Advances in Research on Stellar-Mass Binary Black Hole Spins (Postprint)

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Abstract

Since 2015, 93 gravitational wave events from stellar-mass binary black hole mergers have been detected, enabling measurements of binary black hole parameters such as mass, spin, and distance, as well as the derivation of their statistical distributions. The distributions of binary black hole properties, particularly the spin distribution, arising from different formation mechanisms exhibit significant differences, which can be utilized to constrain the origins of binary black holes. This article briefly reviews binary black hole formation mechanisms—including binary evolution and dynamical interactions—and the origins of spin; surveys general methods and common models for constraining binary black hole origins using spins, along with relevant advances from LIGO/Virgo gravitational wave data; and provides an outlook on future developments.

Full Text

Preamble

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Progress in Studying the Spins of Stellar Binary Black Holes

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Abstract

Since 2015, 93 gravitational wave events from stellar binary black hole mergers have been detected, enabling measurements of binary black hole parameters including masses, spins, and distances, as well as their statistical distributions. Different formation mechanisms for binary black holes produce distinct distributions of their properties, particularly spin distributions, which can be used to constrain their origins. This paper briefly reviews formation mechanisms such as binary star evolution and dynamical interactions, along with the origins of black hole spins. We summarize general methods and common models for constraining binary black hole origins using spin measurements, discuss recent progress using LIGO/Virgo gravitational wave data, and provide an outlook on future prospects.

Keywords: gravitational wave astronomy; gravitational wave sources; black holes

1 Introduction

In 2015, LIGO (Laser Interferometer Gravitational-Wave Observatory) detected GW150914, the first gravitational wave event from a distant binary black hole merger in the universe. This milestone marked humanity’ s ability to observe the cosmos not only through electromagnetic signals but also through gravitational waves [1-4], opening a new window for understanding the universe. The discovery of the binary neutron star merger GW170817 and its electromagnetic counterpart ushered in a new era of multi-messenger astronomy, enabling collaborative studies of important cosmic physical processes from different perspectives—such as the formation and evolution of compact objects, the internal structure and equation of state of neutron stars, and the propagation of gravitational waves [5].

To date, the LIGO-Virgo collaboration has detected 93 gravitational wave events in laser interferometer experiments [6]. Compared to neutron stars, black holes have relatively larger masses and produce stronger gravitational wave signals. Consequently, the vast majority of gravitational wave signals originate from the merger of stellar-mass binary black holes (hereafter referred to as binary black holes). Theoretically, these binary black holes can generally be produced through several mechanisms: (1) evolution of massive binary stars (EMBS channel); (2) dynamical interactions in dense stellar environments (dynamical channel); (3) active galactic nucleus/massive black hole-assisted formation (AGN/MBH-assisted channel); and (4) primordial black holes (PBH channel). Different evolutionary mechanisms produce different distributions of intrinsic physical parameters and some external parameters for binary black holes [7-9].

Gravitational wave observations provide an independent method for studying these mechanisms. The time-series signals from binary black hole mergers can be used to measure numerous physical parameters (including mass, spin, luminosity distance, etc.). As gravitational wave detection progresses, the sample of binary black hole mergers will continue to grow, enabling us to obtain statistical distributions of their physical parameters and external parameters, as well as correlations between different parameters. These statistical properties will greatly aid our understanding of the physical nature of black holes—the most exotic compact objects in the universe—and the formation and evolution of binary black holes. Analysis of the latest O3 gravitational wave data has yielded relatively accurate statistical distributions of binary black hole masses and spins, as well as spin-mass relationships [10,11]. The mass and spin distributions, spin-mass relationships, etc., produced by different mechanisms may differ significantly. Currently, measurements and analyses of binary black hole mass and spin distributions can effectively constrain their formation mechanisms, distinguishing between them and quantifying the contributions of different origins to observed binary black hole merger events. Using spin distributions to constrain binary black hole origins has become a hot topic in recent years, with several studies conducting in-depth investigations [7-9]. On one hand, the spin distributions—particularly the effective spin distribution—produced by different mechanisms can be relatively accurately determined from gravitational wave detection samples. On the other hand, the differences in the relative orientation distributions of spins between components in binary black holes formed through different mechanisms are intuitively clear and straightforward to understand.

This paper focuses on reviewing how spin studies constrain the origins of stellar binary black holes and provides a future outlook. The structure is as follows: Section 2 briefly introduces basic information about black holes, including the formation process of single black holes and the origin of black hole spins. Section 3 describes different formation mechanisms for stellar binary black holes and their theoretically predicted spin-related properties. Section 4 introduces methods for constraining binary black hole origins through spin analysis, including Bayesian parameter estimation methods, an overview of the GWTC dataset from LIGO-Virgo detections, and single-parameter and multi-parameter spin models. Section 5 provides a brief summary and future prospects for binary black hole spin research.

2 Basic Information on Black Holes

According to general relativity, the spacetime background near compact objects is described by Einstein's field equations: $G_{\mu\nu} = \kappa T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the energy-momentum tensor describing matter distribution, and κ is a constant. In 1916, Karl Schwarzschild provided the first exact vacuum solution to Einstein' s field equations—the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where M is the black hole mass ($G/c^2 = 1$). The Schwarzschild metric has a singularity at $r = 2M$; due to gravitational redshift, observers can never receive photons emitted from this location. In 1958, David Finkelstein interpreted the region with $r < 2M$ as an area of such high density that nothing can escape. In 1967, John A. Wheeler named this region “black hole.” Mathematically, it can be proven that the Kerr-Newman black hole is the unique solution for asymptotically flat, stationary, axisymmetric black holes:

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)d\phi - adt\right]^2$$

where $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2Mr + a^2 + Q^2$, and a is the specific angular momentum. Wheeler and others further proved the black hole no-hair theorem: an asymptotically flat, stationary black hole is completely determined by only three parameters—mass M, charge Q, and spin angular momentum $J = aM$. In astrophysics, due to macroscopic charge neutrality, we generally do not consider charged black hole models. Therefore, in this paper, the primary intrinsic physical parameters describing black holes are mass and spin. Based on different masses, black holes can be classified as stellar-mass black holes, intermediate-mass black holes, and supermassive black holes. The main research object of this paper is stellar-mass black holes, which primarily form through the self-gravitational collapse of massive stars in their late evolutionary stages, often accompanied by supernova explosions and/or gamma-ray bursts.

2.1 Formation of Single Black Holes

The most common objects in the universe are stars at various evolutionary stages. In the late stages of stellar evolution, as nuclear fuel in the core is gradually exhausted, the resulting radiation pressure becomes insufficient to balance the inward pressure from the star’ s self-gravity, causing the star to collapse and form a compact object [12]. For relatively low-mass stars ($M \lesssim 9M_\odot$), white dwarfs will form. For slightly more massive stars ($9M_\odot \lesssim M \lesssim 25M_\odot$), the remaining mass in their final evolutionary stage is relatively large, and the star’ s self-gravity exceeds the degeneracy pressure between electrons, causing the core to collapse nearly unimpeded toward the center. The gravitational potential energy released during collapse produces extremely high temperatures, leading to internal neutronization and generating large numbers of neutrinos. If the supernova explosion is non-spherical, the neutron star will receive a kick velocity. Meanwhile, neutron degeneracy pressure resists gravitational collapse. In this case, infalling material impacts the neutron star surface, forming a rebound shock wave that ultimately ejects the outer shell beyond the neutron star, leaving behind a neutron star. If it possesses some rotational angular momentum, it will form a common pulsar system.

For even more massive stars, the neutron degeneracy pressure in the core remains insufficient to support collapse against self-gravity, and the entire star is further “unlimitedly” compressed, ultimately forming the black holes commonly seen in astrophysics. The mass of such black holes generally exceeds the maximum mass of neutron stars—the Oppenheimer static limit [13] (commonly considered to be $2.4M_{\odot}$, though the static assumption is generally not valid, only providing a range of $(2-3)M_{\odot}$). Due to excessive stellar wind loss from particularly massive progenitor stars and possible pair-instability effects in late evolutionary stages, stellar black hole masses are typically limited to below 50-60 $M_{\odot}$. Notably, according to stellar evolution theory simulations, there exists a mass gap for stellar black holes in the $(50-120)M_{\odot}$ range (see Figure 1 [Figure 1: see original paper]). Through electromagnetic observations, more than twenty black holes have been discovered and their masses measured in the Milky Way, with most distributed in the $(5-30)M_{\odot}$ range [14]. However, gravitational wave observations of binary black hole mergers have discovered some primary black holes with masses greater than 50-60 $M_{\odot}$, posing a challenge to existing black hole formation models [15].

2.2 Spin of Single Black Holes

Black hole spin typically originates from the angular momentum of the progenitor star. Depending on different angular momentum dissipation mechanisms in stars, black holes with different spins will be produced. One possible dissipation mechanism involves the transfer of angular momentum within the star through meridional circulation and turbulent interactions. If magnetic fields participate in this process, according to the Tayler-Spruit magnetic dynamo mechanism, stellar angular momentum dissipation becomes more efficient.

Before gravitational waves were detected, the dimensionless spin a_* of black holes in binary systems was typically measured using X-ray observations. In binary systems, matter from the companion star is accreted by the black hole, forming an accretion disk. By observing this accretion disk, it becomes possible to obtain relevant parameters of the black hole. According to predictions from general relativity and accretion disk theory, the inner radius of a black hole’ s accretion disk is truncated at the innermost stable circular orbit (ISCO), and the radius of this orbit R_{ISCO} is a monotonic function of the black hole’ s dimensionless spin parameter $a_* = J/M^2$ (see Figure 2 [Figure 2: see original paper]). Therefore, by fitting the continuous spectrum of the accretion disk to reconstruct its size, the black hole spin can be measured [16]. Another common method involves using the broadening of the Fe Kα emission line due to gravitational redshift and other effects to estimate the position of the accretion disk’ s inner edge, thereby measuring black hole spin [17-20]. Additionally, using quasi-periodic oscillations of X-ray intensity from the accretion disk can also provide information related to ISCO and thus measure black hole spin, though this method has strong model dependence on oscillation modes [18].

Each of these methods has its own problems. For example, fitting the continuous spectrum requires determining the inclination angle of the accretion disk’ s inner edge, which often has particularly large errors; the Fe Kα line broadening method requires precise modeling of the electron corona that produces hard X-rays; and the quasi-periodic oscillation method demands relatively accurate understanding of accretion disk oscillation modes. Moreover, to date, only about 20 sources have been observed using these methods [22], which is not many compared to gravitational wave observations. Therefore, measuring black hole spins through X-ray binaries has limitations. By fitting parameters from the large number of binary black hole merger events observed through gravitational waves, we can measure black hole spins more accurately.

3 Formation Mechanisms of Stellar Binary Black Holes

If two stellar black holes orbit each other along quasi-Keplerian orbits, they form a gravitationally bound stellar binary black hole system. During the inspiral process, as gravitational waves are continuously radiated, their orbital energy gradually decreases, causing the two black holes to gradually approach each other and eventually undergo merger and subsequent ringdown, forming a single merged black hole. For example, the first gravitational wave event discovered by humanity, GW150914, was produced by the merger of stellar black holes with masses of 35.6 $M_{\odot}$ and 30.6 $M_{\odot}$. A particularly notable gravitational wave event, GW190521, has both black hole masses greater than 65 $M_{\odot}$ at 99.68% confidence [23], placing them within the mass gap predicted by stellar evolution numerical simulations. This may imply that these black holes with masses in the $(50-120)M_{\odot}$ range are primarily produced by mergers of first-generation black holes rather than directly from the evolution of massive stars.

Theoretically, there are currently two main mechanisms that can produce stellar binary black holes: (1) evolution of massive binary stars [24,25]; and (2) dynamical interactions in dense stellar clusters (dynamical channel) [26-29]. Binary black holes produced by different mechanisms have statistically distinct characteristics in terms of mass, spin, and orbital properties. This chapter focuses on introducing and discussing these two primary formation mechanisms, while only briefly mentioning other possible mechanisms such as active galactic nuclei and massive black hole mechanisms (AGN/MBH-assisted channel) [30] and primordial black hole mechanisms [31,32].

3.1 Evolution of Massive Binary Stars Mechanism

The evolution of massive binary stars mechanism typically refers to the process where isolated binary stars (not interacting with their environment) evolve through a common envelope phase to ultimately form binary black holes. This theory suggests that when the primary star first ends its main-sequence hydrogen burning, its helium core will contract while the outer hydrogen envelope expands. When the Roche lobe is completely filled, tidal gravity from the companion star will counteract the primary star’ s self-gravity, causing material to transfer from the primary to the companion [33]. Eventually, the primary star will form a bare helium core star and continuously lose mass through stellar winds until, when nuclear energy is exhausted, the primary star directly collapses into a black hole. If natal kicks [25] or mass loss do not disrupt this black hole-companion system, they will continue to evolve until the companion star begins to expand and its material is accreted onto the black hole. Since the companion star has already accreted some material from the primary star initially, and considering mass loss from the primary star itself, possible stellar winds, and supernova explosions, the companion star' s mass at this stage will be much greater than the primary star' s mass. The mass transfer process in this system becomes highly unstable, and this dynamical instability causes the binary to enter the common envelope evolution phase. When differential rotation exists between the inner binary composed of the black hole and helium core and the common envelope, drag forces will dissipate the binary' s orbital energy, potentially causing ejection of the common envelope material. During this phase, the companion star will lose its envelope and become another bare helium star, eventually evolving and collapsing to form a black hole. Assuming the second supernova explosion does not disrupt this binary black hole system, it will further undergo inspiral, merger, and ringdown phases, ultimately producing a new, more massive black hole [25] (see Figure 3 [Figure 3: see original paper]). The gravitational wave signals from each of these processes may be detected by gravitational wave detectors. It should be noted that physical processes such as mass transfer and common envelope evolution in the formation of binary black holes through this mechanism are extremely complex, and current understanding of these processes has significant uncertainties that may affect the results.

In these different massive isolated binary star evolution models, it is generally assumed that the spins of the two progenitor stars in the binary system are parallel to the binary' s orbital angular momentum. If black holes do not experience natal kicks, simple mass exchange and tidal interactions will not significantly change their spin directions. This directly leads to the spins of the two black holes in the formed binary black hole system tending to be aligned. On the other hand, if the black hole mass is relatively small (approximately less than 10 $M_\odot$) and natal kicks from supernova explosions occur, the spin directions will become non-parallel.

3.2 Dynamical Interactions Mechanism

The dynamical interactions mechanism is a relatively general term—any formation process of binary black holes involving gravitational interactions with other stars can be classified under this mechanism. Generally, we can subdivide it based on the environment where the dynamical process occurs, such as evolution in globular clusters and nuclear clusters, or evolution in hierarchical triple systems.

In globular clusters, initially single black holes with relatively large masses will sink to the central region of the cluster due to mass segregation effects. Once in such a dense region, black holes may immediately form self-bound binary black hole systems, or participate in other black hole-star systems through three-body processes, ejecting lighter stars and forming new binary black hole systems. In nuclear clusters, it is generally believed that a supermassive black hole exists (see

Figure 5 [Figure 5: see original paper]). This will create a huge gravitational potential, making mass segregation effects more pronounced and providing a high-density environment more conducive to binary black hole formation [29].

Another common dynamical mechanism is the hierarchical triple systems that are very common in the universe [37]. In such systems, angular momentum is transferred back and forth between the inner binary and the outer third black hole, causing the eccentricity of the inner binary to continuously change. When the eccentricity is excited to very high values, the orbital energy of the inner binary can be dissipated through gravitational waves or tidal forces, promoting their merger (Kozai-Lidov effect). Binary black holes produced through dynamical interactions in dense stellar fields or triple systems are generally considered to have randomly oriented spins. This is because in such complex and random dynamical interaction processes, it is difficult to identify any physically meaningful special direction as the initial spin direction of the black holes. Therefore, it is generally assumed that the initial spins of merging binary black hole systems produced by this mechanism are completely random. Numerical calculations of post-Newtonian orbits show that such randomness will be preserved until the binary black hole system finally merges. Thus, unlike the massive isolated binary star evolution mechanism, the spin directions of binary black holes produced by dynamical mechanisms are typically considered completely random.

3.3 Other Possible Mechanisms

In addition to the two main formation mechanisms mentioned above, stellar-mass binary black holes may also be produced through active galactic nucleus/massive black hole-assisted mechanisms and primordial black hole mechanisms [38]. At the center of active galactic nuclei, there exists a massive accretion disk around the central black hole. The accretion disk has very high temperature and density, is rich in gas, and can easily capture stars or produce stars directly through disk instability. These stars grow to larger masses through interactions with surrounding gas and stars, forming massive binary and binary black hole systems [32]. Primordial black holes originate from density fluctuations in the early universe [31]. Some scholars believe that a significant portion of dark matter is composed of stellar-mass primordial black holes, and their mergers could also produce gravitational wave signals from stellar binary black hole mergers [39].

4 Constraints on Binary Black Hole Origins Based on Spin Analysis

Spin information provides important clues about the formation mechanisms of stellar binary black holes. Gravitational waveforms are highly sensitive to the effective spin χ_{eff} of binary black holes, defined as follows [7]:

$$\chi_{\text{eff}} = \frac{1}{M}(m_{1s}1\cos\theta_1 + m_{2s}2\cos\theta_2)$$

where m_1 and m_2 are the masses of the binary black holes, $M = m_1 + m_2$ is the total mass, S_1 and S_2 are the black hole spin angular momenta, L is the orbital angular momentum of the binary (assumed to be in the z direction), s_1 and s_2 are the corresponding dimensionless black hole spins, and $\cos\theta_1$ and $\cos\theta_2$ are the angles between the spin angular momenta and the orbital angular momentum. During the inspiral phase, the effective spin is nearly conserved.

To further assess the precession effects of binary black holes, a new parameter called the effective precession spin is introduced, defined as:

$$\chi_p = \max(B_{1S_{1\perp}}, B_{2S_{2\perp}}) > 0$$

where $B_1 = 2 + 3q/2$, $B_2 = 2 + 3/2q$, and q is the mass ratio of the binary black holes. We can use gravitational wave detections to measure these two spin-related parameters for binary black holes, thereby constraining the origins of stellar binary black holes—particularly distinguishing between the massive isolated binary star evolution mechanism and the dynamical mechanism. This chapter primarily introduces the general methods for processing GWTC gravitational wave observation data using Bayesian statistics and reviews research progress on constraining stellar binary black hole formation mechanisms through corresponding single-parameter and multi-parameter models.

4.1 Introduction to GWTC Data

GWTC (Gravitational Wave Transient Catalog) is the gravitational wave detection dataset published by the LIGO/Virgo/KAGRA collaboration. Three releases have been made so far, corresponding to the O1 and O2, O3a, and O3b observation runs, with a total of 93 confirmed gravitational wave events, including two neutron star mergers, one black hole-neutron star merger, and the remainder being binary black hole mergers [40,41].

(1) GWTC-1 data includes observations from LIGO and Virgo between September 12, 2015, and January 19, 2016 (O1), and November 30, 2016, to August 2017 (O2). It primarily detected 10 binary black hole merger events including GW150914 and GW151012, plus one binary neutron star merger event GW170817. The total mass range of the detected binary black holes was $18.6^{+3.2}_{-1.1} M_\odot$, and the distance range was 320^{+120}_{-110} Mpc.

Figure 6 [Figure 6: see original paper] shows the posterior distributions of the effective spin $\chi_{\rm eff}$ and effective precession spin χ_p inferred for the gravitational wave sources. It can be seen that the peaks of $\chi_{\rm eff}$ for almost all binary black holes are around 0, with only two sources, GW170729 and GW151226, showing significant deviations from 0: $0.11^{+0.58}_{-0.58}$ and $0.06^{+0.38}_{-0.38}$ (90% confidence interval), respectively. The precession spin χ_p shows a strong correlation with the effective spin $\chi_{\rm eff}$.

(2) GWTC-2 (including 2.1) data includes observations from Advanced LIGO and Advanced Virgo during the O3a run (April 1, 2019–October 1, 2019),

detecting 44 new gravitational wave source candidates. Compared to GWTC-1, the total mass range of binary black holes expanded to $(14-182)M_\odot$.

Figure 7 [Figure 7: see original paper] shows the physical parameters and uncertainties of the gravitational wave sources detected by GWTC-2, with 13 systems having non-zero χ_{eff}, and all χ_{eff} values greater than 0 at 90% confidence. The binary black hole with the largest effective spin was GW190517-055101, with $\chi_{\text{eff}} = 0.52^{+0.19}_{-0.19}$. The precession spin also shows a strong correlation with the effective spin χ_{eff}.

(3) GWTC-3 data includes observations from Advanced LIGO and Advanced Virgo during the O3b run (October 1, 2019—March 27, 2020), adding 38 new gravitational wave sources or candidates detected during O3b (including the first black hole-neutron star merger event). The parameter ranges are similar to GWTC-2 results, as shown in Figure 8 [Figure 8: see original paper]. The effective spins of most gravitational wave sources are close to 0, with 4 greater than 0 (confidence > 89%). Notably, GW191109-010717 and GW200225-060421 have effective spins less than 0 at 90% and 85% confidence intervals, respectively. The correlation between precession spin and effective spin χ_{eff} becomes stronger.

4.2 Introduction to Bayesian Analysis Methods

In gravitational wave data analysis, since the gravitational wave strain $h(f)$ itself is very weak (on the order of 10^{-21} to 10^{-22}) while noise is high, we typically use template matching methods to analyze gravitational wave signals and noise [42]. The signal-to-noise ratio can be calculated using the following expression:

$$\rho = 4\int_{f_{\min}}^{f_{\max}} \frac{|h(f)|^2}{S_n(f)} df$$

where $S_n(f)$ is the one-sided power spectral density of the gravitational wave detector, and f_{min} and f_{max} are the integration limits for gravitational wave frequency. We typically take $\rho = 8$ as the detection threshold (equivalent to a 10% false alarm rate), meaning that values above this threshold define a detected gravitational wave event.

Once a gravitational wave signal is received, Bayesian statistical methods can be used to estimate the posterior distribution $P(\Theta|M)$ of the gravitational wave source' s physical parameters Θ:

$$P(\Theta|M) = \frac{P(M|\Theta)P(\Theta)}{P(M)}$$

where $P(\Theta)$ is the prior distribution of parameters, $P(M)$ is the model evidence, and $P(M|\Theta)$ is the corresponding likelihood function. In gravitational wave data processing, we can obtain:

$$P(M|\Theta) \propto \exp\left[-\frac{1}{2}(s(\Theta)-n, s(\Theta)-n)\right]$$

where s is the detected gravitational wave signal, n is the corresponding Gaussian noise, and the parentheses represent the inner product of the signal relative to the power spectrum. Using the above method, we can estimate the posterior distribution of gravitational wave source parameters. Since the likelihood function is very complex, this process typically requires Markov Chain Monte Carlo (MCMC) sampling or nested sampling to sample the posterior distribution and obtain statistical range estimates for parameters.

To study the spin distributions of binary black hole gravitational waves produced by different mechanisms, we can analyze population information by introducing Bayesian hierarchical models. In Bayesian hierarchical models, we assume that the spins of binary black hole gravitational wave events (totaling N_{obs}) produced by different mechanisms all originate from corresponding higher-level population distributions, which can be described and distinguished by a set of parameters λ [43]. The corresponding hierarchical Bayesian posterior distribution can be expressed as:

$$P(\lambda|\{s_i\}) \propto p(\lambda)\prod_{i=1}^{N_{\mathrm{obs}}}\int d\chi_i P(s_i|\chi_i)\pi(\chi_i|\lambda)$$

where s_i corresponds to the i-th observed gravitational wave signal (corresponding to the i-th source). If we are not interested in the effective spin distribution of individual sources, we can marginalize over them to obtain the distribution of population-level parameters:

$$P(\lambda|\{s_i\}) \propto p(\lambda)\prod_{i=1}^{N_{\mathrm{obs}}}\int d\chi_i P(s_i|\chi_i)\pi(\chi_i|\lambda)$$

Based on the above formula, we can use the distribution of effective spins to distinguish and constrain different binary black hole production mechanism models. For example, assuming the proportion of massive isolated binary star evolution models is λ and the proportion of dynamical mechanism models is $1-\lambda$, the hierarchical distribution of effective spins can be expressed as:

$$P(\chi_{\mathrm{eff}}|\lambda) = P_{\mathrm{EMBS}}(\chi_{\mathrm{eff}})(1-\lambda) + P_{\mathrm{dynamical}}(\chi_{\mathrm{eff}})\lambda$$

Combining hierarchical Bayesian methods with gravitational wave data can yield constraint results on the model proportion λ.

4.3 Spin Models

As mentioned earlier, a simple physical idea is that the spins of binary black holes formed through massive isolated binary star evolution models are usually parallel to the orbital angular momentum. Even if natal kicks occur, this parallelism should not be significantly deviated from. In contrast, the relative spin orientations of the two components in binary black holes produced by dynamical mechanisms are completely random. Methods for constraining binary black hole formation mechanisms based on spin analysis are built upon this foundation. Several different models are generally considered [44].

(1) **DEFAULT Model**: This model assumes that the magnitudes of binary black hole spins follow the same Beta distribution:

$$\pi(\chi_{1,2}|\alpha_\chi, \beta_\chi) = \mathrm{Beta}(\alpha_\chi, \beta_\chi)$$

where α_χ and β_χ are the two parameters of the Beta distribution. The spin tilt angles (relative to orbital angular momentum) follow a distribution containing two mixed components: one portion of binary black holes has spins quasi-parallel to the orbital angular momentum (proportion ξ), corresponding to the massive binary star evolution mechanism; the other portion has completely random spins, corresponding to the dynamical mechanism. In the parallel case, the model assumes the spin tilt angles follow a truncated normal distribution with perfect parallelism as the expectation:

$$\pi(z|\zeta, \sigma_t) = \zeta G_t(z|\sigma_t) + (1-\zeta) I(z)$$

where $z = \cos\theta_{1,2}$ is the cosine of the binary black hole spin tilt angles, I is the random distribution, and $G_t(z|\sigma_t)$ is the truncated Gaussian distribution centered at $z = 0$ with standard deviation σ_t. This model is intuitively the simplest, containing only four parameters (α_χ, β_χ, ζ, σ_t).

(2) **GAUSSIAN Model**: The advantage of this model is that it can simultaneously fit both χ_{eff} and χ_p using a joint Gaussian distribution:

$$\pi(\chi_{\mathrm{eff}}, \chi_p|\mu_{\mathrm{eff}}, \sigma_{\mathrm{eff}}, \mu_p, \sigma_p, \rho) \propto G(\chi_{\mathrm{eff}}, \chi_p|\vec{\mu}, \Sigma)$$

where $\vec{\mu} = (\mu_{\mathrm{eff}}, \mu_p)$. The covariance matrix is:

$$\Sigma = \begin{pmatrix} \sigma_{\mathrm{eff}}^2 & \rho\sigma_{\mathrm{eff}}\sigma_p \\ \rho\sigma_{\mathrm{eff}}\sigma_p & \sigma_p^2 \end{pmatrix}$$

This model has a total of 5 parameters (μ_{eff}, σ_{eff}, μ_p, σ_p, ρ), where σ_{eff} and σ_p help us analyze spin-induced orbital precession effects and some binary black hole systems with anti-parallel spins.

(3) **MULTI SPIN Model**: This model can simultaneously combine different mass distributions and spin distributions, with numerous parameters (12 spin parameters, 10 mass parameters), and is primarily used to analyze spin-mass and mass ratio relationships. This model is typically used for···

4.4 Current Results

This section briefly summarizes relevant results and progress on constraining binary black hole production mechanisms using spin distributions obtained from LIGO/Virgo gravitational wave detections.

4.4.1 GWTC-1 and Before Since the discovery of GW150914 [1], scholars immediately began studying its formation mechanism. For example, some researchers believed that binary black hole gravitational wave sources in the $40M_{\odot}$ to $100M_{\odot}$ range could be formed through isolated binary star evolution [25], while others believed that dynamical evolution could also produce the observed binary black holes [32,45]. Although not many gravitational wave sources were observed before GWTC-1, people had already begun statistical studies of source properties and related models, particularly investigating binary black hole formation mechanisms using spin distribution analysis. For instance, through simple Bayesian analysis of the effective spin χ_{eff} distribution [7], it was found that if the detected black holes were assumed to have large spins, the spin orientation distribution tended more toward isotropy; under the prior condition of small spins, this tendency would be greatly weakened. Figure 9 [Figure 9: see original paper] shows the possible proportion of isotropic distribution in the posterior distribution of binary black hole spins. Notably, due to the small amount of observational data at the time, it was not possible to statistically demonstrate that this conclusion was universal.

4.4.2 GWTC-2 Using the DEFAULT model described above, a non-parallel phenomenon between spin and orbital angular momentum was discovered in the GWTC-2 binary black hole sample, as shown in Figure 10 [Figure 10: see original paper]. This discovery implies that some binary black holes either have completely random spin directions or have spin tilt angles close to 0 but with certain dispersion. Regardless of which scenario, it indicates that some binary black holes have spin components within the orbital angular momentum plane. Figure 10 shows the posterior distributions of binary black hole spin magnitudes and spin tilt angles obtained using the DEFAULT model, where $\chi = |s|$ is the dimensionless spin magnitude.

Although the GWTC-2 data analysis results more strongly support parallel alignment, the dispersion cannot be ignored. Furthermore, if the GAUSSIAN model is used, the case of $\mu_p = \sigma_p = 0$ can be excluded at 99% confidence, indicating the existence of spin-induced orbital precession phenomena, which also verifies from another perspective the existence of non-parallel spin and orbital angular momentum phenomena.

Secondly, examples with anti-parallel alignment were found in the GWTC-2 binary black hole sample, suggesting that more than one binary black hole production mechanism may exist. Using the GAUSSIAN model, it was inferred that some systems might exhibit $\chi_{\rm eff} < 0$, which is equivalent to at least one of the binary black holes having an angle greater than 90° with the orbital angular momentum.

Figure 11 [Figure 11: see original paper] shows the posterior distributions of the mean $\mu_{\rm eff}$ and standard deviation $\sigma_{\rm eff}$ after marginalizing over σ_p and χ_p, with peaks at $0.06^{+0.05}_{-0.05}$ and $0.12^{+0.06}_{-0.04}$, respectively. This requires that some binary black holes have effective spins $\chi_{\rm eff} < 0$. Using the DEFAULT model, we can obtain similar conclusions. Figure 11(a) shows the reconstructed distribution of $\cos\theta_{1,2}$ using this model, where the possibility of $\cos\theta_{1,2} < 0$ still exists, implying the existence of anti-parallel spin phenomena.

Using the posterior distributions of $\mu_{\rm eff}$ and $\sigma_{\rm eff}$ shown in Figure 11, we can obtain the posterior distributions of the proportions f_p ($\chi_{\rm eff} > 0$), f_n ($\chi_{\rm eff} < 0$), and f_v ($\chi_{\rm eff} = 0$) (see Figure 12 [Figure 12: see original paper]). It can be seen that at 99% confidence, $f_p = 0.67^{+0.16}_{-0.16}$, $f_n = 0.27^{+0.17}_{-0.15}$, and $f_v = 0.05^{+0.02}_{-0.01}$. As mentioned earlier, the dynamical mechanism is not the only way to produce binary black holes with effective spins less than zero. If progenitor stars experience a strong natal kick and an inefficient spin realignment process during the supernova explosion phase, approximately less than 10% of binary black holes produced by massive isolated binary star evolution mechanisms may have effective spins less than 0. There are also many complexities, such as the possibility of producing binary black holes with non-parallel spin components in isolated hierarchical triple systems.

Nevertheless, if we simply assume that binary black holes with $\chi_{\rm eff} < 0$ are all produced through dynamical interactions in star clusters, we can use f_n to constrain the fraction of binary black holes with dynamical origins. The corresponding fraction produced by dynamical mechanisms is:

$$f_d = \frac{2f_n}{f_p + f_n}$$

and the fraction from isolated binary evolution mechanisms is:

$$f_i = \frac{f_p - f_n}{f_p + f_n}$$

At 90% confidence, the value of f_d lies between 0.25 and 0.93. This indicates that the binary black holes in GWTC-2 are formed partly through isolated binary mechanisms and partly through dynamical mechanisms in star clusters. More detailed discussions on the generation details of effective spin parameters under dynamical and isolated binary evolution models and why the above parameter descriptions are adopted can be found in the literature [46-50].

Furthermore, if hierarchical mergers exist in the GWTC-2 data, spins will have some relationship with the masses of binary black hole systems—that is, heavier hierarchical binary black hole mergers will have larger spins. However, analyzing GWTC-2 data using the MULTI SPIN model mentioned above did not find evidence of a correlation between spin distribution and mass. This result is similar to that obtained from GWTC-1.

4.4.3 GWTC-3 Compared to GWTC-2, the GWTC-3 binary black hole sample has expanded significantly, but many conclusions obtained from statistical analysis of the spin distributions of these binary black holes are very similar. For example, Figure 13 [Figure 13: see original paper] shows the constraints on binary black hole spin magnitudes and spin tilt angles using the DEFAULT model. It can be seen that compared to GWTC-2, the distribution of the cosine of spin tilt angles $\cos\theta$ for GWTC-3 binary black holes is flatter, more strongly supporting a completely random distribution of spin tilt angles. Similarly, using the GWTC-3 binary black hole sample, the posterior distributions of χ_{eff} and χ_p can be obtained using the GAUSSIAN model, and it is found that μ_{eff} is non-zero (see Figure 14 [Figure 14: see original paper]). This conclusion is consistent with the GWTC-2 results.

Two very important new conclusions have been drawn from the GWTC-3 data. First, the spin distribution is consistent across different mass ranges. Figure 15 [Figure 15: see original paper] shows the relationship between the magnitude of the spin component parallel to the orbital angular momentum s_z and the chirp mass of the binary black hole system. At the low-mass end, the parallel component is almost 0; while at the high-mass end, even with relatively large dispersion, the distribution of parallel component magnitudes remains consistent with 0. The average spin component parallel to the orbital angular momentum for binary black holes with chirp mass less than 30 $M_\odot$ is 0.38 (90% confidence), while at the low-mass end this result is 0.5.

Second, an anti-correlation between mass ratio q and effective spin χ_{eff} was discovered in the GWTC-3 data. Therefore, binary black hole systems with mass ratios approaching 1 tend to have effective spins χ_{eff} close to 0, while systems with large mass ratios tend to have larger positive effective spins. Using the GAUSSIAN model, we can obtain the conditional posterior distribution $p(\chi_{\mathrm{eff}}|q)$ of χ_{eff} on q:

$$p(\chi_{\mathrm{eff}}|q) \propto \exp\left[-\frac{(\chi_{\mathrm{eff}}-\mu(q))^2}{2\sigma^2(q)}\right]$$

where $\mu(q)$ and $\sigma(q)$ describe how the median and standard deviation of the χ_{eff} distribution vary with q. Using the hierarchical Bayesian method described above, we can determine at 97.5% confidence that $\mu(q)$ is a decreasing function of q, which means that the larger the mass ratio, the greater the probability that the binary black hole has an effective spin greater than 0. Figure 16 [Figure 16: see original paper] shows the posterior distribution of q and χ_{eff}.

5 Summary and Outlook

The discovery of gravitational waves has greatly expanded human understanding and knowledge of compact objects in the universe. The detection of a large number of stellar binary black hole mergers through gravitational waves remains an unsolved mystery regarding how they form and evolve to their final merger. Various formation mechanisms for binary black holes have been proposed, each involving numerous astrophysical processes with significant uncertainties in our current understanding. Nevertheless, it is widely believed that binary black holes produced by different mechanisms have distinct distributions of physical parameters such as mass and spin. Ground-based gravitational wave detectors like LIGO/Virgo, with their relatively precise spin measurements, provide an excellent means to constrain different formation mechanisms of binary black holes. Bayesian methods based on spin analysis, using the existing GWTC dataset, have already achieved some preliminary progress, such as discovering non-parallel phenomena between spin and orbital angular momentum in binary black hole samples, finding that spin is almost independent of binary black hole mass but anti-correlated with mass ratio. These findings have profound significance for constraining their origins, particularly for distinguishing between the massive binary star evolution mechanism (where the spins of the two components tend to be parallel) and the dynamical interaction mechanism (where spins tend to be randomly distributed).

As gravitational wave detection continues, we will measure the spin distribution of stellar binary black holes more accurately. In particular, future third-generation ground-based gravitational wave detectors such as the Einstein Telescope (ET) and Cosmic Explorer (CE) will be gradually built in the 2030s. Their extremely high detection sensitivity will greatly expand the gravitational wave detection sample of stellar binary black holes and improve the precision of physical parameter measurements. With the continuous accumulation of data, statistical analysis of binary black hole spins will help us ultimately reveal the origins of stellar binary black holes, potentially constraining numerous important physical processes in black hole formation and uncovering the origin of black hole spins.

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References

[1] Abbott B P, Abbott R, Abbott T D, et al. Phys Rev Lett, 2016, 116(6): 061102
[2] Abbott B P, Abbott R, Abbott T D, et al. Phys Rev Lett, 2016, 116(24):

241102
[3] Abbott B P, Abbott T D, et al. Phys Rev D, 2016, 93(12): 122003
[4] Abbott B P, Abbott R, Abbott T D, et al. ApJ, 2016, 818(2): L22
[5] Abbott B P, Abbott R, Abbott T D, et al. Phys Rev Lett, 2017, 119(16): 161101
[6] The LIGO Scientific Collaboration, the Virgo Collaboration, the KAGRA Collaboration, et al. arXiv e-prints, 2021: arXiv:2111.03606
[7] Farr W M, Stevenson S, Miller M C, et al. Nature, 2017, 548(7667): 426
[8] Mandel I, de Mink S E. MNRAS, 2016, 458(3): 2634
[9] Belczynski K, Ryu T, Perna R, et al. MNRAS, 2017, 471(4): 4702
[10] Abbott B P, Abbott R, Abbott T D, et al. Physical Review X, 2016, 6(4): 041015
[11] Abbott R, Abbott T D, Abraham S, et al. Physical Review X, 2021, 11(2): 021053
[12] Heger A, Fryer C L, Woosley S E, et al. ApJ, 2003, 591(1): 288
[13] O' Connor E, Ott C D. ApJ, 2011, 730(2): 70
[14] Özel F, Psaltis D, Narayan R, et al. ApJ, 2010, 725(2): 1918
[15] Li G P, Lin D B, Yuan Y. Phys Rev D, 2023, 107(6): 063007
[16] Zhang S N, Cui W, Chen W. ApJ, 1997, 482(2): L155
[17] Tanaka Y, Nandra K, Fabian A C, et al. Nature, 1995, 375(6533): 659
[18] Gou L, McClintock J E, Reid M J, et al. ApJ, 2011, 742(2): 85
[19] Uttley P, Cackett E M, Fabian A C, et al. A&A Rev, 2014, 22: 72
[20] McKinney J C, Tchekhovskoy A, Blandford R D. MNRAS, 2012, 423(4): 3083
[21] McClintock J E, Narayan R, Steiner J F. Space Sci Rev, 2014, 183(1-4): 295
[22] Zhang X, Lu Y. ApJ, 2019, 873(2): 101
[23] Abbott R, Abbott T D, Abraham S, et al. ApJ, 2020, 900(1): L13
[24] Belczynski K, Klencki J, Fields C E, et al. A&A, 2020, 636: A104
[25] Belczynski K, Heger A, Gladysz W, et al. A&A, 2016, 594: A97
[26] Portegies Zwart S F, McMillan S L W, Gieles M. ARA&A, 2010, 48: 431
[27] Downing J M B, Benacquista M J, Giersz M, et al. MNRAS, 2010, 407(3): 1946
[28] Ziosi B M, Mapelli M, Branchesi M, et al. MNRAS, 2014, 441(4): 3703
[29] Rodriguez C L, Morscher M, Pattabiraman B, et al. Phys Rev Lett, 2015, 115(5): 051101
[30] McKernan B, Ford K E S, Lyra W, et al. MNRAS, 2012, 425(1): 460
[31] Sigurdsson, Steinn, Hernquist, Lars. Nature, 1993, 364: 423
[32] Stone N C, Metzger B D, Haiman Z. MNRAS, 2017, 464(1): 946
[33] Mapelli M. Frontiers in Astronomy and Space Sciences, 2020, 7: 38
[34] de Mink S E, Mandel I. MNRAS, 2016, 460(4): 3545
[35] Belczynski K, Heger A, Gladysz W, et al. A&A, 2016, 594: A97
[36] Rodriguez C L, Haster C J, Chatterjee S, et al. ApJ, 2016, 824(1): L8
[37] Antonini F, Murray N, Mikkola S. ApJ, 2014, 781(1): 45
[38] Chen Z C, Huang Q G. ApJ, 2018, 864(1): 61
[39] Sasaki M, Suyama T, Tanaka T, et al. Phys Rev Lett, 2016, 117(6): 061101

[40] Abbott B P, Abbott R, Abbott T D, et al. Physical Review X, 2019, 9(3): 031040
[41] Abbott R, Abbott T D, Acernese F, et al. Physical Review X, 2023, 13(1): 011048
[42] Veitch J, Raymond V, Farr B, et al. Phys. Rev. D, 2015, 91(4): 042003
[43] Stevenson S, Berry C P L, Mandel I. MNRAS, 2017, 471(3): 2801
[44] Abbott R, Abbott T D, Abraham S, et al. ApJ, 2021, 913(1): L7
[45] Rodriguez C L, Zevin M, Pankow C, et al. ApJ, 2016, 832(1): L2
[46] Miller S, Callister T A, Farr W M. ApJ, 2020, 895(2): 128
[47] Qin Y, Fragos T, Meynet G, et al. A&A, 2018, 616: A28
[48] Fuller J, Ma L. ApJ, 2019, 881(1): L1
[49] Bavera S S, Fragos T, Qin Y, et al. A&A, 2020, 635: A97
[50] Rodriguez C L, Zevin M, Amaro-Seoane P, et al. Phys Rev D, 2019, 100(4): 043027

Note: Figure translations are in progress. See original paper for figures.