

Correction to the quantum relation of photons involved in the Doppler effect in the framework of a special Lorentz violation model

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Abstract

The possibility of Lorentz symmetry breaking has been discussed in many models of quantum gravity. In this paper we followed the Lorentz violation model in Ref. 14 to discuss the Doppler frequency shift of photons and the Compton scattering process between photons and electrons, pointing out that following the idea in Ref. 14 we have to modify the usual quantum relation of photons involved in Doppler effect. But due to the current limited information and knowledge, we couldn't yet determine the specific expression for the correction coefficient of the quantum relation of photons. However, the phenomenon of spontaneous radiation in a cyclotron maser give us an opportunity to see what the expression for this correction coefficient might look like, as the phenomenon of spontaneous radiation in a cyclotron maser can be explained by the Doppler effect of virtual photons and the Compton scattering process between virtual photons and electrons (or other particles). Therefore, under some restrictive conditions, we construct a very concise expression for this correction coefficient by discussing different cases. And then we used this expression to analyze the wavelength of radiation in the cyclotron maser, which tends to be a limited value at $v \rightarrow c$, rather than to be 0 as in the Lorentz model. And we also discussed the inverse Compton scattering phenomenon and found that there is a limit to the maximum energy that can be obtained by photons in the collision between ultra-high energy particles and low-energy photons, which conclusion is also very different from that obtained from the Lorentz model, in which the energy that can be obtained by the photon tends to be infinite as the velocity of particle is close to c . This paper still inherits the idea in Ref. 14 that the energy and momentum of particles (i.e., any particles, including photons) cannot be infinite, otherwise it will make some physical scenarios meaningless, and this view is also from the idea in some quantum gravity models. When the parameter Q characterizing the degree of deviation from the Lorentz model is equal to 0, all the results and conclusions in this paper will return to the case as in the Lorentz

model, so this paper also provides us with a possible experimental scheme to determine the value of Q in Ref. [14], although it still requires extremely high experimental energy.

Full Text

Correction to the Quantum Relation of Photons Involved in the Doppler Effect Within a Special Lorentz Violation Model

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Abstract

The possibility of Lorentz symmetry breaking has been discussed in many models of quantum gravity. In this paper, we follow the Lorentz violation model in Ref. [14] to discuss the Doppler frequency shift of photons and the Compton scattering process between photons and electrons, pointing out that this approach requires modification of the usual quantum relation of photons involved in the Doppler effect. However, due to currently limited information and knowledge, we cannot yet determine the specific expression for the correction coefficient of the quantum relation of photons. Fortunately, the phenomenon of spontaneous radiation in a cyclotron maser provides an opportunity to infer what this correction coefficient might look like, as this phenomenon can be explained by the Doppler effect of virtual photons and the Compton scattering process between virtual photons and electrons (or other particles). Therefore, under certain restrictive conditions, we construct a concise expression for this correction coefficient by examining different cases. Using this expression, we analyze the wavelength of radiation in the cyclotron maser, which tends to a limited value as $v \rightarrow c$, rather than approaching 0 as in the Lorentz model. We also discuss the inverse Compton scattering phenomenon and find that there exists a limit to the maximum energy that photons can obtain in collisions between ultra-high energy particles and low-energy photons—a conclusion that differs significantly from the Lorentz model, where the photon energy tends to infinity as the particle velocity approaches c . This paper inherits the idea from Ref. [14] that the energy and momentum of particles (including photons) cannot be infinite, as this would render certain physical scenarios meaningless—a view also present in some quantum gravity models. When the parameter Q characterizing the degree of deviation from the Lorentz model equals 0, all results and conclusions in this paper reduce to the Lorentz model case, thus providing a possible experimental scheme to determine the value of Q in Ref. [14], although this still requires extremely high experimental energies.

Key Words: Lorentz model; speed of light; Lorentz violation; Doppler effect; Compton scattering; inverse Compton scattering; quantum relation

1. Introduction

The study of quantum gravity has long been a frontier and hot topic in fundamental physics, primarily because black holes exhibit singularity and information loss problems, and general relativity appears incompatible with quantum theory under the current framework [1-4]. The starting point for many quantum gravity models is the belief that Lorentz symmetry may be violated at ultra-high energy scales, such as near the Planck energy scale.

The most typical model holding this view is Doubly Special Relativity (DSR), which considers not only the speed of light as constant but also introduces another constant called “minimum length or maximum energy,” leading to modification of the energy-momentum dispersion relationship of particles at extremely high energy scales [5,6]. Research on the DSR model appears extensively in the literature, where it is believed capable of solving some difficult problems in the ultra-high energy field [7-12]. However, the DSR model has an obvious disadvantage: it is not concise enough. The equation describing the dispersion relationship of particles usually contains many parameters—for example, at Planck energy scale the dispersion relationship is typically expressed with multiple parameters where the physical meaning of these parameters is not clear [13]. Consequently, many researchers only select one or two of these parameters for study.

For this reason, Ref. [14] proposed another possible Lorentz violation model containing only one parameter in the equation describing the dispersion relationship of particles. More importantly, this model naturally returns to the Lorentz model at low and medium energy scales, and when the particle velocity approaches c , the particle energy tends to a limited value (similar to the DSR model) rather than becoming infinite as predicted by the Lorentz model. To clarify the purpose and viewpoint of this paper, we briefly review the Lorentz violation model proposed in Ref. [14].

In Ref. [14], it is assumed that the speed of light observed by an observer moving with velocity v relative to the light source is nc , where n is a value independent of the direction of v and c , with $n(v=0, c)=1$. Based on this assumption, the coordinate transformation between two inertial systems moving relative to each other at velocity v is given by Eq. (1), where the value of n is independent of the direction of v and c . From Eq. (1), one can obtain that if $dx/dt=c$, then $dx'/dt'=nc$, and if $dx'/dt'=c$, then $dx/dt=nc$, which ensures the equivalence of the two inertial systems. Eq. (1) is similar in form to the Lorentz transformation, and Ref. [14] has shown that Maxwell's equations are also covariant based on Eq. (1). Correspondingly, the dispersion relation for a particle with rest mass m_0 is modified compared to the Lorentz model as shown in Eq. (2), where $E_0=m_0k^2$, $E=\gamma m_0k^2$ denotes the particle's total

energy, $p=\gamma m_0 v$ denotes the particle's momentum, and $\gamma(v,c)=1/\sqrt{(1-v^2/c^2)}$, $k(v,c)=\sqrt{[nvc^2/(nc-c+v)]}$.

At this point, one may notice a problem: if the dispersion relation of particles is modified as in Eq. (2), then the usual quantum relation of photons (see Eq. (3)) should also be modified correspondingly. The reason is that for massless particles, the dispersion relation becomes $E=pk$ rather than $E=pc$ based on Eq. (2). Secondly, Ref. [14] allows the speed of light to vary between inertial systems (especially allowing the observed speed of light to approach 0 when the speed of the light source relative to the observer approaches c), making the usual quantum relation of photons (i.e., Eq. (3)) inapplicable in this case, where h is the Planck constant, and f and λ denote the frequency and wavelength of light, respectively.

This is precisely the issue to be discussed in this paper. One might wonder why we need to modify quantum theory when it has been so successful. The answer is that our current quantum theory is based on Lorentz symmetry, and if the energy is high enough that Lorentz symmetry is obviously broken, quantum theory should be modified. Evidence for this includes various models of quantum gravity that aim to explore new physics emerging at very high energies. However, when attempting to construct or find a quantum relation of photons satisfying Eq. (2), we found it very difficult to determine the specific expression, as the set of expressions satisfying $E=pk$ is large. Fortunately, a physical phenomenon in cyclotron masers called synchrotron radiation can provide insight into what this expression might look like.

In Ref. [15-17], the authors discussed spontaneous radiation and synchrotron radiation in cyclotron masers using the concept of virtual photons. By considering the Doppler effect and Compton scattering of virtual photons, they obtained formulas for spontaneous radiation and synchrotron radiation that match classical theoretical results. Inspired by this idea, we first discuss the restrictive conditions for the modified quantum relation of photons, then attempt to find a possible correction expression satisfying Eq. (2), and further discuss the effect of this modified quantum relation on synchrotron radiation in cyclotron masers. This paper is organized as follows: Section 2 discusses the Doppler effect of photons based on Ref. [14]; Section 3 discusses the corresponding Compton scattering effect; Section 4 analyzes spontaneous and synchrotron radiation phenomena in a cyclotron maser to apply both effects simultaneously, while discussing possible correction coefficients for the quantum relation; Section 5 discusses inverse Compton scattering and points out differences between the Ref. [14] model and the Lorentz model; and finally we summarize the paper.

2. Doppler Effect of Photons

As is well known, the frequency of light emitted by a source differs when measured in a moving inertial system relative to the source—this is called the Doppler frequency shift effect. As shown in Fig. 1 [Figure 1: see original

paper], if inertial system S' moves along the $x(x')$ -axis with velocity v relative to inertial system S , and the frequency and wavelength of light emitted by the source fixed in S' are f_0 and λ_0 respectively, with θ being the angle between the direction of light propagation and the x' -axis, then based on the conclusion in Ref. [14], the clock period of the light source for the observer in S slows down according to Eq. (4), where $\beta=v/c$, T_0 and T represent the clock period measured in S' and S respectively, with $T_0=1/f_0$ and $\lambda_0=cT_0$.

On the other hand, as the light source moves relative to the observer, the observed wavelength of light in S is given by Eq. (5), where $n=n(v\cos\theta, c)$. From Eq. (4) and Eq. (5), we obtain Eq. (6), where f and λ are the frequency and wavelength of light measured in S . Eq. (6) shows the corresponding Doppler effect based on Ref. [14], and when $n=1$, Eq. (6) returns to the usual Lorentz model case.

Based on Eq. (6), we next discuss a special case where $\theta=0$ (if $\theta=\pi/2$, then $n(0, c)=1$ and Eq. (6) returns to the Lorentz model case). In this case, one obtains Eq. (7), where it should be noted that $n=n(v, c)$ due to $v\cos\theta=v$. Based on Eq. (7), one may notice that when $v=nc$, $\lambda=0$ and f tends to infinity, similar to the shock wave property in acoustic Doppler effects. From Ref. [14], we obtain the condition for light shock waves: when $v\sim c$, there is Eq. (8), where Q is a constant whose value must be determined experimentally. As mentioned in Ref. [14], we do not yet know the specific value of Q because current experimental energies are insufficient, but from numerous existing experimental results we know $Q\sim 0$.

As explained in Ref. [14], the reason for choosing n in the form of Eq. (8) is that this function can be well consistent with various experimental results conducted at low or medium energy scales. Importantly, Eq. (8) is very concise with only one parameter Q . Here we re-show the $n\sim v$ curve as Fig. 2 [Figure 2: see original paper]. Since n can be viewed as the degree to which the speed of light observed by the observer in a moving inertial system deviates from c , we see from Fig. 2 that n is almost equal to 1 across a large range of v . This property of n is why it fits well with various low and medium energy experimental results. When $Q=0$, Eq. (1) and Eq. (2) return to the Lorentz model case.

For $v=nc$, there is shock wave condition Eq. (9), where $v_{\text{wave shock}}$ is the solution for $v=nc$, and Eq. (9) is precisely the condition for generating a light shock wave. Similarly, we can express the Mach cone of light as shown in Fig. 3 [Figure 3: see original paper], where α is the cone angle. In Fig. 3, the spatial distribution of the speed of light along the center of the light source satisfies the curve equation $n=n(v\cos\theta, c)$ in polar coordinates. When $v>v_{\text{wave shock}}$, a reverse timing sequence occurs. However, as we know, timing sequence can be reversed in independent events without violating causality (though not in dependent events).

3. Compton Scattering

From the Compton scattering process, we know light can be regarded as a particle (i.e., photon). In special relativity theory, the energy and momentum of photons can be expressed by Eq. (3). However, if we substitute Eq. (7) into Eq. (3), we obtain Eq. (10), which does not formally agree with Eq. (2), because based on Eq. (2), for massless particles there is $E=pc$. Therefore, we should modify the quantum relation of photons if we follow the idea in Ref. [14]. Here we assume the modified quantum relation as Eq. (11), and based on $E=pc$ we have $a_1/a_2=k/n$. Unfortunately, there doesn't seem to be any available information or knowledge to help us specify the expressions for a_1 and a_2 .

Next, we attempt to discuss the Compton scattering process between photons and electrons based on Eq. (11), with the corresponding diagram shown in Fig. 4 [Figure 4: see original paper]. For simplicity, we assume the electrons are stationary relative to the laboratory before collision. According to the interaction diagram between photon and electron in Fig. 4, we can establish the energy conservation and momentum conservation equations in Eq. (12). Note that in Eq. (12), $a_1, a_2, k, n, a_1', a_2', k', n'$ are all functions of v and c . Eq. (12) can be approximately simplified: when $v \ll c$, $k \ll c$, and when $v \sim c$, there is Eq. (13). Due to $Q \sim 0$, based on Eq. (13), when $v \sim c$ there is $k \sim c$. That is, for $v \in [0, c)$, there is $k \ll c$. Therefore, we can assume $k \ll c$ for Eq. (12), and based on Eq. (12) we obtain Eq. (14), where m_0 is the electron mass.

Having obtained the Doppler frequency shift and Compton scattering formulas satisfying Eq. (2), we still don't know the specific expressions for a_1 and a_2 . What we can do next is apply these formulas to real physical situations to infer what a_1 and a_2 might look like. Fortunately, the phenomenon of spontaneous and synchrotron radiation in cyclotron masers provides such a physical application.

4. Spontaneous Radiation in Cyclotron Maser

Cyclotron masers are devices with high peak power and high average power in the millimeter and sub-millimeter bands. After decades of effort, the mechanisms, experimental design, and numerical simulation of cyclotron masers have been extensively analyzed [18-23]. For example, Ref. [15-17] used the concept of virtual photons to analyze spontaneous radiation in cyclotron masers (in free-electron laser theory, the periodic static magnetic field in the oscillator is also regarded as virtual photons, which are then converted into real photons through Compton scattering with high-energy particles [24]). Inspired by this idea, we revisit the discussion of spontaneous radiation in cyclotron masers using Eq. (6) and Eq. (14).

In the laboratory system, we set the charged particle to have charge q , rest mass m_0 , and velocity v perpendicular to magnetic field B . The spiral motion of the charged particle in a magnetic field has period T_c as shown in Eq. (15). As shown in Fig. 5b) [Figure 5: see original paper], we construct an inertial

system S' stationary relative to the particle, with the direction of v parallel to the z' axis. Based on relativistic transformation, both magnetic and electric fields exist in S' . Due to the “moving ruler” effect, in S' the track perimeter of the magnetic field in reverse spiral motion around the charged particle becomes $2\pi R/\gamma$, with corresponding cycle period $T_{c'}$ as shown in Eq. (16).

According to Ref. [15], since the photon in Eq. (16) cannot be observed in the laboratory system but only in S' due to relativistic effects, the photon can be regarded as a virtual photon. Next, we consider the Doppler effect of virtual photons as the periodically varying electric and magnetic fields both move at velocity $-v$ relative to S' . First, we define the angles shown in Fig. 6 [Figure 6: see original paper]: in particle system S' , θ_1' denotes the angle between virtual photon propagation direction and the z' axis, θ_2' denotes the angle between scattered photon propagation direction and the z' axis, and ϕ' denotes the scattering angle. Ω' is the angle between the plane formed by virtual photon propagation direction and the z' axis and the plane formed by scattered photon propagation direction and the z' axis. In the laboratory system, θ_1 denotes the angle between virtual photon propagation direction and the z axis, θ_2 denotes the angle between scattered photon propagation direction and the z axis, and ϕ denotes the scattering angle. Ω is the corresponding angle between planes. Therefore, based on Eq. (6) we obtain the wavelength of virtual photons in particle system S' as Eq. (17), where the source of virtual photons moves at velocity $-v$ relative to the particle system (setting $c=1$), corresponding to Eq. (4) with $n=n(v\cos\theta_1', c)$.

Virtual photons can be scattered and radiated as real photons, and based on Eq. (14) the wavelength of radiated photons satisfies Eq. (18), where λ_2' denotes the wavelength of radiated photons. Using the Doppler formula in Eq. (6), we obtain the wavelength of radiated photons observed in the laboratory system as Eq. (19), where $n'=n(v\cos\theta_2, c)$.

Since $Q \sim 0$, a_1 and a_2 in Eq. (11) should have $a_1 \sim 1$, $a_2 \sim 1$ at low or medium frequency shift (meaning the light source is not moving very fast). Additionally, since the Compton wavelength of the charged particle is generally much smaller than cT_c , we can assume a_2 in Eq. (18). Based on Eq. (14)-(19), we obtain Eq. (20). As stated in Ref. [14], the formal difference between the proposed model and the Lorentz model is that replacing c in the Lorentz model with k yields the Ref. [14] model. Therefore, the formula for aberration of light can be obtained as Eq. (21), where $\theta = \theta_1$ or $\theta = \theta_2$ or $\theta = \Omega$.

According to the geometric definition in Fig. 6, we obtain the geometric relationship in Eq. (22). Substituting Eq. (21) and Eq. (22) into Eq. (20) yields Eq. (23), which is precisely the wavelength of spontaneous radiation from a charged particle in a cyclotron maser observed in the laboratory system. At low or medium frequency shift, $n \sim 1$, $a_2 \sim 1$, $k \sim 1$, so Eq. (23) reduces to Eq. (24), corresponding to the Lorentz model case.

We are more interested in what happens at ultra-high frequency shift. First, we

recall that since Ref. [14] allows $v \in [0, c)$, when $v=nc$ the energy of the scattered virtual photon in Eq. (12) might become infinite because $\omega_1'=0$ unless we assume Eq. (25), where $V=v\cos\theta$, $n=n(V, c)$, and θ is the angle between v and c .

Eq. (25) is motivated by three considerations: First, since the expression for λ in Eq. (6) contains the term $[1-v\cos\theta/(nc)]$, which makes $\lambda=0$ under the condition $v=nc$ and $\theta=0$, and $\lambda=0$ would make the virtual photon energy infinite (rendering Eq. (14) invalid), we hope a_2 in Eq. (11) contains the term $[1-v\cos\theta/(nc)]$ to cancel out this same term in λ . Second, we want to extend Eq. (23) to the case $v \rightarrow c$, where according to Ref. [14] we know that to avoid a_2 becoming infinite as $v \rightarrow c$, we can replace v/c with nv/c . Then for $v \rightarrow c$, the expression $1/(1-nv/c)$ tends to 1 instead of $1/(1-v/c)$ tending to infinity. This argument may seem difficult to understand, but its function becomes clear in the following analysis. Third, as stated above, due to $Q \rightarrow 0$ we should have $a_1 \rightarrow 1$, $a_2 \rightarrow 1$ at low or medium frequency shift, meaning it should return to the Lorentz model case at low or medium frequency shift.

Based on these three key considerations, we construct the expression for a_1 and a_2 as Eq. (25). One might consider other possible expressions, but after many attempts we found Eq. (25) to be the most concise expression satisfying these requirements. Therefore, we use this expression to analyze synchrotron radiation in cyclotron masers.

Now we know that Eq. (23) reduces to Eq. (24) at low or medium frequency shift. Next we discuss the case $v \rightarrow c$, where we can obtain $\omega_1 \approx 1/\gamma \rightarrow 0$, so Eq. (23) can be written as Eq. (28) (setting $c=1$), where $n'=n(v\cos\theta, c) \approx n(v, c)$ and Eq. (29) holds. When $\theta=0$, $\cos\theta=1$, meaning $\theta'=0$, so $n=n(v\cos\theta', c) \approx n(v, c) \approx n(v, c)$ in Eq. (17), yielding Eq. (30), which uses Eq. (31) from Ref. [14] where the expression for n is shown in Eq. (8).

When $\theta=\pi$, $\cos\theta=-1$, meaning $\theta'=\pi$, so $n=n(v\cos\theta', c) \approx n(v, c)$ in Eq. (17), yielding Eq. (32). In Eq. (30) and Eq. (32) we obtain negative values because the timing sequence is reversed due to the light shock wave effect as stated in Section 2. It can be observed that when $v=n$, $\omega_2=0$ (though as shown above, both photon momentum and energy remain limited values), and when $v \rightarrow (n, 1)$, ω_2 increases dramatically and tends to a limited value (due to $Q \rightarrow 0$). This differs obviously from Lorentz model prediction: the observed wavelength approaches 0 regardless of observation location (correspondingly, photon momentum and energy become infinite).

5. Inverse Compton Scattering

Interactions between photons and other particles frequently occur in high-energy fields, such as interactions between gamma photons and nuclei or nucleons. Scattering between extremely relativistic particles and photons is called inverse Compton scattering because photons can obtain energy from particles in this process. Unlike Compton scattering where particles are basically stationary, in

inverse Compton scattering the particles are extremely relativistic and move at very high speeds. However, the method for deriving the relationship between scattered and incident photons is the same, using the energy and momentum conservation laws shown in Eq. (12).

We first build an “electron-rest reference frame” S' moving with the electron (analyzing photon-electron interactions). In S' , the collision satisfies Compton's formula as Eq. (33), where λ_1' and λ_2' represent wavelengths of incident and scattered photons observed in S' , and ϕ denotes the scattering angle. Assuming both observer and light source are in inertial system S moving at velocity v relative to S' , we have Eq. (34) based on Eq. (6) and Eq. (25), where λ_1 and λ_2 represent wavelengths observed in S .

Considering the case $Q=0$ (where photons obtain maximum energy from particles), we have corresponding relations. Since Eq. (33) describes Compton scattering in S' , we can assume $\lambda_2' \geq \lambda_1'$. Then based on Eq. (33)-(34), the observer in S can observe the scattered photon with momentum P_2 as in Eq. (35), which returns to the Lorentz model case. From Eq. (35), when $n=v$, P_2 reaches maximum value as shown in Eq. (36), where $v_{\text{wave shock}}$ comes from Eq. (9). For the observer in S , if the observed electron energy is much greater than the incident photon energy, Eq. (36) simplifies to Eq. (37).

We are particularly interested in the case $v \rightarrow c$, where $n \rightarrow 0$. Based on Eq. (35) we obtain Eq. (38). In the Lorentz model (when $Q=0$, the model returns to the Lorentz model), the maximum photon energy obtainable through inverse Compton scattering has a limit as shown in Eq. (37). Furthermore, when the electron velocity is high enough to approach c , the maximum photon energy instead drops to another non-zero limited value, primarily due to the property of n .

In recent years, the mechanism of high-energy radiation from blazars has been studied extensively [25-31]. One explanatory model is the lepton origin model, which considers that high-energy radiation comes from inverse Compton scattering between relativistic electrons and low-energy photons [29-31]. Since special objects with extreme physical properties, such as blazars, can produce extremely relativistic particle jets, and the cosmic microwave background (CMB) provides natural low-energy photons, we look forward to testing the value of Q in future ultra-high energy radiation experiments, as whether Q equals 0 will have very different effects on the ultra-high energy radiation spectrum, as shown in this section.

6. Summary

Lorentz violation models have been widely studied in the ultra-high energy field in recent years, particularly in attempts to unify general relativity and quantum theory [32-38]. Among these, Ref. [14] proposed a possible Lorentz violation model where the speed of light relates to the speed of the light source and may thus change between inertial systems. The key idea in Ref. [14] is that as $v \rightarrow c$, particle energy or momentum has a limited value (i.e., the time or length scaling

factor limit is $2/(Q-1)/[1-0.5(Q-1)/\ln Q]$, and correspondingly, the energy limit for a particle with rest mass m_0 is $2/(Q-1)m_0c^2/[1-0.5(Q-1)/\ln Q]$ rather than becoming infinite as predicted by the Lorentz model. The Ref. [14] model is similar to the famous rainbow model but differs in that the particle energy limit is related to the particle's rest mass, whereas in the rainbow model it is assumed to be a constant (usually considered to be the Planck energy or near it).

If we follow Ref. [14]'s viewpoint that the speed of light observed by an observer moving with velocity v relative to the light source is nc , we must modify the usual quantum relation of photons involved in the Doppler effect. By discussing the Doppler effect based on Ref. [14], we found that light can exhibit shock phenomena (in vacuum) similar to sound waves, and we obtained the conditions for light shock waves: $v=nc$. When $nc < v < c$, the timing sequence reverses, but this doesn't violate causality as timing sequence can be reversed in independent events (though not in dependent events). We then discussed Compton scattering between photons with the modified quantum relation in Eq. (11) and electrons, finding that unless we define a suitable correction coefficient for the photon quantum relation, photon energy becomes infinite when $v=nc$, making it impossible to discuss Compton scattering across the full range $v \in [0, c)$. To avoid this, we defined a suitable correction coefficient ensuring limited photon energy across $v \in [0, c)$, especially when $v=nc$ or $v \rightarrow c$.

Due to currently limited information, we cannot yet obtain the specific correction coefficients satisfying Eq. (2), but we can establish three key restrictive conditions for possible correction coefficients as shown in Section 4. Many expressions could satisfy these conditions, but after numerous attempts we constructed the concise expression shown in Eq. (25). Based on Eq. (25), we studied spontaneous radiation in cyclotron masers, which can be explained by the virtual photon concept. By analyzing Doppler and Compton effects of virtual and real photons, we obtained the wavelength of spontaneous and synchrotron radiation. The important conclusion is that as $v \rightarrow c$, the wavelength observed in the laboratory does not tend to 0 (which would imply infinite photon energy and momentum in the Lorentz model) but instead tends to a limited value. Even when $v=nc$ where $\hbar\omega=0$, both photon momentum and energy remain limited, ensuring meaningful Compton scattering across $v \in [0, c)$ as allowed by Ref. [14].

Furthermore, we discussed inverse Compton scattering based on Eq. (25), finding a limit to the maximum energy photons can obtain in collisions between ultra-high energy particles and low-energy photons. This occurs when the particle velocity satisfies $v=v_{\text{wave shock}}$. When particle velocity exceeds $v_{\text{wave shock}}$ and approaches c , the photon energy obtainable decreases sharply and tends to another limited value. This differs significantly from Lorentz model predictions where photon energy tends to infinity as particle velocity approaches c .

The fundamental idea of this paper, consistent with Ref. [14], is to avoid infinite energy or momentum for particles (including photons), as this would render real physical situations meaningless in some cases—an idea also inspired by some quantum gravity models [34,35,37]. It is crucial to emphasize that if

$Q=0$, all results and conclusions in this paper return to the Lorentz model case, providing a possible experimental scheme to determine Q , although this still requires extremely high experimental energies.

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