A Novel Imaging Method Based on Reweighted Total Variation for an Interferometer Array on Lunar Orbit Postprint

Authors: Xiaocheng Yang, Mengna Wang, Lin Wu, Jingye Yan, Junbao Zheng and Li Deng

Date: 2024-01-09T00:00:00+00:00

Abstract

Ground-based radio observations below 30 MHz are susceptible to the ionosphere of the Earth and the radio frequency interference. Compared with other space mission concepts, making low frequency observations using an interferometer array on lunar orbit is one of the most feasible ones due to a number of technical and economic advantages. Different from traditional interferometer arrays, the interferometer array on lunar orbit faces some complications such as the three-dimensional distribution of baselines and the changing sky blockage by the Moon. Although the brute-force method based on the linear mapping relationship between the visibilities and the sky temperature can produce satisfactory results in general, there are still large residual errors on account of the loss of the edge information. To obtain the full-sky maps with higher accuracy, in this paper we propose a novel imaging method based on reweighted total variation (RTV) for a lunar orbit interferometer array. Meanwhile, a split Bregman iteration method is introduced to optimize the proposed RTV model so as to decrease the computation time. The simulation results show that, compared with the traditional brute-force method, the RTV regularization method can effectively reduce the reconstruction errors and obtain more accurate sky maps, which proves the effectiveness of the proposed method.

Full Text

Preamble

Research in Astronomy and Astrophysics, 23:125017 (11pp), 2023 December

https://doi.org/10.1088/1674-4527/ad019d

A Novel Imaging Method Based on Reweighted Total Variation for an Interferometer Array on Lunar Orbit

Xiaocheng Yang[1,2], Mengna Wang[1], Lin Wu[2,3], Jingye Yan[2,3], Junbao Zheng[1], and Li Deng[3]

[1] School of Computer Science and Technology, Zhejiang Sci-Tech University, Hangzhou 310018, China

[2] State Key Laboratory of Space Weather, Chinese Academy of Sciences, Beijing 100190, China; yanjingye@nssc.ac.cn

[3] National Space Science Center, Chinese Academy of Sciences, Beijing 100190, China

Received 2023 August 2; revised 2023 September 20; accepted 2023 September 22; published 2023 November 15

Abstract

Ground-based radio observations below 30 MHz are susceptible to Earth's ionosphere and radio frequency interference. Among various space mission concepts, using an interferometer array on lunar orbit represents one of the most feasible approaches due to its technical and economic advantages. However, unlike traditional interferometer arrays, a lunar orbit interferometer array faces complications such as three-dimensional baseline distribution and time-varying sky blockage by the Moon.

Although the brute-force method based on the linear mapping relationship between visibilities and sky temperature can produce generally satisfactory results, large residual errors remain due to loss of edge information. To obtain full-sky maps with higher accuracy, this paper proposes a novel imaging method based on reweighted total variation (RTV) for a lunar orbit interferometer array. Additionally, a split Bregman iteration method is introduced to optimize the proposed RTV model and reduce computation time. Simulation results demonstrate that compared with the traditional brute-force method, the RTV regularization method can effectively reduce reconstruction errors and obtain more accurate sky maps, thereby proving the effectiveness of the proposed approach.

Key words: methods: data analysis – instrumentation: interferometers – techniques: interferometric – space vehicles: instruments – (cosmology:) dark ages – reionization – first stars

1. Introduction

At present, astronomical observations cover most of the electromagnetic spectrum from γ-ray to radio frequencies, with only one frequency band remaining largely unexplored: frequencies below 30 MHz (wavelengths greater than 10 m).

Ground-based radio observations at these low frequencies suffer from strong refraction and absorption by Earth's ionosphere, as well as severe interference from artificial radio frequency sources due to internal reflection within the ionosphere. Consequently, radio observations below 30 MHz are currently regarded as the final unexplored regime in radio astronomy.

Recent years have witnessed renewed interest in low-frequency radio astronomy, particularly motivated by the prospect of observing the redshifted 21 cm line from the cosmic dawn and dark ages (Pritchard & Loeb 2012; Liang et al. 2016). For the aforementioned reasons, space-based low-frequency radio observations are preferable to ground-based observations. From the late 1960s to the 1970s, several satellites such as IMP-6 (Brown 1973) and the RAE missions (Alexander & Novaco 1974; Alexander & Kaiser 1975) were launched to conduct low-frequency radio observations from space. However, due to technological limitations at the time, the resolution of the obtained sky maps was so poor that distinguishing individual celestial objects was difficult (Novaco & Brown 1978). Furthermore, the average spectrum of the whole sky measured by different satellites showed considerable discrepancies (Keshet et al. 2004). Note that low-frequency radio observations with satellites in Earth orbit are strongly affected by radio frequency interference (RFI) from Earth (Zhang et al. 2021). To overcome this challenge, conducting radio observations from the lunar farside surface or lunar orbit is highly desirable.

Although the lunar surface provides a stable platform and can use the Moon to block interference from Earth, it introduces a series of complex technical problems, including landing procedures, installation and deployment, power supply during the lunar night, and data transmission back to Earth (Cecconi et al. 2012; Mimoun et al. 2012; Chen et al. 2021). In contrast, making low-frequency observations using satellites on lunar orbit represents one of the most technically and economically feasible solutions currently available. On one hand, the Moon effectively blocks interference from Earth, and sometimes even from the Sun and planets. On the other hand, satellites orbiting the Moon with a period of about two hours can use solar cells for power. The satellite can transmit observation data back to Earth when the constellation is not shielded by the Moon, eliminating the need for a dedicated relay satellite. Since the 1980s, researchers have proposed numerous space mission concepts of this type, including the Dark Ages Radio Explorer (Plice et al. 2017), the Dark Ages Polarimetry PathfindER (Tauscher et al. 2018), Distributed Aperture Array for Radio Astronomy in Space (Boonstra et al. 2010), and Discovering the Sky at the Longest wavelength (DSL; Boonstra et al. 2016; Chen et al. 2021).

The DSL concept consists of a larger mother satellite and several smaller daughter satellites forming a linear array orbiting the Moon in nearly the same circular orbit. This mission aims to perform interferometric imaging to obtain sky maps below 30 MHz and high-precision global spectral measurements in the frequency range of interest. All daughter satellites are equipped with electrically short antennas for interferometric observations of the sky while on the lunar farside. The mother satellite receives data from the daughter satellites for cross-correlation, stores the resulting visibilities, and transmits them back to Earth when the constellation is on the lunar nearside (the side facing Earth). It is worth noting that the imaging problem for a lunar orbit interferometer array differs from those of previous ground-based interferometer arrays and existing space-ground interferometers such as HALCA (Frey et al. 2000) or RadioAstron (Kardashev et al. 2013). The main complications for low-frequency interferometer arrays on lunar orbit include the whole-sky field of view (FOV) for short dipole antennas, the three-dimensional distribution of antenna array baselines, and the changing portion of the sky blocked by the Moon. Considering practical constraints, electrically short dipole antennas will be used, allowing the array to have a large FOV covering almost the entire sky. To ensure that the directional projection aperture and density are essentially uniform, the 3D distribution of baselines is generated through precession of the satellite orbital plane. Moreover, as the satellites orbit the Moon, the portion of the sky blocked by the Moon changes over time.

Despite these complications, as long as the visibility data are linearly related to the sky brightness distribution, the interferometric imaging problem for lunar orbit can be regarded as a general linear inversion problem. The brute-force mapmaking method, which has been used in the ground-based MITEoR experiment to obtain satisfactory inverse results (Zheng et al. 2017), has been applied to solve this inversion problem (Huang et al. 2018; Shi et al. 2022). However, while the brute-force method produces generally satisfactory results, it suffers from two main disadvantages: the lack of a universal method for selecting the optimal regularization parameter, and the loss of edge information, which leads to large residual errors.

In recent years, total variation (TV) regularization has attracted increasing attention due to its desirable properties such as convexity and ability to preserve sharp edges (Rudin et al. 1992; Vogel & Oman 1998). The TV method has proven to be a very effective and efficient algorithm across a wide range of fields, including image denoising (Yuan et al. 2012), image destriping (Chang et al. 2013, 2015), computed tomography reconstruction (Tian et al. 2011), and reconstruction in synthetic aperture imaging radiometry (Yang et al. 2021). Inspired by these successes, this paper proposes a reweighted total variation (RTV) method based on split Bregman iteration to solve the inverse problem for a lunar orbit interferometer array.

This paper is organized as follows. Section 2 briefly introduces the general formalism for a lunar orbit interferometer array and the brute-force mapmaking algorithm. Section 3 presents the RTV algorithm based on split Bregman iteration. Section 4 provides numerical simulations to illustrate and validate the effectiveness of the proposed method. Finally, Section 5 draws conclusions.

2.1. Principle of Interferometric Imaging

By measuring the complex correlation between signals collected by two spatially separated antennas, interferometers yield the visibility, which can be expressed as

$$V_{ij} = \int_{S^2} \frac{T_k(\hat{s}) A_{kij}(\hat{s})}{\sqrt{\Omega_i \Omega_j}} e^{2\pi i \mathbf{r}_{ij} \cdot \hat{s}/\lambda} d\Omega$$

where $T_k(\hat{s})$ denotes the sky temperature, $A_{kij}(\hat{s})$ represents the combined antenna beam pattern related to the antenna primary beam response, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ denotes the baseline vector between the ith and jth antennas, and $\hat{s}$ represents the wave vector direction pointing toward the observer (Thompson et al. 2017).

By implementing a coordinate transformation from $\hat{s}$ on the spherical surface to its projection on the coordinate axes (l, m, n), Equation (1) can be rewritten as

$$V_{ij} = \int\int T_k(l, m) A_{kij}(l, m) e^{2\pi i [ul + vm + w(n-1)]} \frac{dldm}{n}$$

where (u, v, w) are the components of the vector $\mathbf{r}_{ij}$ in units of the radiation wavelength, and (l, m, n) are the direction cosines relative to the coordinate axes with $l^2 + m^2 + n^2 = 1$.

For ground-based interferometer arrays with small FOVs, the magnitude of the term $w(n-1)$ is much less than 1, which reduces Equation (2) to an ordinary two-dimensional Fourier transform. Consequently, the sky temperature can be reconstructed by a simple inverse Fourier transform. For ground-based interferometer arrays with large FOVs, although the w-term can no longer be ignored, we can still employ various wide-field imaging algorithms for reconstruction, such as 3D Fourier transform (Perley et al. 1989; Cornwell & Perley 1992), faceting (Cornwell & Perley 1992), W-Projection (Cornwell et al. 2008), and W-Stacking (McKinley et al. 2015).

However, these imaging algorithms are not suitable for low-frequency interferometer arrays on lunar orbit. The array of short dipole antennas at low frequencies produces a large FOV covering almost the entire sky, rendering the two-dimensional Fourier transform method inapplicable. Furthermore, to ensure that the directional projection aperture and density are essentially uniform, the 3D distribution of baselines is produced by precession of the orbital plane, requiring an effective 3D algorithm to image the data. Additionally, the effective fraction of the sky blocked varies as the satellites orbit the Moon, making conventional wide-field imaging algorithms inapplicable. In conclusion, conventional algorithms cannot solve the interferometric imaging problem for lunar orbit arrays.

After considering Moon blockage, Equation (1) transforms into the following form (Huang et al. 2018):

$$V_{ij} = \int_{S^2} S_{ij}(\hat{s}) \frac{T_k(\hat{s}) A_{kij}(\hat{s})}{\sqrt{\Omega_i \Omega_j}} e^{2\pi i \mathbf{r}_{ij} \cdot \hat{s}/\lambda} d\Omega$$

where S_{ij} denotes the shade function describing the time-dependent positions in the sky blocked by the Moon.

2.2. Brute-force Mapmaking Algorithm

Despite the complexities described above, the visibility data obtained by a lunar orbit interferometer array are linearly related to the sky brightness distribution. The brute-force method can therefore be used to reconstruct the sky brightness temperature from measured visibilities.

The integral over sky angles is discretized into a sum over sky pixels, allowing Equation (3) to be expressed as

$$V_{ij} = \sum_{p=1}^{N_{\text{pix}}} B_{ij}(n_p) T(n_p) \Delta\Omega$$

where N_{pix} denotes the number of pixels, $\Delta\Omega$ represents the angular size corresponding to a pixel, and $T(n_p)$ is the discrete sky map using the HEALPix scheme (Hansen et al. 2006). Moreover, $B_{ij}(n_p)$ denotes the discrete complex response of the array.

After including measurement noise, Equation (4) can be rewritten in matrix form as

$$\mathbf{V} = \mathbf{B}\mathbf{T} + \mathbf{n}$$

where $\mathbf{V}$ denotes the vector of measured visibilities with dimension $(N_{\text{bl}} \cdot N_t)$, with N_{bl} and N_t representing the number of baselines and observation time points, respectively. Additionally, $\mathbf{T}$ is the vector of the sky map with dimension N_{pix}, $\mathbf{B}$ represents the discrete modeling matrix with dimension $(N_{\text{bl}} \cdot N_t) \times N_{\text{pix}}$, and the noise vector $\mathbf{n}$ has dimension $(N_{\text{bl}} \cdot N_t)$.

In this paper, the noise is modeled as Gaussian noise with zero mean and variance δ^2. Assuming the noise covariance is $\langle \mathbf{n}\mathbf{n}^\dagger \rangle$ (where † denotes the Hermitian conjugate operator), the minimum variance estimator of the sky map is given by

$$\hat{\mathbf{T}} = (\mathbf{B}^\dagger \mathbf{N}^{-1} \mathbf{B})^{-1} \mathbf{B}^\dagger \mathbf{N}^{-1} \mathbf{V}$$

Assuming uniform noise, $\mathbf{N}$ is proportional to the identity matrix $\mathbf{I}$. To solve the inverse problem efficiently, we introduce the singular value decomposition (SVD) of matrix $\mathbf{B}$:

$$\mathbf{B} = \sum_{i=1}^{\min(N_{\mathrm{bl}} \cdot N_t, N_{\mathrm{pix}})} \sigma_i \alpha_i \beta_i^{\dagger}$$

where α_i and β_i are the left and right singular vectors, respectively, and σ_i is the singular value of matrix $\mathbf{B}$ in descending order. Since very small singular values significantly impact error propagation during reconstruction, it is preferable to compute the Moore-Penrose pseudoinverse of matrix $\mathbf{B}$ using truncated singular value decomposition (Xu 1998):

$$\mathbf{B}^{+} = \sum_{i=1}^{m} \frac{1}{\sigma_i} \beta_i \alpha_i^{\dagger}$$

where m represents the number of retained singular values, namely the regularization parameter.

2.3. The Regularization Parameter

The selection of the regularization parameter m is crucial for the performance of the brute-force mapmaking method. Traditional methods estimate the regularization parameter based on a threshold ratio (Huang et al. 2018). Although this can lead to a stable solution, the threshold ratio value is usually set empirically, making it susceptible to the personal experience of the estimator and less precise.

In practice, the choice of regularization parameter is related to the noise level. However, noise information is typically unknown in real systems. When noise characteristics cannot be determined, generalized cross-validation (GCV) provides an effective approach for computing regularization parameters (Golub & Matt 1997). The fundamental idea of GCV is that when a component is removed from the original data (i.e., an equation is removed from the original system), the new system should predict the removed component well. To select an appropriate regularization parameter, the following function must be minimized:

$$G(m) = \frac{\|(\mathbf{I} - \mathbf{B}\mathbf{B}^{+})\mathbf{V}\|^2}{[\mathrm{tr}(\mathbf{I} - \mathbf{B}\mathbf{B}^{+})]^2}$$

where $\mathbf{I}$ is the identity matrix and $\mathrm{tr}(\cdot)$ denotes the trace of a square matrix. Consequently, this paper utilizes the GCV method to select the regularization parameter for the brute-force approach.

3. RTV Regularization

The brute-force mapmaking method described above minimizes the L_2 norm in Hilbert space. TV regularization better preserves edge information compared to L_2 norm regularization methods. However, due to the piecewise constant assumption for images, traditional TV approaches often suffer from oversmoothness at image edges. Consequently, we introduce an RTV regularization method to address this limitation.

According to regularization theory, the inverse problem in Equation (5) can be solved by imposing constraints on the map:

$$\arg\min_{\mathbf{T}} \frac{1}{2}\|\mathbf{V} - \mathbf{B}\mathbf{T}\|_2^2 + \lambda R(\mathbf{T})$$

where the first term represents the data fidelity term ensuring similarity between the desired and reconstructed maps, the second term denotes the regularization term, and λ is the regularization parameter controlling the tradeoff between data fidelity and regularization. For RTV regularization, the regularization term can be expressed as

$$R(\mathbf{T}) = \|\mathbf{W} \circ \nabla\mathbf{T}\|_1 = \|\mathbf{W} \circ (\mathbf{L}_x\mathbf{T})\|_1 + \|\mathbf{W} \circ (\mathbf{L}_y\mathbf{T})\|_1$$

where $\circ$ represents the Hadamard product, ∇ denotes the gradient operation, and matrix $\mathbf{L}$ represents first-order finite-difference operators. Moreover, $\mathbf{W}$ denotes a weight vector with dimension N_{pix} defined as

$$W_i = \frac{1}{|(\nabla\mathbf{T})_i| + \tau}$$

where subscript i denotes the pixel index in the map and $\tau > 0$ is a constant parameter for adjusting the weight (set to 1 in this paper). Compared with conventional TV regularization, RTV regularization can effectively reduce oversmoothness and improve inverse result accuracy by assigning different weights to gradient values at different map locations.

The split Bregman iteration algorithm (Cai et al. 2010; Goldstein & Osher 2009) is an efficient tool for solving l_1-norm regularizations. Compared with conventional optimization methods, split Bregman offers advantages of good numerical stability, fast convergence speed, and reduced memory usage, making it attractive for large-scale problems. Consequently, we introduce the split Bregman algorithm to optimize the RTV model.

We introduce an auxiliary variable $\mathbf{d} = \mathbf{LT}$, transforming Equation (10) into the constrained problem:

$$\arg\min_{\mathbf{T},\mathbf{d}} \frac{1}{2}\|\mathbf{V}-\mathbf{B}\mathbf{T}\|_2^2 + \lambda\|\mathbf{W}\circ\mathbf{d}\|_1 \quad \text{s.t.} \quad \mathbf{d}=\mathbf{L}\mathbf{T}$$

Using Bregman iteration (Goldstein & Osher 2009), Equation (13) can be converted to the following unconstrained optimization problem:

$$\arg\min_{\mathbf{T},\mathbf{d}} \frac{1}{2}\|\mathbf{V}-\mathbf{B}\mathbf{T}\|_2^2 + \lambda\|\mathbf{W}\circ\mathbf{d}\|_1 + \frac{\mu}{2}\|\mathbf{d}-\mathbf{L}\mathbf{T}-\mathbf{b}\|_2^2$$

where μ is the penalty parameter and $\mathbf{b}$ is an auxiliary vector for accelerating iterations. The minimization of Equation (14) with respect to $\mathbf{T}$ and $\mathbf{d}$ can be decoupled and split into two separate subproblems:

$$\arg\min_{\mathbf{T}} \frac{1}{2}\|\mathbf{V}-\mathbf{B}\mathbf{T}\|_2^2 + \frac{\mu}{2}\|\mathbf{d}^k-\mathbf{L}\mathbf{T}-\mathbf{b}^k\|_2^2$$

$$\arg\min_{\mathbf{d}} \lambda\|\mathbf{W}\circ\mathbf{d}\|_1 + \frac{\mu}{2}\|\mathbf{d}-\mathbf{L}\mathbf{T}^{k+1}-\mathbf{b}^k\|_2^2$$

The $\mathbf{T}$-related subproblem in Equation (15) is a differentiable optimization problem that can be directly solved by

$$(\mathbf{B}^\dagger\mathbf{B}+\mu\mathbf{L}^\dagger\mathbf{L})\mathbf{T}^{k+1} = \mathbf{B}^\dagger\mathbf{V}+\mu\mathbf{L}^\dagger(\mathbf{d}^k-\mathbf{b}^k)$$

Using the soft thresholding method (Donoho 1995; Liu et al. 2019), the $\mathbf{d}$-related subproblem in Equation (15) can be solved as

$$\mathbf{d}^{k+1} = \text{shrink}(\mathbf{L}\mathbf{T}^{k+1}+\mathbf{b}^k, \lambda\mathbf{W}/\mu)$$

where $\text{shrink}(\mathbf{x},\mathbf{y})$ denotes the shrinkage operator defined as $\text{sgn}(\mathbf{x})\max(|\mathbf{x}|-\mathbf{y},0)$, which requires only element-wise operations and is computationally efficient. Subsequently, we update the Bregman variable $\mathbf{b}^{k+1}$ by

$$\mathbf{b}^{k+1} = \mathbf{b}^k + \mathbf{L}\mathbf{T}^{k+1} - \mathbf{d}^{k+1}$$

Finally, we select and update parameters λ and μ adaptively during iteration. The penalty parameter μ affects the algorithm's convergence rate. The basic principle for updating μ is that when the constraint violation $\|\mathbf{d}-\mathbf{L}\mathbf{T}\|$ is larger, μ should be larger to promote convergence (Wang et al. 2012; Yang et al. 2021). Therefore, μ is updated as

$$\mu^{k+1} = \min(\mu_{\max}, \eta\mu^k)$$

where $\eta > 1$ is the step constant (set to 1.5 in this paper). Moreover, the regularization parameter λ can be updated by

$$\lambda^{k+1} = \frac{\|\mathbf{V} - \mathbf{B}\mathbf{T}^{k+1}\|_2}{\|\mathbf{W} \circ \nabla \mathbf{T}^{k+1}\|_1} \cdot \mathrm{mean}(\mathbf{W})$$

where mean$(\cdot)$ denotes the mean operator.

The split Bregman approach offers the advantage of splitting a difficult optimization problem into two easily solvable subproblems. Table 1 summarizes the algorithm procedure for the RTV method based on split Bregman iteration.

4. Simulation Results

This section presents numerical simulations to discuss and compare the performance of the RTV regularization method with the traditional brute-force algorithm developed by Huang et al. (2018). Specifically, we perform full-sky reconstruction, part-sky reconstruction, and analyze the impact of satellite failures.

4.1. Input Sky Map

Currently, the radio sky below 30 MHz is poorly characterized. Interpolation and extrapolation of available data are generally used to generate sky maps at frequencies lacking survey data. Since sky intensities follow an approximate power-law spectrum in most radio bands, extrapolation is relatively straightforward and widely employed. Researchers have developed several extrapolated sky models, including the Global Sky Model (de Oliveira-Costa et al. 2008; Rao et al. 2017; Zheng et al. 2017; Kim et al. 2018) and the Self-consistent Sky Model (Huang et al. 2019). However, below 10 MHz, extrapolation becomes very difficult and complicated due to the lack of observational data and absorption by the interstellar medium. To provide a reasonable prediction of sky maps below 10 MHz, the Ultralong-wavelength Sky Model with Absorption (ULSA) developed by Cong et al. (2021) incorporates free-free absorption effects by thermal electrons. Consequently, we generate the input sky map using the ULSA model in this paper.

In the simulations, the sky map is pixelated using HEALPix (Hansen et al. 2006) with $N_{\mathrm{side}} = 64$, corresponding to 1° pixel size. Figure 1 shows the input full-sky maps on a logarithmic scale at 3 MHz and 10 MHz, respectively.

4.2. Simulation System

In our simulation, the DSL mission comprises a larger mother satellite and eight smaller daughter satellites flying around the Moon in the same circular orbit at 300 km altitude, which is stable enough to avoid the lunar surface despite the irregularity of the Moon's gravitational field. The daughter satellites, equipped with short dipole antennas, form a linear interferometer array with a minimum baseline of 1 km and a maximum baseline of 100 km. The orbital plane precesses with an inclination angle of 30° to generate the 3D distribution of baselines, completing a 360° precession in 1.29 years. Table 2 lists the specific parameters of the DSL system used in the simulation.

Over one precession period, the 3D distribution of cumulative baselines is shown in the left panel of Figure 2. Note that the Moon shields a fraction of the sky as the satellites orbit. The right panel of Figure 2 shows the 3D distribution of baselines after accounting for blockage by the Moon.

To quantitatively analyze the performance of imaging methods, we calculate the Root Mean Squared Error (RMSE) and R-squared (RS) values. The RMSE is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{N_{\mathrm{pix}}}\sum_{i=1}^{N_{\mathrm{pix}}}[T(i) - \hat{T}(i)]^2}$$

where $T(i)$ and $\hat{T}(i)$ denote the original and reconstructed brightness temperatures, respectively. Based on the magnitude of sky brightness temperatures, the RMSE unit is set to $\log_{10}$[K] for convenient comparison. The RS is defined as

$$\mathrm{RS} = 1 - \frac{\sum_{i=1}^{N_{\mathrm{pix}}}[T(i) - \hat{T}(i)]^2}{\sum_{i=1}^{N_{\mathrm{pix}}}[T(i) - \bar{T}]^2}$$

where $\bar{T}$ represents the mean value of original brightness temperatures. RS values range from $(-\infty, 1]$, with values closer to 1 indicating better agreement between reconstructed and actual maps.

4.3. Full-sky Reconstruction

To reduce computation time in our first experiment, we randomly select 3×10^4 points from the total baselines shown in the right panel of Figure 2, which is sufficient to reconstruct a high-quality sky map at the given angular resolution. According to Equation (5), measured visibilities are generated by performing forward simulations on the original full-sky maps shown in Figure 1, with noise simulated as Gaussian noise with zero mean and variance δ^2.

With additive Gaussian noise of variance $\delta^2 = 4$ (see Equation (35) and Table 4 in Shi et al. 2022), Figures 3(a)-(b) depict the reconstructed full-sky maps and relative error maps at 3 MHz and 10 MHz using the brute-force method. The relative error map is defined as the difference between the original and reconstructed maps. These results show that the brute-force method loses considerable detailed information and exhibits obvious ripples. In contrast, Figure

4 shows full-sky maps reconstructed using the RTV method. The RTV method produces satisfactory results that match the actual maps well. Compared to the brute-force method, RTV results are visually superior, with smoother reconstructions, clearer edges, and better preservation of details.

To quantitatively evaluate the performance of both approaches, we calculate RMSE and RS values, listed in Table 3. The results demonstrate that the RTV approach markedly outperforms the brute-force approach, showing significantly lower RMSE and higher RS values. Thus, the RTV approach more effectively reduces reconstruction errors.

We also compare the computational speed of both algorithms. As an iterative method, the RTV algorithm's computation time depends primarily on the size of modeling matrix $\mathbf{B}$ and the number of iteration steps. For the reconstructed maps shown above, the MATLAB runtime (MATLAB R2020b on a PC with Intel Core i9-9900k processor and 64 GB RAM) is 4.703 hours for the brute-force algorithm, whereas the RTV algorithm requires only 1.307 hours when $\mathbf{B}$ has size $30,000 \times 49,152$ and the number of iterations is 10. This reveals that the RTV algorithm is substantially faster than the brute-force algorithm.

Figure 5 shows the computation time for both methods with different numbers of baselines. As the number of baselines increases, the brute-force method's computation time rises rapidly, while the RTV method's time increases slowly. This occurs because the brute-force method's computation time is dominated by the SVD of matrix $\mathbf{B}$, whose time complexity grows significantly with matrix size. In contrast, the RTV method's computation time is dominated by the iterative solution process, which is accelerated by the split Bregman algorithm. Consequently, from a computational speed perspective, the RTV method performs better than the brute-force method for large numbers of baselines.

We further analyze the impact of noise on reconstruction performance. The full-sky map at 10 MHz generated by the ULSA model (Cong et al. 2021) is selected as the input sky map (right panel of Figure 1). Measured visibilities corrupted by zero-mean Gaussian noise with different variances are used to retrieve sky maps. Figure 6 shows RMSE and RS values for both methods under various noise levels. The RTV approach demonstrates evident RMSE reduction and apparent RS improvement over the brute-force approach across all noise levels. Specifically, the RTV approach achieves more than 60% RMSE reduction and over 16% RS improvement relative to the nominal brute-force approach. This indicates that the RTV approach is more robust to noise interference than the brute-force approach.

4.4. Part-sky Reconstruction

To further illustrate the comparison between the two approaches, we conduct a second experiment limited to a small portion of the full-sky map. Because full-sky maps contain huge numbers of pixels, the computational load required for retrieving the entire map is substantial. To immensely reduce computational load, part-sky reconstruction has been introduced and proven feasible in Shi et al. (2022). Here, we apply part-sky reconstruction to test and verify the effectiveness of the RTV approach.

From the full-sky map with 1° resolution at 10 MHz, we select a rectangular area (R.A.: $[65^\circ, 25^\circ]$, decl.: $[-35^\circ, 0^\circ]$) as the original part-sky map, shown in Figure 7. Under the same observation conditions as the full-sky reconstruction in Section 4.2, Figure 8 shows the retrieved part-sky maps via the brute-force and RTV approaches. The brute-force algorithm produces reconstruction results with obvious oscillation ripples. In contrast, the RTV algorithm yields excellent inversion results with weaker oscillation ripples, showing better agreement with the reference part-sky map.

RMSE and RS values for the retrieved part-sky maps are calculated for quantitative error analysis. The RMSE values for the brute-force and RTV approaches are 0.156 and 0.053 $\log_{10}$[K], respectively, while the RS values are 0.808 and 0.978, respectively. These results reveal that the RTV approach achieves better performance compared to the brute-force approach, demonstrating its ability to improve reconstruction accuracy.

4.5. Impact of Satellite Failures

This subsection considers reconstruction accuracy under situations with missing visibility data due to satellite failures. In practice, such hardware failures are often unpredictable and may occur (Yan et al. 2023). The simulation system parameters are as described in Section 4.2, but we consider satellite failures occurring after the system operates healthily for two months. The cumulative baseline distribution over two months is shown in Figure 9, containing 47,100 points.

The input map is shown in the right panel of Figure 1. With additive Gaussian noise variance $\delta^2 = 4$, Figure 10 presents relative error maps obtained via the brute-force and RTV methods. The results reveal large reconstruction errors for maps reconstructed by the brute-force method, whereas reconstruction errors for the RTV method are considerably diminished.

RMSE and RS values are calculated to quantitatively appraise reconstruction performance. The brute-force method yields RMSE and RS values of 0.124 $\log_{10}$[K] and 0.592, respectively. In contrast, the RTV method achieves RMSE and RS values of 0.044 $\log_{10}$[K] and 0.949, respectively. These results demonstrate that the RTV method can effectively diminish reconstruction errors even in cases of satellite failures.

5. Conclusions

Currently, low-frequency radio observations below 30 MHz are considered the final unexplored window in radio astronomy. Ground-based observations below 30 MHz are hindered by ionospheric refraction and absorption, as well as RFI.

Consequently, space-based low-frequency observations have become a research hotspot. Among various space mission concepts, an interferometer array on lunar orbit represents one of the most technically and economically feasible options.

Unlike traditional interferometer arrays, a lunar orbit interferometer array faces complications including the whole-sky FOV for short dipole antennas, 3D baseline distribution, and time-varying sky blockage by the Moon. Despite these complexities, the relationship between visibilities and sky brightness temperature can be expressed as linear mapping equations. The brute-force method can solve this linear inversion problem to obtain sky maps, but it cannot preserve edge information well, resulting in large residual errors.

To obtain full-sky maps with higher accuracy, we propose a novel RTV approach to solve the inverse problem for lunar orbit interferometer arrays. Additionally, we introduce the split Bregman iteration method to optimize the RTV regularization model and increase reconstruction speed. Simulation results demonstrate that compared with the traditional brute-force method, the RTV regularization method more effectively reduces reconstruction errors and obtains more accurate full-sky maps, proving the effectiveness of the proposed method.

The presented simulations neglect some practical issues such as calibration errors and baseline measurement errors. In future work, we plan to conduct end-to-end simulations using all baselines and input full-sky maps at nominal resolution while considering these practical issues.

Acknowledgments

This work is supported by the Specialized Research Fund for State Key Laboratories under Grant 202217 and the Key Laboratory of Solar Activity and Space Weather (NSSC) under Grant E26600021S.

References

Alexander, J. K., & Kaiser, M. L. 1975, A&A, 40, 365 Alexander, J. K., & Novaco, J. C. 1974, AJ, 79, 777 Boonstra, A.-J., Garrett, M., Kruithof, G., et al. 2016, Discovering the sky at the Longest Wavelengths (DSL), in 2016 IEEE Aerospace Conf. (Big Sky, MT: IEEE), 1 Boonstra, A.-J., Saks, N., Bentum, M. J., et al. 2010, A Low-Frequency Distributed Aperture Array for Radio Astronomy in Space, in 38th COSPAR Scientific Assembly (Bremen: COSPAR), 11 Brown, L. W. 1973, ApJ, 180, 359 Cai, J.-F., Osher, S., & Shen, Z. 2010, Multiscale Model Sim, 8, 337 Cecconi, B., Hess, S., Hérique, A., et al. 2012, P&SS, 61, 32 Chang, Y., Fang, H., Yan, L., & Liu, H. 2013, OExpr, 21, 23307 Chang, Y., Yan, L., Fang, H., & Luo, C. 2015, ITIP, 24, 1852 Chen, X., Yan, J., Deng, L., et al. 2021, RSPTA, 379, 20190566 Cong, Y., Yue, B., Xu, Y., et al. 2021, ApJ, 914, 128 Cornwell, T., & Perley, R. 1992, A&A, 261, 353 Cornwell, T. J., Golap, K., & Bhatnagar, S. 2008, ISTSP, 2, 647 de Oliveira-Costa, A., Tegmark, M., Gaensler, B., et al. 2008, MNRAS, 388, 247 Donoho, D. L. 1995, ITIT, 41, 613 Frey, S., Gurvits, L. I., Altschuler, D. R., et al. 2000, PASJ, 52, 975 Goldstein, T., & Osher, S. 2009, SIIMS, 2, 323 Golub, G. H., & Matt, U. V. 1997, J. Comput. Graph. Stat., 6, 1 Hansen, F. K., Banday, A., Eriksen, H., Górski, K., & Lilje, P. 2006, ApJ, 648, 784 Huang, Q., Sun, S., Zuo, S., et al. 2018, AJ, 156, 43 Huang, Q., Wu, F., & Chen, X. 2019, SCPMA, 62, 989511 Kardashev, N., Khartov, V., Abramov, V., et al. 2013, ARep, 57, 153 Keshet, U., Waxman, E., & Loeb, A. 2004, ApJ, 617, 281 Kim, D., Liu, A., & Switzer, E. 2018, AAS, 231, 153.09 Liang, J. M., Mao, X. C., & Qin, B. 2016, RAA, 16, 132 Liu, D., Li, S., & Shen, Y. 2019, ITIT, 65, 6650 McKinley, B., Yang, R., López-Caniego, M., et al. 2015, MNRAS, 446, 3479 Mimoun, D., Wieczorek, M. A., Alkalai, L., et al. 2012, ExA, 33, 529 Novaco, J. C., & Brown, L. W. 1978, ApJ, 221, 114 Perley, R. A., Schwab, F. R., & Bridle, A. H. 1989, Synthesis Imaging in Radio Astronomy, in ASP Conf. Ser. 6, Synthesis Imaging in Radio Astronomy: A Collection of Lectures from the Third NRAO Synthesis Imaging Summer School (San Francisco, CA: ASP) Plice, L., Galal, K., & Burns, J. O. 2017, arXiv:1702.00286 Pritchard, J., & Loeb, A. 2012, RPPh, 75, 086901 Rao, M. S., Subrahmanyan, R., Shankar, N. U., & Chluba, J. 2017, AJ, 153, 26 Rudin, L. I., Osher, S., & Fatemi, E. 1992, PhyD, 60, 259 Shi, Y., Xu, Y., Deng, L., et al. 2022, MNRAS, 510, 3046 Tauscher, K., Rapetti, D., & Burns, J. O. 2018, JCAP, 2018, 015 Thompson, A. R., Moran, J. M., & Swenson, G. W. 2017, Interferometry and Synthesis in Radio Astronomy (Berlin: Springer) Tian, Z., Jia, X., Yuan, K., Pan, T., & Jiang, S. B. 2011, PMB, 56, 5949 Vogel, C. R., & Oman, M. E. 1998, ITIP, 7, 813 Wang, J., Ma, J., Han, B., & Li, Q. 2012, SigPr, 92, 2952 Xu, P. 1998, GeoJI, 135, 505 Yan, J., Wu, J., Gurvits, L. I., et al. 2023, ExA, 56, 333 Yang, X., Lu, C., Yan, J., et al. 2022, IGRSL, 19, 3138859 Yuan, Q., Zhang, L., & Shen, H. 2012, ITGRS, 50, 3660 Zhang, Y. J., Li, Y. Z., Cheng, J., & Yan, Y. H. 2021, RAA, 21, 299 Zheng, H., Tegmark, M., Dillon, J. S., et al. 2017, MNRAS, 464, 3486

Note: Figure translations are in progress. See original paper for figures.