Progress and Perspective on the Study of BEC-analog Structure in Nuclei

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The second 0+ excited state at 7.65 MeV in 12C, known as the Hoyle state, is located near the 3-α breakup threshold and possesses a typical BEC-analog structure. This observation has triggered the intensive theoretical studies of the condensation configuration in nuclear systems, typically represented by the THSR wave function. In the mean time, the experimental investigation of the Hoyle-like states in heavier nuclei has been advanced quite slowly, due mostly to the difficulties in detecting multi-fragments in coincidence and the clarification of the reaction-decay mechanisms. We give here a review of the theoretical and experimental progresses in this field by taking into account the latest experimental outcomes for the 4-α resonances in 16O. Some perspectives are also given towards the possible BEC-like states in neutron-rich systems, which would be of particular importance in exploring the properties of the heavier neutron-rich nuclei and also the neutron stars.

Full Text

Preamble

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Progress and Perspective on the Study of BEC-Analog Structure in Nuclei

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Abstract

The second 0^+ excited state at 7.65 MeV in ^{12}C, known as the Hoyle state, is located near the 3-α breakup threshold and possesses a typical Bose-Einstein condensation (BEC)-analog structure. This observation has triggered intensive theoretical studies of condensation configurations in nuclear systems, typically represented by the THSR wave function. Meanwhile, experimental investigation of Hoyle-like states in heavier nuclei has advanced quite slowly, due primarily to difficulties in detecting multiple fragments in coincidence and clarifying the reaction-decay mechanisms.

We review here the theoretical and experimental progress in this field, taking into account the latest experimental outcomes for the 4-α resonances in ^{16}O. Perspectives are also offered regarding possible BEC-like states in neutron-rich systems, which would be of particular importance for exploring the properties of heavier neutron-rich nuclei and neutron stars.

Keywords: cluster structure; BEC-analog state; α-conjugate nuclei; neutron-rich nuclei
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1 Introduction

Nucleon clustering is a general phenomenon in nuclear structure configuration, occurring commonly in light nuclei, at the surface of heavy nuclei (α-particle formation and decay), and frequently in weakly-bound or excited nuclei [1-2]. One famous example is the ${0^+}_2$ (7.65 MeV) state of ^{12}C, also denoted as the "Hoyle state"due to its tremendous significance in the context of carbon synthesis in the universe and hence the origin of life [3]. This state possesses a typical 3-α cluster structure and has attracted intensive experimental and theoretical investigations [1-2,4-9]. It is now commonly understood that the structure of the Hoyle state is basically featured by Bose-Einstein condensation, meaning that all three α clusters are moving in relative s-wave within a volume much larger than that of the ground state [5-6,10-12]. It was recognized that the BEC-like structure can be well described by the Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function [10].

Historically, the BEC concept for a finite nucleus has attracted some debate. The main problem concerns the relation between the finite nucleus and infinite nuclear matter [14-15]. It has been realized that, similar to the successful application of Bardeen-Cooper-Schrieffer (BCS) theory to finite nuclei, the BEC concept is also meaningful for describing systems composed of a finite number of α-particles, all moving basically in s-wave within an expanded low-density volume [6,11,15]. It was demonstrated that the antisymmetrization between nucleons in different α-clusters can be neglected when the average α-α distance is substantially larger than the α-diameter, and thus the localization of the bosons can still be assured [6,15].

Apart from the Hoyle state of ^{12}C, BEC-analog states have also been proposed for heavier α-conjugate nuclei [8,10,16-18], such as ^{16}O [10,16,19-21], ^{20}Ne [22-24], ^{24}Mg [17,25], and ^{40}Ca [26-27]. However, corresponding experimental investigations are challenging, and no conclusive results have been obtained thus far for these heavier systems. The main difficulties lie in the very high excitation of the system, the coincident detection of many decaying α-particles, and the clear selection of the targeted reaction-decay channel among many accompanying channels [11].

In this article, we review the major theoretical approaches that describe BEC-like states in ^{12}C and other nuclei. We then outline the experimental progress in finding evidence for BEC-like structure, which remains quite limited so far. In the final section, we discuss perspectives for possible experimental work in the near future.

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2 Theoretical Approaches

The description of the Hoyle-state structure poses challenging problems for the conventional shell-model approach, within which the $0^+{}_2$ state should appear at an excitation energy much higher than the actually observed 7.65 MeV [6]. On the other hand, early calculations using the cluster-type Orthogonality Condition Model (OCM) for the α + ^{8}Be(0^+) cluster configuration correctly reproduced the energy of the Hoyle state and revealed an s-wave coupling between the two subsystems [28]. Since the two α-particles in ^{8}Be(0^+) also interact via a weakly coupled s-wave, this calculation indicates a weakly coupled 3-α structure in the Hoyle state, resembling a gas-like α condensation [6]. Subsequently, the Resonating Group Method (RGM) [29] and Generator Coordinate Method (GCM) [30-31] were employed as original microscopic calculations for ^{12}C, which further confirmed the gas-like 3-α structure of the Hoyle state. These approaches successfully reproduced the basic properties of the Hoyle state, including its excitation energy and the E0 and E2 transition strengths. It is worth noting that, as early as the 1970s [32], studies with 3-α RGM had already reasonably reproduced the excitation energy of the Hoyle state [29-31].

In recent years, Antisymmetrized Molecular Dynamics (AMD) [33-34] and Fermion Molecular Dynamics (FMD) [35] models have further confirmed the gas-like clustering structure in the $0^+{}_2$ (7.65 MeV) state of ^{12}C [5]. The AMD model is particularly noteworthy as it provides a good description of nucleon clustering without assuming the preformation of cluster structure within the nucleus [2]. This advantage relies on using wave packets, instead of angular-momentum eigenstates, to represent the nucleon distributions. In addition, full antisymmetrization, in line with the Pauli exclusion principle, and the orthogonal property between quantum states also drive the system into strongly correlated substructures [33]. FMD is essentially the same as the AMD approach but with a variable width parameter for the Gaussian-type wave packet [12]. For the Hoyle state in ^{12}C, both AMD and FMD demonstrate that the second 0^+ state has a dominant gas-like 3-α configuration with an extended large size [36-38].

The Algebraic Cluster Model (ACM), originated from the algebraic theory of molecules, is another approach that describes the excited states in ^{12}C quite successfully [9,39]. ACM places the 3-α system as an equilateral triangle [39-41]. By employing a general quantization technique based on the Lie algebra U(+ 1) for systems with degrees of freedom, ACM achieves a quantization of the Jacobi variables. Within the ACM framework, the energy levels of some Nα-conjugated nuclei, such as ^{12}C [39], ^{16}O [42], and ^{20}Ne [43], have been calculated.

The idea of α-condensate in nuclear systems was initially proposed by Röpke et al. in their study of strongly coupled fermion systems [44]. Later, the THSR wave function was formulated to represent BEC-analog states in nuclei [10]. It may be regarded as a variation from the Brink wave function [45]. While the Brink wave function places subsystems (clusters) at random positions inside the nucleus, the THSR wave function requires all subsystems (clusters) to move in s-wave around the center of mass of the whole system [10]. When all subsystems are bosonic, the THSR wave function naturally describes the BEC-analog structure of a nucleus [6,10,12].

Taking the 3-α clustering system as an example, the THSR wave function can be written as [6,12]:

$$\Psi_{\mathrm{THSR}} = \mathcal{A}\left\{\exp\left[-\left(\frac{\xi_1^2}{3B^2}+\frac{\xi_2^2}{B^2}\right)\right]\Phi(3\alpha)\right\}$$

where $\Phi(3\alpha) = \phi_{\alpha 1}\phi_{\alpha 2}\phi_{\alpha 3}$ is the 3-α wave function, $\mathcal{A}$ is the antisymmetrization operator, and B is a parameter characterizing the overall size of the nucleus. X_i is the center of mass of the ith α-cluster. The Jacobi coordinates ξ are related to $\mathbf{X}$ according to [6,46-47]:

$$\xi_1 = \mathbf{X}_1 - \frac{1}{2}(\mathbf{X}_2 + \mathbf{X}_3), \quad \xi_2 = \mathbf{X}_2 - \mathbf{X}_3$$

It has recently been demonstrated that THSR wave functions with flexible size parameters can be used to describe various cluster structures, including not only gas-like but also non-gas-like cluster structures, such as extremely deformed or asymmetric clustering states [6,11,20,23,48-49]. For instance, the inversion-doublet band (headed by $J^\pi = 0^+$ and $J^\pi = 0^-$ states) for the asymmetric ^{16}O + α structure in ^{20}Ne can also be described by a single THSR-type wave function with different size parameters for the core ^{16}O and for the overall ^{20}Ne system, respectively [6,23,49]. This implies that the THSR wave function with flexible size parameters represents the fundamental physical properties of clustering states. The size parameter is essential here to characterize the cluster-mean-field.

The “container” model was then proposed, referring to cluster motion inside the mean-field potential characterized by the size parameter subject to full antisymmetrization among all nucleons [6,20]. Unlike conventional cluster model wave functions like the Brink wave packet, the container model describes the relative motion of clusters through the dynamic evolution of the container itself, and the size of the container evolves from smaller to larger values with increasing excitation energies. The generally nonlocalized wave function can result in localized density distribution for clusters due to the Pauli blocking effect [6,48]. Due to its relatively simple picture and parameter settings, the container model may be regarded as a powerful and promising approach that has the potential to replace traditional RGM and Brink-GCM models in describing cluster dynamics [6,11]. This is particularly true for intrinsically BEC-like states which overlap perfectly with THSR wave functions.

As an example, Fig. 1(a) schematically shows the α + ^{8}Be cluster structure in ^{12}C and the α + ^{12}C cluster structure in ^{16}O within the “container” picture, where 2-α or 3-α clusters are confined in a “container”characterized by parameter β_1, and the remaining α cluster is confined in a larger “container” characterized by β_2 (β_2 is related to the parameter B defined in the equation). Fig. 1(b) and Fig. 1(c) show comparisons of the Hoyle band in ^{12}C between experimental data and theoretical results calculated within the “container” approach. The theoretical results agree reasonably well with the experimental data.

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3 Experimental Observations

Although theoretical studies on BEC-like structure in nuclei have achieved much progress, related experimental evidence remains very scarce. Actual observations are mostly concentrated on the Hoyle state of ^{12}C. Reference [5] provides a summary of measurements for the properties of the Hoyle state, including excitation energy, decay modes and widths, β and γ transition strengths, inelastic scattering form factor, and so on. These measurements are basically indirect and need to be combined with theoretical interpretations to reveal the internal structure of the Hoyle state. For instance, the observed inelastic scattering form factor was found to be very sensitive to the size of the final state and provided strong support for an expanded gas-like structure in the Hoyle state [1,11,38,50-52].

One interesting issue is the possible appearance of the Hoyle state in heavy-ion collisions, where many nuclear species can be produced. This was evidenced by von Oertzen et al. [11,52], who conducted experiments involving the reaction ^{28}Si + ^{24}Mg → ^{40}Ca + 3α. This experiment applied an extensive array of particle detectors to achieve coincident measurements of many charged particles. The γ-ray spectra related to the decay of heavier fragments, such as ^{40}Ca, ^{39}K, and ^{36}Ar, were collected in coincidence with three α-particles either randomly emitted to all angles or closely correlated to hit only one small-size detector. The latter case could be interpreted as the appearance of the Hoyle state during the collision. Interestingly, a strong γ-ray peak corresponding to excited ^{36}Ar was observed, which is in coincidence with compact 3-α emission (“Hoyle state”) but not with random α emission. The interpretation of this finding is that the 3-α condensation state at the surface of a compound nucleus exhibits larger diffuseness and radial extension, which allows lowering of the Coulomb barrier and thus leads to strongly correlated 3-α emission. To address this kind of phenomenon, it would be advantageous to use inverse kinematics, meaning employing heavier projectiles on lighter targets, allowing detection of fast-moving decay products such as multiple α-particles at forward angles with high efficiency [52]. This detection technology has been implemented in numerous recent experiments [53].

Theoretically, BEC-like structures have also been predicted for α-conjugate nuclei heavier than ^{12}C, such as ^{16}O, ^{20}Ne, etc. Since more particles must be detected and identified in coincidence, corresponding experimental work has progressed quite slowly.

For ^{16}O, the sixth 0^+ state at 15.1 MeV, just above the 4-α separation threshold at 14.4 MeV, has been considered a candidate for the BEC-analog state [20-21,54]. Table 1 summarizes inclusive measurements for this state. However, experimental determination of the BEC-like structure must come from multiple α-decay measurements [55]. The various channels of 4-α decay from a condensation state in ^{16}O can be expressed as [8]:

$^{16}\mathrm{O(BEC)} \rightarrow 4\alpha$
$\rightarrow {}^{8}\mathrm{Be}(0^{+}{}_{1}) + {}^{8}\mathrm{Be}(0^{+}{}_{1}) \rightarrow 4\alpha$
$\rightarrow {}^{12}\mathrm{C}(0^{+}{}_{2}) + {}^{8}\mathrm{Be}(0^{+}{}_{1}) \rightarrow 4\alpha$

Due to the Coulomb barrier effect, the latter channel should be favored [55]. So far, searches for α-decay from the ~15.1 MeV state in ^{16}O have been either unsuccessful or characterized by large uncertainties, most likely because its resonance energy is too close to the threshold, resulting in a decay probability as small as the nominal background rate [54,56-58].

Experimental efforts have also been devoted to investigating Hoyle-analog resonances above 15.1 MeV, considering possible rotational band members associated with the BEC-like 0^+ band head. Experiments can be divided into two categories. One is resonant scattering between ^{12}C and ^{4}He at center-of-mass energies covering the relevant ^{16}O resonant energies [63-65]. This method suffers from low energy resolution and relatively high background due to shortcomings such as inevitable mixing of non-resonant reaction mechanisms and limited energy-step size, which often leads to inconsistent results [64-65]. Another category includes transfer reactions or inelastic excitation followed by 4-α decay from intermediate ^{16}O resonances [66-68]. This method has the advantage of controlling the reaction mechanism, but excitation-energy resolution and background contamination depend sensitively on how the four α particles are measured. Practically, it is very difficult to measure all 4-α particles with clear particle identification (PID). Previous experiments [66-68] relied on counting the number of hits without PID. Consequently, background in the excitation energy (E__x) spectra was generally quite high, which deteriorated sensitivity to obscured resonant states [58,66-68].

Very recently, a new experiment was performed by the Peking University group at the HI-13 tandem accelerator at the China Institute of Atomic Energy, focusing on clear PID for all four α-particles decaying from ^{16}O [69]. The ^{16}O projectile at 96 MeV was inelastically excited by a carbon target. An array of eight charged-particle telescopes was used to detect the four α particles. Each telescope consisted of double-sided silicon strip detectors with various thicknesses, providing excellent position and energy resolutions. A large number of 4-α events were recorded in coincidence and with full PID. If three out of four α-particles could be combined to form the Hoyle state of ^{12}C, the event was selected as belonging to the $^{16}\mathrm{O} \rightarrow {}^{12}\mathrm{C}({0^+}_2$, Hoyle state) $+ \alpha \rightarrow 4\alpha$ decay channel. The selected events were then used to reconstruct Hoyle-like states of ^{16}O. For the first time, four narrow resonances just above the 15.1 MeV state were identified from this $\alpha + {}^{12}\mathrm{C}({0^+}_2)$ decay channel with high significance, as demonstrated in Fig. 2 (lower right panel). Theoretically, some calculations coincide with these experimental observations. One is the unified scattering and structure model, which predicted a $^{12}\mathrm{C}({0^+}_2) + \alpha$ rotational band with 0^+, 2^+, 4^+, and 6^+ members at energies in excellent agreement with the observed resonance peaks [70]. Here the 0^+ member should be a BEC-analog state. Another model, within the field theory superfluid approach, predicted two condensation-type 0^+ resonances in the energy range covered by the observed resonances [71]. This new observation is certainly very encouraging regarding the existence of BEC-like states in ^{16}O, similar to those identified in ^{12}C. Further measurements to clarify the spins of the currently observed 4-α resonances in ^{16}O would be essential [69].

There have also been a few experimental studies of BEC-like states in nuclei heavier than ^{16}O, such as in ^{20}Ne [22,24], ^{24}Mg [25,56], and ^{28}Si [72]. However, related condensation signatures remain quite ambiguous.

4 Discussion and Perspective

As outlined above, experimental work on BEC-analog structure in nuclei is still far from satisfactory compared to theory [11]. Apart from investigations of α-conjugate nuclei as exemplified above for ^{16}O, we particularly pay attention here to neutron-rich unstable nuclei, which represent the actual frontier of nuclear physics studies.

Immediately associated with the well-known ^{12}C(Hoyle state) would be the neutron-rich carbon isotopes, such as ^{14,16}C. If BEC-like resonances exist in ^{14}C, the favored decay channel would be $^{14}C \rightarrow {}^{8}Be(0^{+}{}_{1}) + {}^{6}He \rightarrow \alpha + \alpha + {}^{6}He$, as learned from ^{12}C and ^{16}O [5,13,69]. Therefore, this kind of experiment could be conducted by measuring the three decay fragments $\alpha + \alpha + {}^{6}He$, and then selecting reconstructed $^{8}Be(0^{+}{}_{1})$ events which can be combined with ^{6}He to form ^{14}C resonances, as depicted in Fig. 3 [Figure 3: see original paper]. Since the ground state of ^{8}Be is already in α-α relative s-wave, the remaining step is to check the total angular momentum of the reconstructed ^{14}C, which is the same as the relative orbital angular momentum of the ^{6}He fragment. To achieve this measurement, a high excitation energy is required. Fig. 4 [Figure 4: see original paper] illustrates fragment-separation thresholds for various decay channels from ^{14,16}C, revealing the possible appearance of BEC-analog states at around 19-24 MeV. In addition, sufficient event statistics would be essential but difficult, considering the low excitation cross section and the need for clear detection and identification of the three decay fragments [53,69]. Fortunately, for either ^{14}C or ^{16}C, the three decay helium fragments should all be in their ground states, which simplifies the decay paths and thus the Q-value selection [53,69]. Finally, the spin of the mother nucleus, ^{14,16}C, can be determined by angular correlation analysis or relative momentum analysis [13,73].

It is worth emphasizing that two neutrons in a low-density environment may form a compact dineutron pair (2n) which behaves like a boson (Ref. [74] and references therein). The latest experimental work has achieved evidence for the existence of the α + 2n + 2n BEC-like configuration in $^{8}He(0^{+}{}_{2})$ [75]. This opens a new research direction to search for BEC-like structure and properties of low-density neutron matter, such as at the surface of very neutron-rich heavy nuclei or in neutron stars.

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Note: Figure translations are in progress. See original paper for figures.