

Fast Star Image Matching Algorithm for Wide Field-of-View: Postprint

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Abstract

In astronomical observations based on CCD images, star image matching is a fundamental task. This paper proposes a star image matching algorithm based on k-d tree and k-means clustering algorithms, which utilizes triangle invariance tuples to perform blind matching of similar triangles; the algorithm can indirectly calculate the scale of CCD images. The k-d tree is employed three times to optimize computations, and the k-means clustering algorithm is used to segment the image, thereby enhancing the accuracy of star image matching. The star image matching algorithm was tested using sparse star fields captured by the 1 m telescope and dense star fields captured by the 2.4 m telescope at Yunnan Observatories. Experimental results demonstrate that the proposed method can effectively adapt to minor variations in image scale while simultaneously improving the accuracy of star image matching.

Full Text

A Fast Star Matching Algorithm for Large Field of View

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Abstract: Star matching is a fundamental task in astronomical observations based on CCD images. This paper proposes a star matching algorithm that employs triangle invariant tuples for blind matching of similar triangles, inspired by the Astroalign image alignment algorithm. The algorithm utilizes k-means clustering to indirectly calculate image scale and performs three rounds of kd-tree

optimization to improve computational efficiency. By segmenting the image into multiple regions, the method enhances matching precision. We tested the algorithm using dense star field images captured by the 2.4 m telescope at Yunnan Observatories. Experimental results demonstrate that the method effectively adapts to subtle variations in image scale while improving matching accuracy.

Keywords: star matching; CCD image; star catalogue; kd-tree; k-means clustering algorithm

1. Introduction

In astronomical observations, star matching—aligning stellar positions and photometric information from images with corresponding reference catalogue data—is essential for data reduction and analysis. Common approaches include vector-based methods [?], radial and cyclic feature methods [?], triangle-based methods [?], and locality-sensitive hashing techniques [?]. A particularly successful implementation is Astroalign [?], which employs quadrilaterals instead of triangles and uses Bayesian decision criteria to select the correct image transformation.

While some studies suggest quadrilateral matching outperforms triangle matching [?], triangle-based methods remain widely used due to their structural simplicity [?, ?, ?]. Vector-based methods generally surpass radial and cyclic feature approaches in efficiency. However, conventional triangle methods require constructing congruent triangles, necessitating prior knowledge of image scale—a significant limitation since scale varies due to factors like atmospheric refraction, image distortion, and tube flexure.

Inspired by Astroalign’s image alignment algorithm [?], we propose a star matching method using k-means clustering and kd-tree optimization. This approach performs blind matching of similar triangles using triangle invariant tuples, enabling indirect scale calculation. The kd-tree’s nearest-neighbor search accelerates matching, while k-means segmentation allows local transformations to replace global ones, improving precision and adapting to subtle scale variations across the field.

2. Methodology

2.1 Triangle Invariant Tuples for Blind Matching Our method uses similar rather than congruent triangles to address scale variations and reduce triangle generation. Triangle invariant tuples enable blind matching of similar triangles through kd-tree nearest-neighbor searches.

For a triangle with side lengths L_1 , L_2 , and L_3 , the triangle inequality theorem yields the relationship $L_1 + L_2 > L_3$. We define an invariant tuple (x, y) where $x = L_1/L_3$ and $y = L_2/L_3$. This tuple representation is scale-invariant, allowing direct matching of similar triangles without prior scale knowledge. Figure

1 shows characteristic plots of these invariant tuples generated from astronomical images and reference catalogues, while Figure 2 illustrates the generation workflow.

The matching process involves: (1) generating invariant tuples from triangles formed by stars in both the image and reference catalogue, (2) using a kd-tree to find nearest-neighbor tuples between the two sets, and (3) verifying potential matches. Each matched similar triangle pair can solve for a transformation, indirectly yielding the image scale.

2.2 K-means Clustering for Local Transformations A single global transformation often proves insufficient for large images due to distortion effects. Since local distortion variations are relatively small, we employ k-means clustering to partition stars into spatially coherent groups. The algorithm iteratively assigns stars to the nearest cluster center, enabling region-specific local transformations.

After obtaining a global transformation, we identify corresponding subregions in the reference catalogue for each star cluster and solve for local transformations. If a cluster contains too few stars or insufficient bright stars, the global transformation is used instead. We then convert all stellar pixel coordinates to celestial coordinates using either global or local transformations and identify catalogue matches via kd-tree nearest-neighbor search using the distance metric:

$$s^2 = (\Delta\alpha \cos \delta)^2 + (\Delta\delta)^2$$

where $\Delta\alpha$ and $\Delta\delta$ are right ascension and declination differences, respectively. This squared formulation avoids computationally expensive square root operations.

3. Experimental Results

We validated our algorithm using both sparse and dense star field images from Yunnan Observatories. The sparse field images were captured by the 1 m telescope (scale $0.286''/\text{pixel}$, 1900×1900 pixels, ~ 100 stars per image). The dense field images were from the 2.4 m telescope (scale $0.234''/\text{pixel}$, 4096×4112 pixels, ~ 1000 stars per image). Figure 3 shows matching results for representative images.

For one sparse field image, our algorithm achieved a matching rate of 87.324% with 110 detected stars, yielding a scale of $0.23410''/\text{pixel}$ and an average residual of $0.121''$ (calculated without square root for efficiency). Using local transformations with k-means clustering, the residual decreased to $0.105''$ while maintaining the same number of matched stars.

For dense field images, we tested various cluster counts (1, 4, 9, 16, 36, 49, 100). With a global transformation ($k=1$), the matching rate was 72.403%

with an average residual of 0.202". At $k=49$ clusters, the residual minimized at 0.173", though the local transformation matching rate decreased because some clusters lacked sufficient stars for reliable local solutions. The scale varied only slightly across partitions—on the order of 10^{-4} to 10^{-3} arcseconds per pixel—with smaller variations in adjacent regions. Figure 4 shows the relationship between cluster count, residual, and local transformation success rate, while Figure 5 depicts scale variations across 49 clusters.

4. Discussion

The proposed algorithm offers significant advantages. Using kd-trees reduces time complexity from $O(n^2)$ for exhaustive search to $O(n \log n)$. For our dense field with 1,290 image stars and 5,000 catalogue stars, exhaustive search required 1.288 seconds, whereas kd-tree search completed in under 50 seconds.

Unlike grid-based partitioning, which struggles with non-uniform star densities, k-means naturally adapts to local density variations. While other clustering algorithms like DBSCAN or mean-shift could be used, k-means provides reliable performance for this application. The method requires at least three stars per partition to solve for a four-constant model, making it suitable for fields with moderate stellar density.

Parameter tuning involves two thresholds: (1) pixel tolerance for congruent triangle matching (an empirical value depending on image resolution), and (2) the minimum number of matching triangles required to validate a transformation. The second parameter should be set high initially and decreased gradually if no valid transformation is found. Setting it too low risks accepting incorrect transformations.

5. Conclusion

We present a feasible star matching algorithm for large fields of view that adapts to local scale variations without requiring precise telescope scale parameters. By combining k-means clustering with kd-tree optimization and triangle invariant tuples, the method improves matching precision while maintaining computational efficiency. The approach is particularly effective for images with significant distortion.

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