CNPC2023 Observation of Possible Bose-Einstein Condensate in 16O via 4-α Decay

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Date: 2024-02-06T00:00:00+00:00

Abstract

Recently, a 16O + 12C inelastic scattering experiment conducted at the Beijing Tandem Accelerator Nuclear Physics National Laboratory of the China Institute of Atomic Energy has provided new evidence for the existence of a 4-α Bose-Einstein condensate state in 16O. The experiment employed multiple sets of double-sided silicon microstrip charged-particle telescopes, achieving for the first time accurate identification of four α particles (Particle Identification, PID) and coincidence measurements in 16O decay. On this basis, a high-resolution reaction Q-value spectrum was obtained and clear 4-α resonant states were reconstructed. Among these, four resonant states with high statistical significance (mostly above 5σ) were observed near the threshold, which decay via the characteristic pattern of 12C(Hoyle state) + α, consistent with theoretical predictions of Hoyle-BEC-like structures and their rotational band characteristics. This observational result will drive further theoretical research, and additional measurements of physical quantities for these resonant states are also required experimentally.

Full Text

Preamble

Nuclear Physics Review, Vol. 31, No. 1, Feb. 2024
Article ID: 1007-4627(2024)01-0001-08
CNPC2023

Searching for possible Bose-Einstein condensate states in 16O via its 4-α decay

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Abstract

Recent inelastic scattering experiments of 16O + 12C conducted at the Beijing Tandem Accelerator Nuclear Physics National Laboratory of the China Institute of Atomic Energy have provided new evidence for the existence of a 4-α Bose-Einstein condensate state in 16O. Using multiple double-sided silicon microstrip charged-particle telescopes, this experiment achieved, for the first time, accurate particle identification (PID) and coincidence measurement of four α particles in the decay of 16O. Based on this capability, high-resolution reaction Q-value spectra were obtained and clear 4-α resonance states were reconstructed. Near the 4-α separation threshold, four highly significant resonance states were observed (most with significance above 5σ), which decay through the characteristic pattern of 12C(Hoyle state) + α, consistent with theoretical predictions of Hoyle-BEC-like structures and their rotational band features. These observations will stimulate further theoretical investigations, and additional measurements of physical quantities for these resonance states are needed experimentally.

Keywords: Cluster structure; Hoyle-like states; 16O resonances; High excitation
Classification: O571.21
Document code: A
DOI: 10.11804/NuclPhysRev.31.01.01

1. Introduction

Cluster structures in atomic nuclei represent exotic states that commonly appear in low-density nuclear regions and have been a focal point of interest in nuclear physics and nuclear astrophysics in recent years [1-6]. According to the Ikeda diagram, cluster structure states generally appear near the corresponding nuclear breakup thresholds [7]. As excitation energy increases, multi-α structures can emerge, culminating in all-α structures at the highest order. If all α particles occupy the lowest S-wave and the overall spatial distribution of the system expands, the nuclear system originally based on independent motion of fermions (nucleons) can transform into a Bose–Einstein condensation (BEC) state governed by bosonic motion [8]. This evolution has important implications for nuclear matter forms and nuclear astrophysical processes.

Previous experimental and theoretical studies have essentially confirmed that the Hoyle state of 12C (7.65 MeV, 0+), which plays a crucial role in stellar evolution, is a BEC-like state composed of 3-α particles [9-13]. The Hoyle state primarily decays via α + 8Be $\rightarrow$ 3α [14]. In heavier systems such as 16O, 20Ne, and 24Mg, although various theoretical predictions exist, reliable experimental evidence for such states remains lacking due to the experimental difficulty of directly measuring multiple α particles and determining their states [15-16].

For 16O, theoretical work suggests that a 4-α condensate state similar to the Hoyle state may appear near the 4-α separation threshold (14.437 MeV), though different theoretical models predict varying energy levels [12,17-18]. The theoretically predicted 4-α condensate state has substantial overlap with the 12C(0+2) + α configuration and decays primarily through a two-step process: 16O $\rightarrow$ 12C(0+2) + α $\rightarrow$ 4α [19-20]. The first 4-α condensate state above the threshold is generally believed to correspond to the 15.1 MeV excited state of 16O observed in inclusive experiments [18,21-23]. However, reliable cluster decay measurements for this 15.1 MeV state in 16O remain unavailable [15,20,24-25], mainly because α particles have difficulty penetrating the Coulomb barrier in the energy region very close to the separation threshold. Some signals observed in previous experiments were likely just statistical fluctuations of background [20,24].

Therefore, experimental measurements of 4-α decay in 16O need to target excited states higher than 15.1 MeV. Since 1967, several experiments have investigated multi-α decay of 16O, which can be categorized into resonance scattering experiments [26-28] and breakup reaction experiments [15,29-31]. Resonance scattering experiments typically use thick gas targets, allowing selection of the excitation energy region by adjusting the beam energy. These experiments generally employ reactions between 12C and gaseous 4He, measuring the final-state multiple α particles to reconstruct the resonance spectrum of 16O. However, they inevitably introduce non-resonant components, and the reaction point is difficult to determine, resulting in relatively high background and poor energy resolution, with inconsistent results across different experiments. Breakup reactions are typically two-step processes: first, the excited target nucleus is obtained through projectile-target reactions, then the target nucleus decays into 4-α. In this case, the reaction Q-value and excitation energy of the parent nucleus can be derived primarily from the decay products, with the resolution of Q-value and excitation energy spectra depending on the identification and measurement of final-state fragments. The aforementioned breakup reaction experiments all derived reaction Q-values and excitation energies by counting detected particle hits [15,29-31] and measuring their energies. This approach without particle identification (PID) introduces significant background and affects the judgment and selection of reaction channels.

This paper primarily introduces the inelastic excitation and decay experiment conducted at the HI-13 tandem accelerator of the China Institute of Atomic Energy to search for Hoyle-like states in 16O, along with its main data processing procedures. This experiment analyzed the reaction mechanism and reconstructed 16O resonance states using as many identified α particles as possible [32].

2. Experimental Setup

Since the physics goal of this experiment required multi-coincidence detection of final-state products and the highest possible energy and position resolution, an array of eight silicon microstrip detector telescopes was employed to detect reaction products, as shown in [Figure 1: see original paper]. The T0 and T1 telescopes (L0, R0, L1, R1) each consisted of three layers of silicon detectors: the first layer was a W1-type silicon microstrip detector approximately 40 m thick with an active area of 50 mm $\times$ 50 mm, segmented into 16 independent readout strips on both front and back sides; the second layer was a BB7-type silicon microstrip detector approximately 500 m thick with an active area of 64 mm $\times$ 64 mm, segmented into 32 independent readout strips on both sides; the final layer was an MSX40-type large-area silicon detector approximately 1500 m thick with an active area of 64 mm $\times$ 64 mm. The T2 (L2, R2) and T3 (L3, R3) telescopes each consisted of two silicon detector layers. T2 comprised one 40 m W1-type silicon microstrip detector and one 1500 m large-area silicon detector, while T3 comprised one 40 m BB7-type silicon microstrip detector and one 1500 m large-area silicon detector. More detailed information about the detector array can be found in [32].

In the experiment, 16O beams bombarded a 0.98 m 12C target, inelastically exciting 16O above the 4-α separation threshold. These highly excited states could then decay into four α particles through various pathways. The α particles and recoil 12C nuclei were detected by the telescope system. Particle identification (PID) was performed using the ΔE-E method based on energy deposition in the two silicon layers. Since the position of PID curves depends on the thickness of the first silicon layer, and the actual W1-type microstrips used had significant thickness non-uniformity, PID analysis was performed on a pixel-by-pixel basis. As an example, the PID for individual pixels in the first two silicon layers of the L0 telescope is shown in [Figure 2: see original paper].

Although the experimental setup avoided the strong direct beam particles near zero degrees, some particles could still originate from nuclear reactions occurring within the silicon detectors themselves. To minimize such background effects, particle trajectories were reconstructed using position information recorded in the two silicon layers. Events whose trajectories could not be traced back to the target region were rejected. This method is illustrated in [Figure 3: see original paper]. Additionally, time correlation constraints among detection signals were applied to exclude accidental coincidence events as much as possible.

As mentioned, this experiment placed particular emphasis on α particle identification. First, events where all four α particles in the final state were identified (full 4-α PID) were selected, as shown in Figure 4: see original paper. Assuming the recoil nucleus to be 12C, the momentum and energy of 12C could be deduced from the beam momentum/energy and the measured momenta/energies of the four α particles based on momentum-energy conservation [32-34]. The validity of assuming the recoil nucleus to be 12C could be verified by the corresponding peak in the Q-value spectrum mentioned in the next section. To increase the statistics of good events, events where some α particles stopped in the first silicon layer and thus lacked PID could be included. For example, cases where only three of the four α particles penetrated the first silicon layer and had PID, while the fourth α did not have enough energy to penetrate the first layer and thus lacked PID. To minimize background from particle misidentification, measurement and verification of the recoil 12C were required—that is, events had to include a fifth "hit" (5- "hit" events), as shown in Figure 4: see original paper. Since 12C rarely penetrates the first silicon layer, there would be two particles lacking PID in this case.

We first required that the measured energy of the fourth "hit" be less than the penetration energy of α particles in the first silicon layer (approximately 9–10 MeV). Then we required that the energy of the detected fifth "hit" [Tdet(12C)] be consistent with the kinetic energy of the recoil 12C [Tcal(12C)] deduced from momentum-energy conservation using the beam and the other four "hits". In other words, events within the red-line region in [Figure 5: see original paper] were selected. These 5-"hit" events satisfying such constraints could also produce clean Q-value peaks (see later analysis), and selecting this peak ensured that the assumed PID was equivalent to correct PID. These added events together with the full 4-α PID events are collectively referred to as "good events with addition" and were used for subsequent physics analysis.

3. Data Analysis and Results

The reaction Q-value is the mass difference before and after the reaction, as well as the difference between the total kinetic energy of final-state particles and the beam energy, which can be expressed as:

$$Q = \sum_{i=1}^{4} T_{\alpha_i} + T_{12C} - E_{\text{beam}}$$

[Figure 6: see original paper] shows a comparison between our results and those obtained by Freer et al. [29]. The peak around -14.4 MeV corresponds to final-state particles of four α particles plus recoil 12C. Any other final-state mass combinations with more than five particles are far from this peak, indicating that this Q-value peak is equivalent to complete PID for 4α + 12C without background interference. The peak around -18.9 MeV also corresponds to the same five final-state nuclei, but with 12C in its 4.44 MeV excited state. The

Q-value peaks corresponding to our "good events with addition" are very clear with essentially no background. Since background tends to concentrate near the 4-α separation threshold, maintaining low background in the Q-value peaks is particularly important for reconstructing resonances near the threshold.

The decay modes of multi-α resonant states are generally not simultaneous breakup of all α members, but rather multi-step processes. For 16O, the typical processes are 16O(excited) $\rightarrow$ 12C(excited) + α $\rightarrow$ 4α or 16O(excited) $\rightarrow$ 8Be + 8Be $\rightarrow$ 4α. In such cases, exclusive methods can be used to select two or three α particles and search for parent resonances using the invariant mass (IM) method, as illustrated in [Figure 7: see original paper]. Examples of spectra reconstructed from 3-α or 2-α particles in our "good events with addition" are shown in [Figure 8: see original paper]. The known 7.65 MeV and 9.64 MeV resonances of 12C and the ground state of 8Be are clearly visible, with their widths and positions reflecting the good resolution of this experiment and the reliability of the data analysis.

We then selected events where any three α particles could combine to reconstruct the 7.65 MeV excited state (Hoyle state) of 12C, while simultaneously selecting the peak near -14.44 MeV in Figure 6: see original paper. In this case, the final-state products are 12C(Hoyle state $\rightarrow$ 3α), the remaining fourth α particle, and the recoil ground-state 12C. The fourth α can then combine with the 12C(Hoyle state) to reconstruct the resonance state of the projectile 16OP after inelastic excitation, or alternatively combine with the recoil ground-state 12C to reconstruct a target-like 16OT formed by the target 12C capturing a transferred α. Clearly, the former process is the desired 4-α decay physics process, while the latter is a background process. Since transfer reaction cross sections can be large, this interfering background process can have significant probability (cross section). With identical final-state particles, these two processes cannot be distinguished by Q-value spectra, but can be separated through Dalitz plot analysis [29,35-36], as shown in [Figure 9: see original paper]. The strong horizontal band in the figure corresponds to known excited states of 16OT around 12 MeV.

To avoid the influence of 16OT, data with 16OT > 12 MeV (above the red line in [Figure 9: see original paper]) were selected. The projected invariant mass spectrum of 16OP is shown in [Figure 10: see original paper], which also displays the efficiency curve obtained from Monte Carlo simulations. The efficiency of the spectrum reconstructed from only 4-α PID events approaches zero near 19 MeV, precisely because the inclusion of 5- "hit" events improved the detection efficiency distribution near the threshold, enabling observation of the crucial resonance states. Data from other decay channels (such as via 8Be + 8Be decay) or with Q-values near -18.88 MeV (recoil 12C excited) can be processed similarly, but are not presented here as they are less relevant to the Hoyle-like resonances in 16O discussed in this paper.

Five narrow resonance peaks are clearly visible near the 4-α breakup threshold in [Figure 10: see original paper]. Considering the blocking effect of the Coulomb barrier in the near-threshold region, the appearance of these peaks more prominently reflects the corresponding 4-α structure in 16O. The spectrum was fitted using Breit-Wigner functions convolved with the detection resolution to describe each peak, with results summarized in . The significance level (SL) of each peak was obtained through maximum likelihood analysis [37], comparing the difference in maximum likelihood function values when including the peak (LH1) or not including it (LH0). The results show that the four peaks at 16.18, 17.46, 18.47, and 19.74 MeV have high significance, particularly the latter three with significance greater than 5σ. The peak at 15.63 MeV has lower significance due to limited statistics and requires confirmation in future experiments.

[Figure 11: see original paper] compares the results of this experiment with previous decay experiments. The near-threshold resonance peaks obtained in this work are clearly structured and highly significant, whereas previous results suffered from large background interference that made resonance peaks difficult to identify. This is mainly because previous measurements lacked PID for multiple decay α particles and effective rejection of interfering background reaction channels [15,29,31]. This experiment, for the first time, discovered four 4-α resonance peaks in 16O with high significance, decaying through the characteristic path of 12C(Hoyle state) + $\alpha \rightarrow 4\alpha$. This decay channel itself experimentally demonstrates a close relationship with the BEC-like state of 16O. Dynamical analysis shows that if 16O is in a BEC-like state, it will preferentially decay via the 12C(Hoyle state) + α mode [19]. This is partly due to Coulomb barrier effects (a single α penetrating the barrier is relatively easy), and partly because the decay process tends to preserve the parent nucleus structure as much as possible (structural link) [38-40]. Since the three α particles inside 12C(Hoyle state) are already in S-waves, only the orbital of the fourth α needs to be considered.

Theoretically, Ohkubo et al. studied the 12C + α scattering system using the coupled-channel method and predicted a rotational band formed by the 12C(Hoyle state) + α system, with its 0+, 2+, 4+, and 6+ members located at 16.61, 17.04, 18.38, and 19.95 MeV [41]. This is remarkably close to the positions of the four resonance peaks obtained in this experiment. Although spin-parities of these states could not be measured through angular correlations due to limited statistics in this experiment, their characteristic decay mode through the Hoyle state already demonstrates their close association with the theoretically predicted Hoyle-like states. For each member of this rotational band, the three intrinsic α particles are already in S-orbitals, leaving only the fourth α with possible different angular momenta, which likely includes cases where the fourth α is also in an S-orbital—i.e., a BEC-like state of 16O. Funaki et al. also obtained similar results through calculations, finding a rotational band above the 4-α separation threshold whose bandhead (0+ state) corresponds to the fourth α particle falling into the Hoyle state [42]. This relationship is illustrated in [Figure 12: see original paper]. Of course, other theoretical work suggests that several 0+ states (not necessarily forming a rotational band) related to BEC structure may appear near the 4-α separation threshold [43]. Clearly, these measurements will stimulate further theoretical research, and experimental determination of the spin-parities of the resonance states listed in is also needed.

4. Conclusion

Using an array of eight telescopes based on silicon microstrip detectors, we measured the inelastic excitation of 16O on a 12C target and its 4-α decay process, obtaining for the first time a large number of events with complete identification of all four α particles. To effectively utilize events without full α particle identification, we added selection of a fifth particle "hit" consistent with the properties of recoil 12C, thereby increasing the number of good events with clear Q-value resolution (equivalent to full final-state particle identification) by approximately a factor of two, particularly improving detection efficiency near the 4-α separation threshold. Through analysis of these "good events with addition," four resonance states in 16O near the 4-α separation threshold were observed with high significance, decaying via the characteristic pattern 12C(Hoyle state) + α $\rightarrow 4\alpha$. Since 12C(Hoyle state) already has a BEC-like structure and the configuration of the fourth decay α is also consistent with theoretically predicted states containing S-orbit components, this experiment provides important experimental evidence for the appearance of BEC-like states in 16O. Future experiments need to directly determine the spin-parities of the observed resonance states, and theoretical model analysis of the intrinsic structure, decay paths, and widths of these resonance states is also required.

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Note: Figure translations are in progress. See original paper for figures.