A Rendezvous Mission to the Second Earth Trojan Asteroid 2020 XL5 with Low-Thrust Multi-Gravity Assist Techniques postprint

Authors: Shi-Hai Yang, Bo Xu and Xin Li

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Abstract

As the second of Earth's Trojan asteroids, 2020 XL5 is worthy of rendezvous and even sample return missions in many aspects. In this paper, a rendezvous mission to Earth's second Trojan asteroid 2020 XL5 is proposed. However, due to its high inclination and large eccentricity, direct impulsive transfer requires large amounts of fuel consumption. To address this challenge, we explore the benefits of electric propulsion and multi-gravity assist techniques for interplanetary missions. These two techniques are integrated in this mission design. The design of a low-thrust gravity-assist (LTGA) trajectory in multi-body dynamics is thoroughly investigated, which is a complex process. A comprehensive framework including three steps is presented here for optimization of LTGA trajectories in multi-body dynamics. The rendezvous mission to 2020 XL5 is designed with this three-step approach. The most effective transfer sequence among the outcomes involves Earth–Venus–Earth–Venus-2020 XL5. Numerical results indicate that the combination of electric propulsion and multi-gravity assists can greatly reduce the fuel consumption, with fuel consumption of 9.03%, making it a highly favorable choice for this rendezvous mission.

Full Text

Preamble

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A Rendezvous Mission to the Second Earth Trojan Asteroid 2020 XL5 with Low-Thrust Multi-Gravity Assist Techniques

Shi-Hai Yang, Bo Xu, and Xin Li
School of Aeronautics and Astronautics, Sun Yat-sen University, Shenzhen 518107, China; xubo27@mail.sysu.edu.cn

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Abstract

As the second of Earth's Trojan asteroids, 2020 XL5 represents a compelling target for rendezvous and potential sample return missions. This paper proposes a rendezvous mission to Earth's second Trojan asteroid 2020 XL5, addressing the significant challenge posed by its high inclination and large eccentricity, which would require prohibitive fuel consumption for direct impulsive transfer. To overcome this obstacle, we explore the integration of electric propulsion and multi-gravity assist techniques—two powerful methods for interplanetary mission optimization. The design of low-thrust gravity-assist (LTGA) trajectories in multi-body dynamics is thoroughly investigated through a comprehensive three-step optimization framework. Applying this approach to the 2020 XL5 rendezvous mission yields an optimal transfer sequence of Earth–Venus–Earth–Venus–2020 XL5. Numerical results demonstrate that the combination of electric propulsion and multi-gravity assists substantially reduces fuel consumption to just 9.03%, making it a highly favorable option for this challenging rendezvous mission.

Key words: minor planets – asteroids: general – celestial mechanics – Planetary Systems

1. Introduction

In 1772, Lagrange proved that two groups of small bodies could stably share a planet's orbit if they remained near the triangular points 60° ahead (L4) or behind (L5) the planet \cite{Connors_{2011}}. The first Trojan asteroid, 588 Achilles, was discovered in 1906 by Max Wolf, orbiting 60° ahead of Jupiter \cite{Nicholson_{1961}}. Since then, more than 10,000 Trojans have been identified for Venus \cite{De_{{{la}}*{{Fuente}}*}{{{Marcos}}*{{de}}*}{{{la}}*{{Fuente}}*}{{{Marcos}}*{{2014}}}*
Mars \cite{de{{{La}}*{{Fuente}}*}{{{Marcos}}*{{de}}*}{{{La}}*{{Fuente}}*}{{{Marcos}}*{{2013}}}},*
Jupiter \cite{Dvorak{{{Schwarz}}*{{2005}}}}, Uranus \cite{de*{{{la}}*{{Fuente}}*}{{{Marcos}}*{{de}}*}{{{la}
and Neptune \cite{Sheppard{{{Trujillo}}_{{2006}}}}, with Jupiter Trojans comprising the vast majority.

Despite this abundance, no Earth Trojan (ET) was identified until 2011, when asteroid 2010 TK7 was discovered by the Wide-field Infrared Survey Explorer (WISE) mission \cite{Connors_{2011}}. Current observational limits suggest a population of perhaps 10^3 ETs with diameters greater than 100 m ($H \approx 23$) \cite{Wiegert_{{{et}}*{{al}}*}{2000}}, yet dedicated surveys have failed to detect additional members due to unfavorable viewing geometry from Earth \cite{Malhotra_{2019}, Wiegert_{{{et}}*{{al}}*}{1997}, Whiteley_{{{Tholen}}*{{1998}}}*, *Markwardt*{{{et}}*{{al}}*}{2020}, Lifset_{{{et}}*{{al}}*}{2021}}.

Asteroid 2020 XL5 was discovered by the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS) on 2020 December 12[1] and was subsequently confirmed as the second ET asteroid through follow-up investigations \cite{Hui_{{{et}}*{{al}}*}{2021}, Santana-Ros_{{{et}}*{{al}}*}{2022}}. Both 2010 TK7 and 2020 XL5 are classified as transient ETs, with orbital stability around L4 on the order of thousands of years—far shorter than the stability timescale of a theoretical primordial ET population \cite{Santana-Ros_{{{et}}*{{al}}*}{2022}}.

With an inferred diameter of 1.18 ± 0.08 km (assuming an albedo of 0.06 ± 0.03) \cite{Santana-Ros_{{{et}}*{{al}}*}{2022}}, 2020 XL5 is significantly larger than 2010 TK7 ($ $0.3 km) \cite{Connors_{2011}}. Non-gravitational effects such as the Yarkovsky effect couple orbital motion to physical and geological properties, making the determination of these properties crucial for understanding the asteroid's origin and evolution. Furthermore, because ETs share Earth's orbit, their chemical composition can provide valuable insights into the space environment of Earth's vicinity, making them prime candidates for rendezvous and sample return missions.

However, the high inclination and considerable eccentricity of both 2020 XL5 and 2010 TK5 render direct impulsive transfer from low Earth orbit (LEO) using chemical propulsion (CP) extremely costly. For 2020 XL5, the minimum total delta-v for a ballistic rendezvous trajectory ranges from 7.9 to 10.3 km s^{-1} depending on launch conditions, while for 2010 TK7 the range is 6 to 8.5 km s^{-1} \cite{Santana-Ros_{{{et}}*{{al}}*}{2022}}. Consequently, a rendezvous mission to 2020 XL5 is substantially more expensive than one to 2010 TK7, necessitating advanced trajectory optimization techniques.

Previous studies on 2010 TK7 transfer trajectories have demonstrated the effectiveness of gravity-assist techniques in reducing launch energy and rendezvous delta-v by altering spacecraft inclination \cite{Lei_{{{et}}*{{al}}*}{2017}, Gao_{{{et}}*{{al}}*}{2019}}. In two-body dynamics, an impulsive trajectory with a Venus-Earth-Venus swingby sequence achieves 41% fuel consumption with a flight time of 1694 days \cite{Lei_{{{et}}*{{al}}*}{2017}}. Electric propulsion (EP) offers another efficient method for reducing fuel consumption \cite{Chen_{{{et}}*{{al}}*}{2018}}. When EP was applied to the final leg of an impulsive transfer from Venus to 2010 TK7, converting it to a low-thrust leg, fuel consumption decreased to 4.59% with a flight time of 1684 days (assuming an initial mass of 800 kg and maximum thrust of 120 mN) \cite{Lei_{{{et}}*{{al}}*}{2017}}.

Building upon these advances, this work addresses the design of a rendezvous mission to 2020 XL5 by combining EP and multi-gravity assist techniques to

[1] https://www.minorplanetcenter.net/mpec/K20/K20XH1.html

optimize the transfer trajectory, given the asteroid's high inclination and large eccentricity.

2. Methodology

2.1. Determination of Swingby Number and Sequence

In a rendezvous mission to an asteroid, the spacecraft departs from LEO, executes a series of planetary swingbys, and culminates in rendezvous with the target asteroid. The delta-v required for LEO departure is assumed to be provided by the rocket's upper stage \cite{Xu_{{{et}}*{{al}}*}{2007}}. Deep-space maneuvers (DSMs) are introduced to accommodate cases where relative velocities at swingby are not perfectly matched \cite{Lei_{{{et}}*{{al}}*}{2017}, Gao_{{{et}}*{{al}}*}{2019}}. For an impulsive maneuver during swingby, periapsis serves as a useful starting point because it is easily identified and often near the optimal solution \cite{Gobetz_{1963}}, as illustrated in [Figure 1: see original paper].

When a DSM occurs at periapsis, the angles δ_1, δ_2, and θ can be expressed as \cite{Gao_{{{et}}*{{al}}*}{2019}}:

$$\delta_1 = \arcsin\left(\frac{1}{1+\frac{r_p v_\infty^-}{\mu_P}}\right)$$

$$\delta_2 = \arcsin\left(\frac{1}{1+\frac{r_p v_\infty^+}{\mu_P}}\right)$$

$$\theta = 2\arcsin\left(\frac{1}{\sqrt{1+\frac{r_p v_\infty^-}{\mu_P}}}\right) - 2\arcsin\left(\frac{1}{\sqrt{1+\frac{r_p v_\infty^+}{\mu_P}}}\right)$$

where μ_P is the planet's gravitational constant, r_p is the periapsis radius, and v_∞^- and v_∞^+ are the hyperbolic excess velocities before and after the DSM. The hyperbolic excess velocities are calculated by:

$$v_\infty^\pm = v_{sc}^\pm - v_P(t_{GA})$$

where v_P is the planet's velocity vector in the heliocentric ecliptic reference frame and t_{GA} is the swingby date. When no DSM occurs at swingby, the angles satisfy $\delta_1 = \delta_2 = \theta/2$. Given a range for r_p, Equation (3) is solved using the bisection method \cite{Gao_{{{et}}*{{al}}*}{2019}}. The spacecraft velocities relative to the planet before and after the DSM, $v_{sc}^\pm$, are obtained using the method from \cite{Gao_{{{et}}*{{al}}*}{2019}}. The DSM impulse is then:

$$\Delta v_{DSM} = v_{sc}^{+} - v_{sc}^{-}$$

The departure delta-v is given by:

$$\Delta v_1 = \sqrt{v_{E\infty}^2 + v_{LEO}^2} - v_{LEO}$$

where $v_{E\infty}$ is the hyperbolic excess velocity at Earth departure, $v_{LEO} = \sqrt{\mu_E / r_{LEO}}$ is the circular velocity at the parking orbit with $r_{LEO} = 6578.137$ km, and μ_E is Earth's gravitational constant.

The rendezvous delta-v is:

$$\Delta v_f = v_a(t_f) - v_{sc}(t_f)$$

where v_{sc} and v_a are the velocity vectors of the spacecraft and target asteroid in the heliocentric ecliptic reference frame at rendezvous epoch. The total delta-v budget is:

$$\Delta v_{total} = \Delta v_1 + \sum_{k=1}^{n_0} \Delta v_{DSM,k} + \Delta v_f$$

With the total delta-v budget established, the impulsive transfer trajectory can be optimized using the global optimization algorithm ICEA. Since swingby altitude is constrained, the objective function is:

$$J_{sequence} = \Delta v_{total} + p_r \sum_{k=1}^{n_0} \max(0, h_{min} - h_k) + p_t \max(0, t_f - t_{f,max})$$

where n_0 is the number of swingbys, p_r and p_t are penalty factors, and $t_{f,max}$ is the maximum allowed flight time. While mission planning typically imposes constraints on DSM impulse due to engine limitations and departure delta-v due to upper stage capacity, these constraints are neglected in this phase because: (1) the optimization focuses on determining swingby number and sequence, and (2) the required DSM impulses and departure delta-v are feasible with EP.

The decision variables include the departure epoch t_0, the flight time from Earth to the first swingby planet ΔT_0, and the flight times from the k-th swingby planet to the next body ΔT_k ($k = 1, 2, \dots, n_0$). From these, the swingby epochs $t_{GA,k}$ and rendezvous epoch t_f are determined. The position vectors of swingby planets are calculated from Keplerian elements\footnote{https://ssd.jpl.nasa.gov/planets/approx_{pos}.html}. Spacecraft velocity vectors at various epochs are evaluated by solving the

Lambert problem \cite{Gooding_{1990}}, and the total delta-v budget is derived using Equations (1), (5), (6), and (7).

The candidate swingby planets are denoted as $\{P_1, P_2, \dots, P_{N_P}\}$, where N_P is the number of candidates. The maximum number of swingbys is N_0. The search approach is depicted in [Figure 2: see original paper]. For a given swingby number n_0, the total number of sequence combinations is $N_P^{n_0}$. Each sequence can be represented as an integer j between 1 and $N_P^{n_0}$, which corresponds to a base-N_P representation. The ICEA algorithm then optimizes every transfer trajectory for the sequence Earth $\rightarrow P_{p_1} \rightarrow \cdots \rightarrow P_{p_{n_0}} \rightarrow$ asteroid.

2.2. Optimization of LTGA Trajectory in Two-Body Dynamics

2.2.1. Time-Free Fuel-Optimal Problem for LTGA Trajectory In the second step, only solar gravitation is considered. Most derivations follow \cite{Jiang_{{{et}}{{al}}}{2012}}. The spacecraft motion is governed by EP thrust and solar gravity. The equations of motion (EOM) are:

$$\dot{r} = v$$

$$\dot{v} = -\frac{\mu_s}{r^3} r + \frac{T_{\max}}{m} \alpha$$

$$\dot{m} = -\frac{T_{\max}}{I_{sp} g_0} \gamma$$

where r and v are the spacecraft's position and velocity vectors in the heliocentric ecliptic frame, $r = \|r\|$, m is spacecraft mass, μ_s is the solar gravitational constant, $T_{\max}$ and I_{sp} are the EP system's maximum thrust and specific impulse, g_0 is Earth's sea-level gravitational acceleration, $\gamma \in [0, 1]$ is the thrust ratio, and α is the thrust direction unit vector.

In LTGA trajectory optimization, length quantities are nondimensionalized by astronomical unit (au = 149,597,870 km), mass by initial spacecraft mass m_0, and time by $\sqrt{au^3/\mu_s}$. The fuel-optimal problem maximizes final mass (or minimizes fuel consumption):

$$J = m(t_f)$$

subject to boundary conditions:

$$r(t_0) = r_E(t_0)$$

$$v(t_0) = v_E(t_0) + v_{E\infty}$$

$$r(t_f) = r_a(t_f)$$

$$v(t_f) = v_a(t_f)$$

and swingby constraints:

$$\|r(t_{GA,k}) - r_{P,k}\| = R_{SOI,P,k}, \quad k = 1, 2, \dots, n_0$$

where $r_{P,k}$ is the position vector of the k-th swingby planet.

To handle equality and inequality constraints, numerical multipliers ν, ξ, χ, and κ are introduced. The fuel-optimal problem features a bang-bang control structure that complicates convergence. A numerical continuation method addresses this challenge \cite{Bertrand_{{{Epenoy}}{{2002}}}}. *A homotopic parameter ε perturbs the performance index* \cite{Jiang{{{et}}{{al}}}{2012}}:

$$J_\varepsilon = \lambda_0 m(t_f) + \varepsilon \int_{t_0}^{t_f} \frac{T_{\max}}{m(t)} \gamma(t) dt$$

where λ_0 normalizes the solution. The normalization condition is:

$$\|\lambda(t_0)\| = 1$$

with free initial and final times. The boundary conditions include:

$$\lambda_m(t_f) = \lambda_0$$

$$\lambda_r(t_0) = -\nu$$

$$\lambda_v(t_0) = -\xi$$

$$\lambda_m(t_0) = \lambda_0$$

where λ is the costate vector. When $\varepsilon = 1$, the performance index matches the energy-optimal problem, which is easier to solve due to control continuity. Applying Pontryagin's Maximum Principle (PMP) transforms the problem into an MPBVP. The Hamiltonian is:

$$H = \lambda_r \cdot v + \lambda_v \cdot \left(-\frac{\mu_s}{r^3} r + \frac{T_{\max}}{m}\alpha\right) - \lambda_m \frac{T_{\max}}{I_{sp} g_0}\gamma + \sum_{k=1}^{n_0} \nu_k \cdot (r - r_{P,k}) + \sum_{k=1}^{n_0} \xi_k \cdot (v - v_{P,k})$$

The optimal thrust direction and magnitude minimizing the Hamiltonian are \cite{Jiang_{{{et}}{{al}}}{2012}}:

$$\alpha^* = -\frac{\lambda_v}{\|\lambda_v\|}$$

$$\gamma^* = \begin{cases} 1 & \text{if } S_F > 0 \\ 0 & \text{if } S_F < 0 \\ [0,1] & \text{if } S_F = 0 \end{cases}$$

where the switching function is:

$$S_F = \frac{T_{\max}}{m}\|\lambda_v\| - \lambda_m \frac{T_{\max}}{I_{sp} g_0}$$

Substituting the optimal control law into the Hamiltonian yields the Euler-Lagrange equations for costate variables:

$$\dot{\lambda}_r = \frac{\partial H}{\partial r}, \quad \dot{\lambda}_v = -\frac{\partial H}{\partial v}, \quad \dot{\lambda}_m = -\frac{\partial H}{\partial m}$$

Transversality conditions for swingbys are \cite{Jiang_{{{et}}{{al}}}{2012}}:

$$\lambda_r(t^+_{GA,k}) = \lambda_r(t^-_{GA,k}) + A_k$$

$$\lambda_v(t^+_{GA,k}) = \lambda_v(t^-_{GA,k}) + B_k$$

where A_k and B_k are derived from gravity-assist constraints. Complementary slackness conditions for inequality constraints are:

$$\chi_k \geq 0, \quad \chi_k \cdot g_k = 0$$

Unlike \cite{Jiang_{{{et}}{{al}}}{2012}} where initial and final times are fixed, this work allows temporal flexibility. For the time-free problem, the transversality conditions are \cite{Hull_{2013}}:

$$H(t_0) = 0, \quad H(t_f) = 0$$

For the energy-optimal problem ($\varepsilon = 1$), the differential equations are integrated using the RKF7(8) integrator \cite{Fehlberg_{1968}}. As $\varepsilon \to 0$, ODE right-hand sides vary abruptly near switching points, so the bisection method detects these points \cite{Martinon_{{{Gergaud}}{{2010}}}, Zhang{{{et}}{{al}}}{2015}}.

At $\varepsilon = 1$, the MPBVP has $10 + 9n_0$ unknowns: λ_0, seven initial costates $\lambda(t_0)$, $3n_0$ gravity-assist impulse vectors, n_0 swingby dates, $4n_0$ Lagrange multipliers χ_k, and n_0 multipliers κ_k. Correspondingly, there are $10 + 9n_0$ equations: 6D boundary conditions, 1D normalization, 1D transversality, $3n_0$ impulse constraints, $4n_0$ inequality conditions, and n_0 complementary slackness conditions.

2.2.2. Global Search with ICEA Algorithm The second step aims to obtain feasible basic parameters for subsequent LTGA optimization rather than precise MPBVP solutions. These parameters include initial/final times t_0 and t_f, swingby dates $t_{GA,k}$, launch $v_{E\infty}$, hyperbolic excess velocities $v^{\pm}_{k,\infty}$, and periapsis radii $r_{p,k}$. Given the difficulty of solving the MPBVP directly, a global optimization method is employed using the energy-optimal control law ($\varepsilon = 1$) to reduce computation time.

The normalization condition transforms the search space into a hypersphere \cite{Jiang_{{{et}}{{al}}}{2012}}. The $8 + 5n_0$ variables λ_0, $\lambda(t_0)$, χ_k, and κ_k are normalized using $7+5n_0$ angle variables β. Launch $v_{E\infty}$ is also a decision variable. Penalty factors convert the constrained problem into an unconstrained one:

$$J_{search} = m_p + p_f \cdot \Gamma^*$$

where Γ^* includes boundary and transversality constraints, with penalty factors $p_f = 1000$ and $p_t = 1000$.

2.3. Design of LTGA Trajectory in Multi-Body Dynamics

2.3.1. Time-Fixed Fuel-Optimal Problem Without Swingby in Multi-Body Dynamics The third step incorporates third-body perturbations from solar system planets. Given that orbital elements from the Minor Planet Center (MPC) are heliocentric, the EOM are described in the heliocentric ecliptic reference frame. Most derivations follow the time-free fuel-optimal problem in two-body dynamics, except for the EOM and Euler-Lagrange equations. The spacecraft motion is governed by EP thrust and gravitation from the Sun, Earth, and other planets:

$$\dot{r} = v$$

$$\dot{v} = -\frac{\mu_s}{r^3} r + \sum_{i=1}^{10} \mu_i \frac{r_i - r}{\|r_i - r\|^3} + \frac{T_{\max}}{m} \alpha$$

$$\dot{m} = -\frac{T_{\max}}{I_{sp} g_0} \gamma$$

where μ_i ($i = 1, \ldots, 8$ for planets, $i = 9$ for Pluto, $i = 10$ for the Moon) are gravitational constants, and r_i are position vectors. Planetary states use JPL DE421 ephemeris \cite{Folkner_{{{et}}{{al}}}{2009}}. Boundary conditions are:

$$r(t_0) = r_E(t_0), \quad v(t_0) = v_E(t_0) + v_{E\infty}$$

$$r(t_f) = r_a(t_f), \quad v(t_f) = v_a(t_f)$$

The performance index and Hamiltonian with homotopic parameter ε are:

$$J_\varepsilon = \lambda_0 m(t_f) + \varepsilon \int_{t_0}^{t_f} \frac{T_{\max}}{m(t)} \gamma(t) dt$$

$$H = \lambda_r \cdot v + \lambda_v \cdot \left(-\frac{\mu_s}{r^3} r + \sum_{i=1}^{10} \mu_i \frac{r_i - r}{\|r_i - r\|^3} + \frac{T_{\max}}{m} \alpha \right) - \lambda_m \frac{T_{\max}}{I_{sp} g_0} \gamma$$

Optimal thrust magnitude and direction follow Equations (19)-(21). The Euler-Lagrange equations in multi-body dynamics are:

$$\dot{\lambda}_r = -\frac{\partial H}{\partial r}, \quad \dot{\lambda}_v = -\frac{\partial H}{\partial v}, \quad \dot{\lambda}_m = -\frac{\partial H}{\partial m}$$

For this TPBVP at $\varepsilon = 1$, normalization yields seven unknown angle variables β transformed from λ_0 and $\lambda(t_0)$. Seven constraints include 6D boundary conditions and the mass costate transversality condition. When $\varepsilon < 1$, λ_0 remains fixed at the energy-optimal value, leaving seven unknown initial costates. The normalization condition no longer applies.

Compared to the MPBVP for multi-gravity assists, obtaining a good initial guess for this TPBVP is easier via global optimization. The ICEA algorithm searches for initial guesses for the energy-optimal problem:

$$J_{guess} = \|\Gamma^*(\beta^*)\|$$

where β^* are angle variables from normalization, and Γ^* includes boundary and transversality constraints. A hybrid solver from the GNU Scientific Library (GSL) solves the nonlinear equations \cite{Powell_{1970}}.

The fuel-optimal solution process follows \cite{Jiang_{{{et}}{{al}}}{2012}}:

1. Apply ICEA to find approximate normalized costates and λ_0 for the energy-optimal problem ($\varepsilon = 1$) using RKF7(8).
2. Use these approximations as initial guesses to solve the accurate energy-optimal problem with RKF7(8) and GSL hybrid solver. Return to step 1 if non-convergent.
3. Fix λ_0 and gradually decrease ε to zero, using previous solutions as initial guesses. RKF7(8) with bisection detects switching points. If non-convergent, reduce the step size.
4. Output the fuel-optimal solution.

2.3.2. Patched-Arc Model for Optimization of LTGA Trajectory The patched-arc model is commonly used for interplanetary missions \cite{Melbourne_{{{Sauer}}{{1965}}}, Pierson{{{Kluever}}{{1994}}}}. *While swingby planets are often treated as massless with instantaneous swingbys \cite{Johnson*{1969}}, this assumption is inadequate for multi-body dynamics. To handle rapid costate variations during swingby, the hyperbolic trajectory within the planet's sphere of influence (SOI) is modeled as unpowered, subject only to gravity. The SOI radius for planet P is \cite{Bate_{{{et}}*{{al}}*}{2020}}:

$$R_{SOI,P} = a_P \left(\frac{m_P}{M_s} \right)^{2/5}$$

where a_P is the planet's semimajor axis, and m_P and M_s are planet and Sun masses.

For the k-th low-thrust leg between SOI boundaries, boundary conditions are:

$$r(t_{SOI,P}^{out}) = r_{P,k}(t_{SOI,P}^{out}) + R_{SOI,P} \frac{r(t_{SOI,P}^{out}) - r_{P,k}(t_{SOI,P}^{out})}{\|r(t_{SOI,P}^{out}) - r_{P,k}(t_{SOI,P}^{out})\|}$$

$$v(t_{SOI,P}^{out}) = v_{P,k}(t_{SOI,P}^{out}) + v_{k,\infty}^{-}$$

$$r(t_{SOI,P}^{in}) = r_{P,k}(t_{SOI,P}^{in}) + R_{SOI,P} \frac{r(t_{SOI,P}^{in}) - r_{P,k}(t_{SOI,P}^{in})}{\|r(t_{SOI,P}^{in}) - r_{P,k}(t_{SOI,P}^{in})\|}$$

$$v(t_{SOI,P}^{in}) = v_{P,k}(t_{SOI,P}^{in}) + v_{k,\infty}^{+}$$

The patched-arc model is illustrated in [Figure 3: see original paper]. Algorithm 1 outlines the general procedure.

Algorithm 1. Algorithm of patched-arc model

Input: Initial time t_0, final time t_f, launch $v_{E\infty}$, swingby dates $t_{GA,k}$, hyperbolic excess velocities $v^{\pm}_{k,\infty}$, periapsis radii $r_{p,k}$

Output: Fuel-optimal solutions for all low-thrust legs, complete trajectory, optimal thrust profile

1. Calculate parking orbit elements from launch $v_{E\infty}$ in Earth-centered inertial frame; transform to state vector.
2. Integrate unpowered EOM forward to Earth SOI boundary; transform to planetocentric ecliptic frame.
3. For each swingby $k = 1$ to n_0:
   - Calculate periapsis elements using $v^{\pm}_{k,\infty}$ and $r_{p,k}$.
   - Obtain SOI entry/exit states via forward/backward integration.
4. For each low-thrust leg $k = 1$ to $n_0 + 1$:
   - Set boundary conditions using Equations (46)-(47).
   - Solve fuel-optimal problem with given boundary conditions.

The complete design process is:

1. **Swingby selection:** Compute impulsive transfer performance for all possible sequences using Lambert problems ([Figure 2: see original paper]).
2. **Parameter search:** Use global optimization (ICEA) to obtain feasible basic parameters (not precise MPBVP solutions) under energy-optimal control.
3. **Refinement:** Repeat step 2 to estimate LTGA trajectory performance via patched-arc approach (two-body dynamics for low-thrust legs, multi-body for planetocentric phases). Select best-performing parameters for final multi-body LTGA optimization.

3. Results

3.1. Swingby Sequence for Rendezvous to 2020 XL5

The orbital elements of 2020 XL5 from MPC[2] are listed in . Given Earth's synodic periods with Venus (1.6 yr) and Mars (2.1 yr) \cite{Takao_{{{et}}{{al}}}{2021}}, the departure epoch spans 2024 January 1 to 2028 January 1—approximately twice the synodic period width. Flight times ΔT_0 and ΔT_k range from 10 to 1200 days. Previous work identified Venus and Earth as significant swingby planets for 2010 TK7 \cite{Lei_{{{et}}{{al}}}{2017}}. Considering 2020 XL5's large eccentricity (0.3874) and heliocentric distance range of 0.6133-1.3881 au, Mars is also favorable. Therefore, Venus, Earth, and Mars are designated as candidate swingby planets, with minimum swingby altitudes of 200 km. Their Keplerian elements are in .

[2] https://www.minorplanetcenter.net/data

The maximum total flight time is 1200 days. ICEA optimization uses PSO subpopulation $NP_1 = 400$, DE subpopulation $NP_2 = 100$, and $NG = 3000$ generations. Penalty factors are $p_r = 10^5$ and $p_t = 10^3$. With $N_P = 3$ candidate planets and maximum swingbys $N_0 = 5$, the best 10 results are in (E=Earth, V=Venus, M=Mars). Direct Earth-to-2020 XL5 transfer requires 5.93 km s^{-1} launch delta-v and 4.40 km s^{-1} rendezvous delta-v—both prohibitively large. Multi-gravity assist techniques significantly reduce total delta-v. Mars proves inadequate for this mission, while Venus as the final swingby is optimal. The best sequence is Earth–Venus–Earth–Venus–2020 XL5 ($n_0 = 3$) with 6.26 km s^{-1} total delta-v.

The impulsive trajectory for this sequence is further optimized with $NG = 5000$, yielding parameters in . The propellant mass ratio is 44.51% using chemical propulsion with $I_{spc} = 300$ s \cite{Mingotti_{{{et}}{{al}}}{2012}}. These results provide bounds for launch, swingby, and rendezvous dates for LTGA optimization.

3.2. Global Search for Basic Parameters

Mission parameters: $T_{\max} = 120$ mN, $I_{sp} = 3000$ s, $m_0 = 800$ kg, $g_0 = 9.80665$ m s^{-2}. Planetary states use JPL DE421 ephemeris \cite{Folkner_{{{et}}{{al}}}{2009}}. The search uses $NG = 100,000$, $NP_1 = 160$, $NP_2 = 40$. Based on impulsive results: launch date ranges 2024 October 1 to 2025 March 1; ΔT_0 ranges 50-100 days; ΔT_k ranges 300-500 days. The maximum gravity-assist impulse is limited to local circular speed at periapsis (0-10 km s^{-1}) \cite{Sims_{1996}}. Departure delta-v is capped at $V_C = 5$ km s^{-1} for 800 kg payload \cite{Lei_{{{et}}{{al}}}{2017}}, limiting $\|v_{E\infty}\| \leq 6.50$ km s^{-1}. Maximum total flight time is 1200 days.

Forty search runs yield three feasible parameter sets. Estimated LTGA trajectories using patched-arc (two-body dynamics for low-thrust legs) show minimum fuel consumption of 8.95% with 1157 days flight time, and worst-case 10.4% with 1142 days. Convergence curves in [Figure 4: see original paper] show objective functions reaching 20-30. The best parameters are in .

3.3. LTGA Trajectory from Earth to 2020 XL5 in Multi-Body Dynamics

The parking orbit has $a_{park} = 6578.137$ km, $e_{park} = 0$, and preferred inclination $i_0 = 28.5^\circ$. Gravitation from the Sun, Venus, Earth, Mars, and Jupiter is considered. Using the basic parameters from Section 3.2, the LTGA trajectory is optimized with Algorithm 1. Parking orbit elements and departure delta-v are in ; hyperbolic trajectory elements at periapsis are in ; LTGA parameters are in .

The heliocentric trajectory is shown in [Figure 5: see original paper]. Because 2020 XL5 has large ecliptic inclination, the final Venus swingby must occur when Venus crosses the asteroid's nodal line—an opportunity every 112.35 days

(Venus orbital period 224.7 days). Thrust magnitude, direction, and spacecraft mass histories are in [Figure 6: see original paper] and [Figure 7: see original paper]. Orbital element histories are in [Figure 8: see original paper].

The total flight time is 1156.84 days with 9.03% propellant mass ratio, leaving 728 kg final mass. For comparison, a direct low-thrust transfer () requires 966.27 days (190.57 days shorter) but consumes 20.8% propellant mass—11.8% higher, corresponding to 94 kg more propellant. Thus, multi-gravity assist is crucial for fuel savings.

Force model accuracy is assessed by applying control laws from two-body and multi-body (Sun, Venus, Earth, Mars, Jupiter) models to the full solar system model. Position and velocity errors at each leg's end are in (nondimensionalized). Two-body control laws yield errors of $\sim 10^{-2}$, while multi-body laws keep errors near 10^{-5}. Including Mercury, outer planets, and the Moon would increase computation time without significant accuracy gains. Therefore, the adopted multi-body model provides an optimal balance.

The synodic coordinate system perspective ([Figure 9: see original paper]) reveals that both transfers approach 2020 XL5 near the L1 point. In the Circular Restricted Three-Body Problem (CR3BP), trajectories naturally approaching L4 lie along unstable manifolds of L1 orbits \cite{Elliott_{{{et}}*{{al}}*}{2020}}. The Jacobi constant, conserved in CR3BP, indicates spacecraft energy \cite{Sood_{{{Howell}}_{{2019}}}}. Earth-orbiting spacecraft have high Jacobi constants ($ 3), *while2020XL5atL4haslowvalues*($2.8), making the transfer energetically challenging. [Figure 11: see original paper] shows that direct transfer requires continuous thrusting to achieve the required Jacobi constant reduction, while multiple swingbys reduce it more efficiently, with Venus's first swingby decreasing it by $ $0.1 and the second by 0.04. Orbital element histories ([Figure 8: see original paper]) show that the first Venus swingby primarily changes semimajor axis, eccentricity, and longitude of ascending node, while the second primarily changes inclination.

4. Conclusions

This paper proposes a rendezvous mission to Earth's second Trojan asteroid 2020 XL5 using low-thrust multi-gravity assist techniques. A three-step approach designs LTGA trajectories in multi-body dynamics. Key conclusions are:

1. **Direct impulsive transfer** from Earth to 2020 XL5 requires 10.33 km s^{-1} total delta-v, exceeding current technological capabilities.
2. **Multi-gravity assist optimization** reveals the best sequence as Earth–Venus–Earth–Venus–2020 XL5, reducing total delta-v to 6.26 km s^{-1} and fuel consumption to 44.51%. This highlights the technique's potential for reducing launch energy and rendezvous delta-v, with Venus playing a crucial role.

3. **Direct low-thrust transfer** requires 20.8% fuel consumption, demonstrating EP's significance for interplanetary fuel savings.
4. **LTGA trajectory** for the Earth–Venus–Earth–Venus–2020 XL5 sequence achieves 9.03% fuel consumption, proving the effectiveness of combining EP and multi-gravity assists. This integrated approach is recommended for the 2020 XL5 rendezvous mission.

The multi-body dynamics model including Sun, Venus, Earth, Mars, and Jupiter gravitation ensures accuracy while saving computation time compared to full solar system models.

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Appendix

This appendix provides expressions for converting between launch v_∞ (transformed from $v_{E\infty}$ in Earth-centered ecliptic to inertial frame) and orbital elements of the parking orbit or swingby hyperbolic trajectory.

Parking orbit minimal admissible inclination:

$$i_{min} = \arccos\left(\frac{v_{E\infty,z}}{\|v_{E\infty}\|}\right)$$

Hyperbolic trajectory at swingby:

Semimajor axis: $a_{GA} = -\frac{\mu_P}{\|v_\infty\|^2}$

Eccentricity: $e_{GA} = 1 + \frac{r_p\|v_\infty\|^2}{\mu_P}$

Inclination: $\cos i_{GA} = \frac{W_{GA,z}}{\|W_{GA}\|}$

Longitude of ascending node: $\tan\Omega_{GA} = \frac{W_{GA,x}}{-W_{GA,y}}$

Argument of periapsis: $\omega_{GA} = \arccos\left(\frac{e_{GA}\cdot W_{GA}}{\|e_{GA}\|\|W_{GA}\|}\right)$

True anomaly at periapsis: $f_{GA} = 0$

The unit angular momentum vector $W_{GA} = r \times v$ is calculated at SOI entry/exit. Quadrants for Ω_{GA} and ω_{GA} are determined by component signs. Note that Section 3 solutions may have small errors between $\|v_\infty^-\|$ and $\|v_\infty^+\|$, resembling DSM swingbys ([Figure 1: see original paper]). Hyperbolic elements are evaluated using the outbound trajectory where the spacecraft departs from the planet.

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Note: Figure translations are in progress. See original paper for figures.