1016/j.physrep.2004.10.001.

[31] MENG J, TOKI H, ZHOU S G, et al. Prog Part Nucl Phys, 2006, 57: 470. DOI: 10.1016/j.ppnp.2005.06.001.

[32] MENG J, GUO J Y, LI J, et al. Progress in Physics, 2011, 31: 199 (in Chinese).

[33] NIKŠIĆ T, VRETENAR D, RING P. Prog Part Nucl Phys, 2011, 66(3): 519. DOI: 10.1016/j.ppnp.2011.01.055.

[34] MENG J, PENG J, ZHANG S Q, et al. Front Phys, 2013, 8: 55. DOI: 10.1007/s11467-013-0287-y.

[35] MENG J, ZHOU S G. J Phys G, 2015, 42(9): 093101. DOI: 10.1088/0954-3899/42/9/093101.

[36] ZHOU S G. Phys Scr, 2016, 91(6): 063008. DOI: 10.1088/0031-8949/91/6/063008.

[37] SHEN S, LIANG H, LONG W H, et al. Prog Part Nucl Phys, 2019, 109: 103713. DOI: 10.1016/j.ppnp.2019.103713.

[38] MENG J, ZHAO P. AAPPS Bull, 2021, 31(1): 2. https://doi.org/10.1007/s43673-

021-00001-8.
[39] MENG J. Nucl Phys A, 1998, 635: 3. DOI: 10.1016/S0375-9474(98)00178-X.
[40] MENG J, RING P. Phys Rev Lett, 1998, 80: 460. DOI: 10.1103/PhysRevLett.80.460.
[41] LÜ H F, MENG J, ZHANG S Q, et al. Eur Phys J A, 2003, 17: 19. DOI: 10.1140/epja/i2002-10136-3.
[42] ZHOU S G, MENG J, RING P, et al. Phys Rev C, 2010, 82: 011301. https://link.aps.org/doi/10.1103/PhysRevC.82.011301.
[43] LI L, MENG J, RING P, et al. Phys Rev C, 2012, 85: 024312. DOI: 10.1103/PhysRevC.85.024312.
[44] LI L, MENG J, RING P, et al. Chin Phys Lett, 2012, 29: 042101. DOI: 10.1088/0256-307X/29/4/042101.
[45] CHEN Y, LI L, LIANG H, et al. Phys Rev C, 2012, 85: 067301. DOI: 10.1103/PhysRevC.85.067301.
[46] ZHOU S G, MENG J, RING P. Phys Rev C, 2003, 68: 034323. DOI: 10.1103/PhysRevC.68.034323.
[47] COTTLE P D, KEMPER K W. Physics, 2012, 5: 49. DOI: 10.1103/Physics.5.49.
[48] BAUMANN T, AMTHOR A M, BAZIN D, et al. Nature, 2007, 449(7165): 1022. DOI: 10.1038/nature06213.
[49] YANG Z H, KUBOTA Y, CORSI A, et al. Phys Rev Lett, 2021, 126: 082501. https://link.aps.org/doi/10.1103/PhysRevLett.126.082501.
[50] SUN X X. Phys Rev C, 2021, 103: 054315. https://link.aps.org/doi/10.1103/PhysRevC.103.054315.
[51] SUN X X, ZHAO J, ZHOU S G. Phys Lett B, 2018, 785: 530. DOI: 10.1016/j.physletb.2018.08.071.
[52] SUN X X, ZHAO J, ZHOU S G. Nucl Phys A, 2020, 1003: 122011. DOI: 10.1016/j.nuclphysa.2020.122011.
[53] ZHONG S Y, ZHANG S S, SUN X X, et al. Sci China Phys Mech Astron, 2022, 65(6): 262011. DOI: 10.1007/s11433-022-1894-6.
[54] ZHANG K, YANG S, AN J, et al. Phys Lett B, 2023, 844: 138112. https://www.sciencedirect.com/science/article/pii/S037026932300446X. DOI: https://doi.org/10.1016/j.physletb.2023.138112.
[55] AN J L, ZHANG K Y, LU Q, et al. Phys Lett B, 2024, 849: 138422. https://www.sciencedirect.com/science/article/pii/S0370269323007554. DOI: https://doi.org/10.1016/j.physletb.2023.138422.
[56] ZHANG K, CHEOUN M K, CHOI Y B, et al. Phys Rev C, 2020, 102: 024314. https://link.aps.org/doi/10.1103/PhysRevC.102.024314.
[57] ZHANG K Y, PAN C, ZHANG S Q, MENG J. Chin Sci Bull, 2021, 66: 3561 (in Chinese).
[58] PAN C, CHEOUN M K, CHOI Y B, et al. Phys Rev C, 2022, 106: 014316. https://link.aps.org/doi/10.1103/PhysRevC.106.014316.
[59] GUO P, CAO X, CHEN K, et al. arXiv, 2024. DOI: 10.48550/arXiv.
[60] ZHANG K Y, WANG D Y, ZHANG S Q. Phys Rev C, 2019, 100: 034312. DOI: 10.1103/PhysRevC.100.034312.
[61] IN E J, KIM Y, PAPAKONSTANTINOU P, et al. J Korean Phys Soc,

2020, 77(11): 966. https://doi.org/10.3938/jkps.77.966.
[62] IN E J, PAPAKONSTANTINOU P, KIM Y, et al. Int J Mod Phys E, 2021, 30(02): 2150009. DOI: 10.1142/S0218301321500099.
[63] ZHANG K, HE X, MENG J, et al. Phys Rev C, 2021, 104: L021301. https://link.aps.org/doi/10.1103/PhysRevC.104.L021301.
[64] PAN C, ZHANG K Y, CHONG P S, et al. Phys Rev C, 2021, 104: 024331. https://link.aps.org/doi/10.1103/PhysRevC.104.024331.
[65] HE X T, WANG C, ZHANG K Y, et al. Chin Phys C, 2021, 45: 101001. DOI: 10.1088/1674-1137/ac1b99.
[66] SUN W, ZHANG K Y, PAN C, et al. Chin Phys C, 2022, 46: 064103. DOI: 10.1088/1674-1137/ac53fa.
[67] ZHANG X Y, NIU Z M, SUN W, et al. Phys Rev C, 2023, 108: 024310. https://link.aps.org/doi/10.1103/PhysRevC.108.024310.
[68] MUN M H, KIM S, CHEOUN M K, et al. Phys Lett B, 2023, 847: 138298. https://www.sciencedirect.com/science/article/pii/S0370269323006329. DOI: https://doi.org/10.1016/j.physletb.2023.138298.
[69] GUO P, PAN C, ZHAO Y C, et al. Phys Rev C, 2023, 108: 014319. https://link.aps.org/doi/10.1103/PhysRevC.108.014319.
[70] SUN X X, ZHOU S G. Sci Bull, 2021, 66(20): 2072. DOI: 10.1016/j.scib.2021.07.005.
[71] SUN X X, ZHOU S G. Phys Rev C, 2021, 104: 064319. https://link.aps.org/doi/10.1103/PhysRevC.104.064
[72] MISU T, NAZAREWICZ W, ABERG S. Nucl Phys A, 1997, 614(1): 44. DOI: 10.1016/S0375-9474(96)00458-7.
[73] WATANABE S, MINOMO K, SHIMADA M, et al. Phys Rev C, 2014, 89: 044610. https://link.aps.org/doi/10.1103/PhysRevC.89.044610.
[74] LI X, HEENEN P H. Phys Rev C, 1996, 54: 1617. https://link.aps.org/doi/10.1103/PhysRevC.54.1617.
[75] PEI J, XU F, STEVENSON P. Nucl Phys A, 2006, 765(1): 29. https://www.sciencedirect.com/science/article/pii/S0375947405011607. DOI: https://doi.org/10.1016/j.nuclphysa.2005.10.004.
[76] NAKADA H, TAKAYAMA K. Phys Rev C, 2018, 98: 011301. https://link.aps.org/doi/10.1103/PhysRevC.98.011301.
[77] KASUYA H, YOSHIDA K. Prog Theor Exp Phys, 2020, 2021(1). https://doi.org/10.1093/ptep/ptaa163.
[78] GENG J, LONG W H. Phys Rev C, 2022, 105: 034329. https://link.aps.org/doi/10.1103/PhysRevC.105.034
[79] GENG J, NIU Y F, LONG W H. Chin Phys C, 2023, 47(4): 044102. https://dx.doi.org/10.1088/1674-1137/acb7cd.
[80] CALCI A, NAVRÁTIL P, ROTH R, et al. Phys Rev Lett, 2016, 117: 242501. https://link.aps.org/doi/10.1103/PhysRevLett.117.242501.
[81] ZHANG K Y, ZHANG S Q, MENG J. Phys Rev C, 2023, 108: L041301. https://link.aps.org/doi/10.1103/PhysRevC.108.L041301.
[82] KUCHAREK H, RING P. Z Phys A, 1991, 339(1): 23. DOI: 10.1007/BF01282930.
[83] XIANG Y, LUO Q, YANG S, et al. Symmetry, 2023, 15(7). https://www.mdpi.com/2073-8994/15/7/1420. DOI: 10.3390/sym15071420.
[84] ZHANG K Y, PAN C, ZHANG S Q. Phys Rev C, 2022, 106: 024302. https://link.aps.org/doi/10.1103/PhysRevC.106.024302.
[85] PAN C, ZHANG K, ZHANG S. Int J Mod Phys E, 2019, 28(09): 1950082.

DOI: 10.1142/S0218301319500824.
[86] PEREZ-MARTIN S, ROBLEDO L M. Phys Rev C, 2008, 78: 014304. https://link.aps.org/doi/10.1103/PhysRevC.78.014304.
[87] KOBAYASHI N, NAKAMURA T, KONDO Y, et al. Phys Rev Lett, 2014, 112: 242501. https://link.aps.org/doi/10.1103/PhysRevLett.112.242501.
[88] TAKECHI M, SUZUKI S, NISHIMURA D, et al. Phys Rev C, 2014, 90: 061305(R). https://link.aps.org/doi/10.1103/PhysRevC.90.061305.
[89] SAKURAI H, AOI N, GOTO A, et al. Phys Rev C, 1996, 54: R2802. https://link.aps.org/doi/10.1103/PhysRevC.54.R2802.
[90] HAMAMOTO I. Phys Rev C, 2007, 76: 054319. https://link.aps.org/doi/10.1103/PhysRevC.76.054319.
[91] REN Z, ZHU Z, CAI Y, et al. Phys Lett B, 1996, 380(3): 241. https://www.sciencedirect.com/science/article/pii/0370269396004625. DOI: https://doi.org/10.1016/0370-2693(96)00462-5.
[92] HORIUCHI W, INAKURA T, NAKATSUKASA T, et al. Phys Rev C, 2012, 86: 024614. https://link.aps.org/doi/10.1103/PhysRevC.86.024614.
[93] SHARMA M K, MEHTA M S, PATRA S K. Int J Mod Phys E, 2013, 22(01): 1350005. https://doi.org/10.1142/S0218301313500055.
[94] XU X D, ZHANG S S, SIGNORACCI A J, et al. Phys Rev C, 2015, 92: 024324. https://link.aps.org/doi/10.1103/PhysRevC.92.024324.
[95] FANG Z, SHI M, GUO J Y, et al. Phys Rev C, 2017, 95: 024311. https://link.aps.org/doi/10.1103/PhysRevC.95.024311.
[96] SUN T T, QIAN L, CHEN C, et al. Phys Rev C, 2020, 101: 014321. https://link.aps.org/doi/10.1103/PhysRevC.101.014321.
[97] URATA Y, HAGINO K, SAGAWA H. Phys Rev C, 2017, 96: 064311. https://link.aps.org/doi/10.1103/PhysRevC.96.064311.
[98] CHOUDHARY V, HORIUCHI W, KIMURA M, et al. Phys Rev C, 2021, 104: 054313. https://link.aps.org/doi/10.1103/PhysRevC.104.054313.
[99] XIONG X Y, PEI J C, ZHANG Y N, et al. Chin Phys C, 2016, 40(2): 024101. https://dx.doi.org/10.1088/1674-1137/40/2/024101.
[100] BÜRVENICH T, MADLAND D G, MARUHN J A, et al. Phys Rev C, 2002, 65: 044308. https://link.aps.org/doi/10.1103/PhysRevC.65.044308.
[101] ZHAO P W, LI Z P, YAO J M, et al. Phys Rev C, 2010, 82: 054319. DOI: 10.1103/PhysRevC.82.054319.
[102] LALAZISSIS G A, KARATZIKOS S, FOSSION R, et al. Phys Lett B, 2009, 671(1): 36. https://www.sciencedirect.com/science/article/pii/S0370269308014500. DOI: 10.1016/j.physletb.2008.11.070.
[103] LONG W, MENG J, VAN GIAI N, et al. Phys Rev C, 2004, 69: 034319. DOI: 10.1103/PhysRevC.69.034319.
[104] THOENNESSEN M. The discovery of isotopes[M]. Springer, New York, 2016.
[105] ZHANG Z Y, GAN Z G, YANG H B, et al. Phys Rev Lett, 2019, 122: 192503. https://link.aps.org/doi/10.1103/PhysRevLett.122.192503.
[106] AHN D S, FUKUDA N, GEISSEL H, et al. Phys Rev Lett, 2019, 123: 212501. https://link.aps.org/doi/10.1103/PhysRevLett.123.212501.
[107] AHN D S, AMANO J, BABA H, et al. Phys Rev Lett, 2022, 129: 212502. https://link.aps.org/doi/10.1103/PhysRevLett.129.212502.

[108] BASTIN B, GRÉVY S, SOHLER D, et al. Phys Rev Lett, 2007, 99: 022503. https://link.aps.org/doi/10.1103/PhysRevLett.99.022503.
[109] DOORNENBAL P, SCHEIT H, TAKEUCHI S, et al. Phys Rev Lett, 2013, 111: 212502. https://link.aps.org/doi/10.1103/PhysRevLett.111.212502.
[110] FRAUENDORF S, MENG J. Nucl Phys A, 1997, 617: 131. DOI: 10.1016/S0375-9474(97)00004-3.
[111] BOHR A, MOTTELSON B R. Nuclear structure: II[M]. Benjamin, New York, 1975.
[112] LU B N, ZHAO E G, ZHOU S G. Phys Rev C, 2012, 85: 011301. https://link.aps.org/doi/10.1103/PhysRevC.85.011301.
[113] GIACALONE G, JIA J, ZHANG C. Phys Rev Lett, 2021, 127: 242301. https://link.aps.org/doi/10.1103/PhysRevLett.127.242301.
[114] BALLY B, BENDER M, GIACALONE G, et al. Phys Rev Lett, 2022, 128: 082301. https://link.aps.org/doi/10.1103/PhysRevLett.128.082301.
[115] UZAWA K, HAGINO K, YOSHIDA K. Phys Rev C, 2021, 104: L011303. https://link.aps.org/doi/10.1103/PhysRevC.104.L011303.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.