Quark Matter Symmetry Energy and Quark Star Tidal Deformability

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Abstract

Studies in the past decade have shown that quark matter symmetry energy has a significant influence on the equation of state of quark matter. In this study, we investigate the stability window of quark matter by adopting the effective mass model and introducing isospin-dependent terms in the quark mass scaling. The results show that a sufficiently large isospin strength dependence parameter C_I can significantly enlarge the stability window of strange quark matter, thus enabling the calculated results to simultaneously satisfy the constraints from astronomical observational data of PSR J1614-2230 with $1.928pm0.017$ solar masses and the tidal deformability $70leLambda_{1.4}le580$ obtained from the binary neutron star merger event GW170817. In contrast to the case of strange quark matter, the stability window of $_{\text{u-d}}$ quark matter decreases with increasing isospin strength dependence parameter, making $_{\text{u-d}}$ quark stars unable to support the corresponding astronomical observational data. Finally, we find that the symmetry energy of strange quark matter is much larger than that of $_{\text{u-d}}$ quark matter, and the one-gluon exchange interaction between quarks leads to a softening of the symmetry energy of strange quark matter.

Full Text

Preamble

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Study on Symmetry Energy of Quark Matter and Tidal Deformability of Quark Stars

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Abstract

Research over the past decade has demonstrated that the symmetry energy of quark matter exerts a significant influence on its equation of state. In this study, we investigate the stability window of quark matter using an equivparticle model with an isospin-dependent term introduced into the quark mass scaling. Our results indicate that a sufficiently large isospin dependence parameter C_I can substantially enlarge the absolute stability region of strange quark matter, enabling the calculated results to simultaneously satisfy the constraints from astrophysical observations: the mass of PSR J1614-2230 at 1.928 ± 0.017 solar masses and the tidal deformability $70 \leq \Lambda_{1.4} \leq 580$ obtained from the binary neutron star merger event GW170817. In contrast to strange quark matter, the stability window of *u*-*d* quark matter decreases as the isospin dependence parameter increases, rendering *u*-*d* quark stars incompatible with the corresponding observational data. Finally, we find that the symmetry energy of strange quark matter is far greater than that of *u*-*d* quark matter, and the one-gluon exchange interaction between quarks causes the symmetry energy of strange quark matter to soften.

Keywords: symmetry energy; quark matter; quark star; tidal deformability; stability window

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1. Introduction

According to current theory, when matter reaches sufficiently high densities, the quarks confined within protons and neutrons undergo a deconfinement phase transition, forming what we call quark matter. If quark matter consists only of u and d quarks (along with electrons e and muons μ to satisfy charge neutrality conditions), we refer to it as *u*-*d* quark matter. If s quarks are present in addition to u and d quarks, we call it strange quark matter.

Witten, Bodmer, and others have conjectured that strange quark matter might be the true ground state of QCD [1-4], allowing it to exist stably in the universe. Quark matter is a system dominated by strong interactions, and theoretically, we can study it in detail starting from quantum chromodynamics, the fundamental theory of strong interactions. However, direct perturbative calculations are unreliable due to non-perturbative effects of quarks at low densities (relevant to neutron star interiors). Additionally, lattice QCD calculations at finite chemical potential are hindered by the sign problem. Consequently, researchers currently employ various phenomenological models to investigate quark matter, such as the mass-density-dependent model [5-9], quark cluster model [10-11], quasiparticle model [12-13], and NJL model [14].

When using phenomenological models, determining the parameter range is crucial. Based on the hypothesis that quark matter can exist stably, the most common constraint is that the average baryon energy of quark matter must be less than 930 MeV. Furthermore, when considering the stability of strange quark matter, we must require that the average baryon energy of u-d quark matter exceeds 930 MeV within the same theoretical framework. Recent years have witnessed significant progress in astronomical observations, particularly precise measurements of pulsar masses. The mass of PSR J1614-2230 has been measured at $1.97 \pm 0.04 M_\odot$ [15], with more precise measurements yielding $1.928 \pm 0.017 M_\odot$ [17]. Additionally, recent measurements of PSR J0740+6620 indicate a mass of $2.14^{+0.20}_{-0.18} M_\odot$ [16], with updated data giving $2.08^{+0.07}_{-0.07} M_\odot$ [19]. In summary, these masses approach or even exceed two solar masses. Moreover, with upgrades to gravitational wave detection equipment and techniques, an increasing number of gravitational wave events have been discovered. In particular, the binary neutron star merger event GW170817 [20] measured a tidal deformability of $\Lambda_{1.4} = 190^{+390}_{-120}$ [21], providing crucial information for further constraining the equation of state of dense matter. In the following study, we will use these astrophysical observational data to constrain the equation of state of quark matter.

For nuclear matter, although the density behavior of symmetry energy below and near saturation density has been well determined, determining its behavior at supra-saturation densities remains a significant challenge. At supra-saturation densities, strongly interacting matter may undergo phase transitions, and various new forms of matter may emerge. Indeed, isospin effects are quite significant due to different symmetry energy terms in different phases [22]. Compared to nuclear matter, studies on the symmetry energy of quark matter are relatively scarce [23-24].

In this paper, we consider isospin effects among quarks within the equivparticle model [25] and investigate the stability window and symmetry energy of quark matter in conjunction with the aforementioned constraints. The paper is organized as follows: In Section 2, we briefly introduce the equivparticle model incorporating isospin effects. Section 3 presents calculations of the stability window and symmetry energy of quark matter, followed by a discussion of the results. Finally, we provide a brief summary.

2. Introduction to the Equivparticle Model with Isospin Effects

In the equivparticle model, we introduce an effective chemical potential μ^* to express the thermodynamic potential density of quark matter simply as:

$$\Omega_0(\mu_i^*, m_i) = -\sum_i \frac{g_i}{24\pi^2}\left[\mu_i^*\nu_i(\mu_i^{*2} - \frac{5}{2}m_i^2) + \frac{3}{2}m_i^4 \ln\frac{\mu_i^* + \nu_i}{m_i}\right]$$

where $f(x) = x\sqrt{x^2-1}(x^2-5) + \frac{3}{2}\ln(x+\sqrt{x^2-1})$, g_i is the particle degeneracy factor ($g_q = 6$ for quarks, $g_e = 2$ for electrons), and m_i is the isospin-dependent effective particle mass. We define $\nu_i = \sqrt{\mu_i^{*2} - m_i^2}$ as the Fermi momentum of the particles. From the fundamental thermodynamic relation $\rho_i = -d\Omega_0/d\mu_i^*$, we can easily obtain the particle number density:

$$\rho_i = \frac{g_i}{6\pi^2}(\mu_i^{*2} - m_i^2)^{3/2}$$

To account for strong interactions between quarks, various parameterizations of the effective particle mass have been proposed [5,8,25,27]. Generally, in the equivparticle model, the particle mass m_i is a function of baryon density ρ_b. Therefore, to incorporate isospin effects into the equation of state of quark matter, the isospin asymmetry parameter $\delta \equiv 3(\rho_d - \rho_u)/(\rho_d + \rho_u)$ can be introduced into the mass scaling. In previous literature [27], m_i was parameterized as $m_i(\rho_b, \delta) = m_{i0} + D\rho_b^{-1/3} - \tau_i \delta D_I \rho_b^{\alpha} e^{-\beta\rho_b}$, where τ_i is the third component of isospin for the quark. In the present study, we adopt the quark mass scaling used in previous work [26]:

$$m_i(\rho_b, \delta) = m_{i0} + m_{Ii}(\rho_b, \delta) = m_{i0} + D\rho_b^{-1/3} + C\rho_b^{1/3} + C_{Ii}\delta^2\rho_b$$

where m_{i0} is the current quark mass, D is the confinement parameter characterizing linear confinement effects, $C < 0$ represents one-gluon exchange interactions between quarks [27], $C > 0$ represents leading-order perturbative interactions [25], and C_{Ii} ($C_I \equiv C_{Iu} = C_{Id}, C_{Is} = 0$) is the isospin strength dependence parameter, whose magnitude indicates the dependence of particle mass on isospin asymmetry. Additionally, we note that this quark mass scaling satisfies the exchange symmetry between u and d quarks when electromagnetic interactions are neglected.

The energy density E and pressure P of the system are given by:

$$E = \Omega_0 - \sum_i \mu_i^* \frac{\partial\Omega_0}{\partial\mu_i^*} = -\Omega_0 + \rho_b \sum_i \mu_i \frac{\rho_i}{\rho_b}$$

$$P = -\Omega_0 + \rho_b \sum_i \mu_i \frac{\partial\rho_i}{\partial\rho_b}$$

To obtain the equation of state of quark matter, we solve the following equations:

1. Charge neutrality condition: $2\rho_u - \rho_d - \rho_s - 3\rho_e = 0$

2. Baryon number conservation: $\rho_b = \frac{1}{3}(\rho_u + \rho_d + \rho_s)$
3. β-equilibrium condition: $\mu_u + \mu_e = \mu_d = \mu_s$

It should be emphasized that the chemical potentials in the β-equilibrium condition are the true chemical potentials of quarks. If only u-d quark matter is studied, the contributions from strange quarks s need not be considered in the above equations. To solve these equations, we need to specify the relationship between effective chemical potential and true chemical potential, i.e., $\mu_i = \mu_i^* + \Sigma_i$, where $\Sigma_i = \partial m_i/\partial \rho_b \cdot \partial \Omega_0/\partial m_i$ is the scalar self-energy.

Regarding symmetry energy, by definition:

$$E_{\text{sym}}(\rho_b, \rho_s) = \frac{1}{2}\frac{\partial^2(E_{uds}/\rho_b)}{\partial \delta^2}\bigg|_{\delta=0}$$

where E_{uds} is the energy density of three-flavor quark matter. From the isospin asymmetry definition δ, we can easily obtain the derivatives of δ with respect to u-quark number density ρ_u and d-quark number density ρ_d:

$$\frac{\partial \delta}{\partial \rho_u} = -\frac{6\rho_d}{(\rho_d + \rho_u)^2}, \quad \frac{\partial \delta}{\partial \rho_d} = \frac{6\rho_u}{(\rho_d + \rho_u)^2}$$

In the pressure calculation formula, the third term on the right side after summation equals zero. Therefore, the pressure formula can be simplified to:

$$P = -\Omega_0 + \rho_b \sum_i \mu_i \frac{\rho_i}{\rho_b}$$

To calculate the symmetry energy of quark matter, we rewrite the Fermi momenta of u and d quarks as:

$$\nu_u = (1 - \delta/3)\nu, \quad \nu_d = (1 + \delta/3)\nu$$

where ν is the quark Fermi momentum in symmetric u-d quark matter at quark number density $\rho = 2\rho_u = 2\rho_d$. From these formulas, we calculate:

$$E_{\text{sym}}(\rho_b, \rho_s) = \frac{9C_I\rho_b m}{\sqrt{\nu^2 + m^2}}\frac{3\rho_b - \rho_s}{(3\rho_b - \rho_s)(1 + P/E)}$$

where $m = m_0 + D\rho_b^{-1/3} + C\rho_b^{1/3}$ and we assume equal current masses for u and d quarks: $m_0 \equiv m_{u0} = m_{d0}$. If we consider only the symmetry energy of u-d quark matter, we can set $\nu_s = 0$ and $\rho_s = 0$ in the formulas.

After calculating the equation of state of quark matter, the structure of quark stars can be obtained by solving the TOV equations:

$$\frac{dP}{dr} = -\frac{G(E+P)(m+4\pi r^3 P)}{r(r-2Gm)}$$

$$\frac{dm}{dr} = 4\pi r^2 E$$

where $G = 6.707 \times 10^{-45}\text{MeV}^{-2}$ is the gravitational constant. The dimensionless tidal deformability $\Lambda \equiv \frac{2}{3}k_2 C^{-5}$ can be calculated in conjunction with the TOV equations [28-32], where $C \equiv GM/R$ is the compactness of the star and k_2 is the Love number.

3. Numerical Results and Discussion

To calculate the equation of state of quark matter, we need to specify model parameter values. In the following calculations, we set the current masses of u, d, and s quarks to $m_{u0} = m_{d0} = 7.5$ MeV and $m_{s0} = 100$ MeV. For the model parameters C, D, and C_I, we must constrain them according to other conditions. Based on the hypothesis that strange quark matter can exist in an absolutely stable state, we require its average baryon energy to be less than 930 MeV. Additionally, since no stable *u*-*d* quark matter has been discovered in nature, we assume the average baryon energy of *u*-*d* quark matter exceeds 930 MeV. Furthermore, based on recent measurements of massive pulsars showing that compact stars can reach or exceed two solar masses, we require our model parameters to support this observational result. In addition to mass constraints from pulsar observations, the rapid development of gravitational wave astronomy has provided stringent constraints on the equation of state of matter under extreme conditions. In particular, the tidal deformability measurement $70 \leq \Lambda_{1.4} \leq 580$ from the binary neutron star merger event GW170817 is significant for studying the nuclear matter equation of state. Based on these considerations, we first present the stability windows of quark matter according to the above constraints.

[Figure 1: see original paper] shows the stability window of strange quark matter, with the horizontal and vertical axes representing model parameters C and $\sqrt{D}$ (in MeV). Subfigures (a), (b), and (c) correspond to isospin strength dependence parameter values of 0, 400 $\text{MeV} \cdot \text{fm}^{-3}$, and 104 $\text{MeV} \cdot \text{fm}^{-3}$, respectively. The solid black line indicates where the average baryon energy of strange quark matter equals 930 MeV, while the dashed black line indicates where the average baryon energy of *u*-*d* quark matter equals 930 MeV. According to the hypothesis of absolutely stable strange quark matter, the region between the solid and dashed lines represents the absolutely stable region for strange quark matter. The figures demonstrate that as the isospin strength dependence parameter increases, the absolutely stable region of strange quark matter expands, indicating that isospin effects among quarks can increase the probability of strange quark matter being absolutely stable.

The black line with solid dots shows the relationship between C and $\sqrt{D}$ that satisfies the two-solar-mass constraint for strange quark stars, with its leftmost endpoint marked by a star symbol. Beyond this point (i.e., with further decrease in C), the effective quark mass becomes negative as the central density of the strange star continues to increase, preventing the existence of two-solar-mass strange stars. The black line with upward triangles shows the relationship satisfying $\Lambda_{1.4} = 580$, while in subfigure (c), the black line with downward triangles shows the relationship satisfying $\Lambda_{1.4} = 70$. Thus, the observed tidal deformability range $70 \leq \Lambda_{1.4} \leq 580$ from the binary neutron star merger lies between the two triangular lines, and a large portion of the absolute stability region also falls between these lines, suggesting that calculations in the current model support the hypothesis that the merger object in GW170817 was a strange star.

[Figure 1: see original paper] also reveals that without isospin effects (subfigure (a)) or with small isospin strength dependence (subfigure (b)), only a narrow parameter range supports the two-solar-mass constraint, and the tidal deformability condition $70 \leq \Lambda_{1.4} \leq 580$ cannot be simultaneously satisfied. Only when the isospin strength dependence parameter is sufficiently large (subfigure (c)) can both the two-solar-mass constraint and the tidal deformability constraint be met simultaneously, specifically in the star-marked region at the leftmost end of the two-solar-mass line in subfigure (c). We conclude that isospin effects among quarks are crucial for satisfying the hypothesis of absolutely stable strange quark matter and current astrophysical observations.

It should be noted that further increasing the isospin strength dependence parameter does not significantly enlarge the absolutely stable region of strange quark matter. Therefore, in this model, supporting even more massive strange stars cannot be achieved by simply increasing the isospin strength dependence parameter, implying that other effects among quarks need to be considered.

Recent studies have suggested that the true ground state of QCD might be u-d quark matter rather than strange quark matter [33]. Based on this, the average baryon energy of u-d quark matter should be less than 930 MeV. We now investigate the effect of isospin on the stability window of u-d quark matter, with results shown in [Figure 2: see original paper]. The meaning of each line is the same as in [Figure 1: see original paper], with the shaded region representing the absolutely stable region for u-d quark matter. Contrary to the case of strange quark matter, the stable region (shaded area) gradually decreases as the isospin strength dependence parameter increases. In subfigure (a) with $C_I = 0$, a considerable portion of the parameter space satisfies both the two-solar-mass constraint and the tidal deformability condition $70 \leq \Lambda_{1.4} \leq 580$. However, when the isospin strength dependence parameter increases to 104 $\mathrm{MeV} \cdot \mathrm{fm}^{-3}$ (subfigure (c)), no model parameters can simultaneously satisfy both constraints. Therefore, the hypothesis of absolutely stable u-d quark matter and current astrophysical observations (two-solar-mass compact stars and tidal deformability $70 \leq \Lambda_{1.4} \leq 580$) do not support large isospin effects among quarks.

We now examine the symmetry energy of quark matter. Based on the equation of state and formula (14), we select two sets of model parameters. For strange quark matter, we choose $(C, \sqrt{D}/\text{MeV}, C_I/\text{MeV} \cdot \text{fm}^3) = (-0.6, 164.5, 104)$. For u-d quark matter, we select $(C, \sqrt{D}/\text{MeV}, C_I/\text{MeV} \cdot \text{fm}^3) = (-0.5, 170, 40)$. These parameter values are chosen because the resulting equations of state satisfy both the hypothesis of absolutely stable quark matter and the constraints from astrophysical observations mentioned above. compares the minimum average baryon energy $(E/\rho_b)_{\text{min}}$, maximum quark star mass M_{max}, and tidal deformability $\Lambda_{1.4}$ for strange quark matter and u-d quark matter under these typical model parameters. The table shows that both have average baryon energies below 930 MeV, maximum masses near two solar masses, and tidal deformabilities within the range of 70–580.

Based on these two parameter sets, we calculate the symmetry energy of quark matter, with results shown in [Figure 3: see original paper]. The solid line represents the symmetry energy of strange quark matter, while the dashed line represents that of u-d quark matter (multiplied by 10 for comparison). [Figure 3: see original paper] demonstrates that at densities below approximately 10 times nuclear saturation density ρ_0 ($\rho_0 \approx 0.16$ fm^{-3}), the symmetry energy of strange quark matter is far greater than that of u-d quark matter. This occurs because satisfying the two-solar-mass constraint and astrophysical observations requires, as shown in [Figure 1: see original paper], a sufficiently large isospin dependence parameter for strange quark matter ($C_I = 104$ MeV$\cdot$fm^3), whereas for u-d quark matter, [Figure 2: see original paper] shows the opposite situation requiring a small isospin dependence parameter ($C_I = 40$ MeV $\cdot$ fm^3). Since quark matter symmetry energy is proportional to C_I (as seen in formula (14)), we obtain a huge difference between the symmetry energies of strange quark matter and u-d quark matter in [Figure 3: see original paper]. Additionally, unlike the symmetry energy of u-d quark matter, which increases with density, the symmetry energy of strange quark matter decreases at higher densities. As we previously pointed out, this is caused by the one-gluon exchange effect between quarks [32]; when the model parameter $C < 0$, strange quark matter exhibits softer symmetry energy.

4. Summary

In this paper, we have studied the stability windows of strange quark matter and u-d quark matter using the equivparticle model with isospin effects among quarks. Combining the hypothesis of absolutely stable quark matter, the two-solar-mass observational constraint, and the tidal deformability constraint $70 \leq \Lambda_{1.4} \leq 580$ from the binary neutron star merger event GW170817, we find that isospin effects among quarks are extremely important. Such effects can significantly enlarge the absolutely stable region of strange quark matter, enabling the equation of state to satisfy all three constraints. In contrast, isospin effects have the opposite impact on u-d quark matter, reducing its stable region, and excessively large isospin strength dependence parameters make its equation of state incompatible with these constraints. Finally, we calculated the symmetry energy of quark matter and found that the symmetry energy of strange quark matter is much larger than that of u-d quark matter, and the one-gluon exchange effect between quarks causes the symmetry energy of strange quark matter to soften.

In conclusion, based on current theory and astrophysical observations, more comprehensive and extensive astronomical observations and more mature and reliable theoretical calculations are needed to further determine the magnitude of isospin effects among quarks.

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Note: Figure translations are in progress. See original paper for figures.