Postprint: Numerical Simulation of Bimaterial Interface Crack Problems Based on the Extended Finite Element Method

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Abstract

The extended finite element method (XFEM) exhibits substantial advantages over the conventional finite element method in addressing strong and weak discontinuity problems through the introduction of enrichment functions in discontinuous regions. This study derives the displacement approximation equations for bimaterial interface cracks based on XFEM, and presents the numerical discretization scheme for interaction integrals along with the integration strategy for element stiffness matrices. The weak discontinuity at the material interface is simulated via an improved extended finite element formulation, the region of crack penetration is modeled using the strongly discontinuous Heaviside function, and the crack tips are respectively enriched by two distinct asymptotic crack-tip enrichment functions. Corresponding XFEM programs have been developed in Matlab. Numerical examples demonstrate that the simulation results agree favorably with those reported in the literature.

Full Text

Preamble

This work presents a comprehensive mathematical framework for analyzing complex systems. The foundation of our approach rests on several key equations that describe the underlying relationships between system components.

We begin by establishing the core mathematical relationships. The primary formulation is given by:

$ # %”% & $

This is supplemented by additional parameters defined through $%&%’() ∗ $, $$ # 2 & ()+!” ,)+$, and $$ 0’&“30”2 WXT-(cid:142)(cid:143)(cid:132)J(cid:157)(cid:158)(cid:140)(cid:159)(cid

*(cid:148)<!+=(cid:134) ! t.´SD•¸‘&´SDE&&!1”“!$. The complete system description incorporates $$+’##3@ABC!S 345+”’+”22$ through $$89%8”(0 ! C/:))*)NS.L)8@I9/8*6\TU\textless{}9<..L9<>&4:.<>?:)@ G<9W.LK9IX)NS.L)<8@I9/K&!1””!$, which together capture the essential dynamics.

The theoretical development proceeds through several key statements:

B. The relationship OPJ&&Q is characterized by: $$1# %&%’“%%%”“%3#%&%”’%%!”“&2” +,-.!(cid:242)£&(cid:219)Y’ 60789*(P!’P!&3”=;;+/)7 /012!(cid:242)£&8ˆ¥+’ “IN’(cid:255)s.˝D¸@(cid:140):;ø

B. For the configuration E:9<.K.O)@L<8*)N8M0 M*9.A 7./:8<9/K, we have: $$” (’&”30’&’!+ CGV9&ED6,R9<X@8<+,@7.L9/8*K97*@8I9)<)N\978I.L98*9<I.LN8/./L8/]K@K9<>.HI.<A.A N9<9I...*7.<I7.I:)A$

B. The general case yields: $$” (’&”30’&’!+ g $

The analysis further considers special cases and extensions. The parameterized forms are given by:

$’% +( )øs.$, $$!% B˝D1 ;-.’ ’PPP (cid:237)&b6FVQCED5_(cid:214)6$, and $$ % ’ “(cid:141)c ºz.{’(cid:255)sx’7](cid:228)(cid:217)>IN’(cid:255)s.’ (cid:158) f/{c(cid:192)(cid:142)]…£(cid:152)S0(cid:201)k¢ • (cid:238)—Rœ(cid:140)9 ˝¿(cid:139)(cid:140)&(cid:137)(cid:138)a(cid:137)Pxk\(cid:157)(cid:244)“UVm(cid:152)_‘ øW&¤(cid:129)jk>I§ØŒB,OJ•k\2ºx ¨a&{¸@I§(cid:228) • /EW|(cid:192)(cid:181)…k\3F2 º’ IN’(cid:255)s.{(cid:139)‘TH+”>BS0(cid:201)k¢ ˜¯]xˆ4(cid:137)(cid:158)#?(cid:228)(cid:217)¿(cid:144)(cid:1 >–”x(cid:133)N(cid:154)_‘xOP’ CG5GcSJB$

Computational aspects are addressed through $$3% œ(cid:140)9˝¿(cid:139)(cid:140):;øW˜¯&— ˝D1;U–(cid:209) (cid:201))’ (cid:176)–$ and $$ (cid:181) z (cid:158) º : ; ø W’ œKLP(cid:176)–$, which enable practical implementation.

The framework encompasses numerous derived quantities, including:

$$1 % x -.>I§ºÆI§ØŒ(cid:254)(cid:154)I§yc(cid:254)&(cid:158)WP D.8W9K9A.9˝(cid:139)(cid:140)I§yc(cid:2 Œc(cid:192)(cid:142)(cid:139)(cid:140)IØŒ(cid:254)’ 4P(cid:176)–$ through $$ #” &],˘W &(cid:132),˘ ’ (cid:156)P /ß](cid:153)&E*OJ(cid:142) !8@H /0 (cid:143)BE*O(cid:134) #8@H /0 (cid:223)(cid:138)’ @?(cid:217)(cid:155)-0(cid:156)>Q(cid:229)æ(cid:238)} { fˆ6cW(cid:137)>˙)x(cid:252)P}(cid:156)/P¡º&S !%!” (cid:143)(cid:144)?¨z(cid:137) 3 ! !8@H /0 I0&’ (!/0I8@H 0&’ ’ /0 #0<)’/” W&04 TV5VT_T SW(S;v¡º • (cid:238) ;x(cid:141)sx˝~#TS{(cid:141)sW¡ º4x˝~# (cid:127)& (cid:127)T (cid:137)‹ß%(cid:253)„S#_T (cid:137)¡º4 T x(cid:142)3’ (cid:138)Jc(cid:192)x(cid:157)˝(cid:137) I/&E $

These formulations establish the complete mathematical basis for subsequent analysis.

Application-specific models are introduced as follows. The primary application case is defined by:

$$P&’%Y’&% h(cid:217)>¡ºV” ’ {œKL W&(cid:139)‘(cid:128)7(cid:141)s4P…(cid:253),”¡º&(cid:135)6(cid:141)s (cid:252)(cid:253),”¡º&I§“(cid:148)(cid:141)s&(cid:181)z(cid:219)ºÆ5(cid:239)(cid:160)q

(cid:141)s&(cid:129)(cid:151)(cid:127)(cid:136)5(cid:239)(cid:160)q(cid:141)s4P(cid:239)(cid:253),”¡º&(cid:129) I§ØŒ(cid:141)s&(cid:219)ºÆ5(cid:239)(cid:160)q(cid:141)s&(cid:127)(cid:136)5(cid:239)(cid:160)q ijkl(:IIM(TT/O87+HOI@+.A@+/<#mnopq(OPJ&&Q’&’%# g !” h (cid:141)s4Pfi(cid:253),”¡º&:;øW(cid:141)s4P4P(cid:252) (cid:253),”¡º’ B?(cid:229)æ0d B@>?(cid:157)(cid:143)(cid:144)(cid:158)<(cid:159)^/H’ F(cid:240) ! a æ& ˛ 2 v , (cid:137) %” g%”” 77& e (cid:137) %6g’”” 77x(cid:140):;(cid:139)&(cid:158)WX’˛øWI§&(cid:139)(cid:212) (cid:221)(cid:143)(cid:144)(cid:130)o !” gP+3 cZ8x/P&I§•(cid:137) 4 ,6 g ¡‘+!&”+3” &:;‹›% ;’ g;% g“+&&3(cid:224)(cid:140):;(cid:139) (cid:139)“ ˘ W O J (cid:147) (cid:148)& : ; ’ x (cid:211) (cid:209) ˛ ~ -’ g %@”13 j’“1 cZ8&-’ ,-% g’ e”’“’ P~1;(cid:138)(cid:221)x CY^Kf//(cid:144) &” g! $

Reference implementations utilize parameters from $$’!% (cid:154) ,SUSCDYcS$ and $$ (cid:143)(cid:144)8(cid:217)&œKLPS!$, with computational verification through $$’!% + ,SUSCDYcS$ and $$” (cid:154)S!2” ˙(cid:143)(cid:144)/(cid:137)(cid:140):;øWI§ I§ØŒœ(cid:140)9˝&}%(cid:254)(cid:132)&S!2” U ˚ (^&T v(cid:215)(cid:137)S !2 ” rx()(cid:135)˛ @ ø W I § Ø Œ c (cid:192) (cid:142)’ =’ x´…D(cid:190)º *>! -’ T-% ” x—R(cid:129)«X&(cid:129) =% x´…D(cid:190)(cid:254)— R’ =/x´…D(cid:190)I§•Nx— R(cid:129)—R’ (cid:240) 1#WXøWI§ =’ x(cid:134)7 ^9>+1#=’)N8/.<I.L9<I.LN8/./L8/] (cid:240) $.

The extended model incorporates:

$#=%)N8/.<I.L9<I.LN8/./L8/] B+@?(cid:160)(cid:255)¡(cid:157)(cid:143)(cid:144)(cid:158)<(cid:159)^/H’¢ F(cid:240) 2 aæ&˛(cid:136)…Y(cid:211)(cid:209)(cid:140):;˘(cid:139)](cid:181)ø (cid:154)(cid:132)(cid:181)ø(cid:212)(cid:141)P(cid:221)¥(cid:143)(cid:144)o(cid:130) !g’ cZ8/P&…¢ (cid:181)ø(cid:212)(cid:221)(cid:153)˘c(cid:192)(cid:127)r IYg” 77&{˛2WT’• N(cid:137) %4 x W X I §’ (cid:139) « ‹ (cid:137) %” j%“& %” g $, $$!% xKL&’ ” ,4 }%” ¤&2(cid:139) ?M(cid:137)v(cid:192)(cid:255)R(cid:139)’ R’ gR% g“@&&3(cid:224)(cid:139)(cid:139)“˘W O(cid:134)(cid:147)(cid:148)’ 1;I§ØŒOJSN(cid:215)5 &/&(cid:192)~‡ 7 =/g&/,&” &(cid:158)W &” g! $, and $$!% (cid:154) JYE6$

Optimization procedures are governed by $$ % U ˚ fœKLx1;U ˚ M(cid:253)%d’ ¤(cid:240) 3 eP (cid:143)(cid:144)8(cid:217)&œKLPS!$ and $$ ” ’ (cid:190) ijkl(:IIM(TT/O87+HOI@+.A@+/<#mnopq(OPJ&&Q g $, yielding the final computational framework.

Systematic evaluation yields the following key results:

B. For configuration OPJ&&Q: $$ (cid:136)œ(cid:140)9˝1; ~R&(cid:129)P % (cid:136)1;~X&ON´’rR’ (cid:136)(cid:137)3@! (cid:240) 2#(cid:140):;(cid:139)møWI§! ˘WO(cid:134)” $

B. Under conditions B)@L<8)N ,8I9)<8G<9W.LK9IX)N-.N.<K.Q./:<)))>X: $$’% #¤(cid:215)5&(cid:221)/(cid:215)&(cid:131)]Y+(cid:140):;øWI§xR;’(cid:255)s- . 8\978I.L98M8I.! M8<.KIL89<” (cid:240) 3#WXøWI§ =’ x(cid:134)7 ^9>+3#=’)N8/.<I.L9<I.LN8/./L8/] (cid:240) P#WXøWI§ =% x(cid:134)7 ^9>+P#=%)N8/.<I.L9<I.LN8/./L8/] .7.<I7.I:)A N)LA9KK9798L78I.L989<I.LN8/./L8/]$

B. For the generalized case 6<>9<..L9<>NL8/I@L. 7./:8<9/K: $$

%% #D6a B&b_FS,-6JB6BJ&FY,-C&.I8+S\)@<A8LX..7.<I N)L/L8/]
8<8XK9K8I8\978I.L989<I.LN8/.$

B. The OPJ&&Q configuration extends to: $$ &% #NŁ&¡(cid:134)2&(cid:155)^ª&B+(cid:140):;øWI§OJSN(cic
øsºÆ$

B. The E:9<.K.O)@L<8)N8MM9.A 7./:8<9/K case yields: $$!’” (%&0%3+
4DS,Uc9<>& VS_ 4:.<:8<& -G R9<>:@8& .I8+b)@<A8LX..0 7.<I8<8XK9K)NKIL.KK9<I.<K9IXN8/I)LF
$

The complete parameter space is explored through $$! ’ ” (%&0%3 ! 9<
E:9<.K.” + $ through $$P&&$, providing comprehensive coverage of possible configurations.

Advanced computational methods are introduced, including:

$$1”0$ and $$’%% FSb_J-6Z&Z_ccY6JB&J6,SJ-V&.I8*+D9>:0)LA.L.HI.<A0
$

These enable efficient solution of the core equations $$ B% +Y<I.L<8I9)<8* ″$
through $$’”% (cid:242)£&@(cid:228)(cid:204)&-(cid:158)y+P___IN’(cid:255)s1;øWI§OJ
$.

The final implementation integrates:

$$ B% +(cid:237) (cid:238) ´ S ´ ˆ R & & Q& %“’$, $$” (8<8XK9K)N9<I.LN8/./L8/]
@K9<>\)@<A8LX..7.<I7.I:)A (SMM90 ’’$, and $$3+ /8I9)<)NW9LI@8/L8/]
.HI.<K9)< 7.I:)A $

with specialized cases given by $$ @K9<».<.L89?.A .HI.<A.A N9<9I...7.<I7.I:)A
$ through $$ B% +E)7M@I.L 7.I:)AK9< 8MM*9.A 7./:8<9/K8<A
.<>9<..L9<>&%””&&’P%!%3 T%P T $.

This mathematical foundation provides the necessary tools for analyzing the systems discussed in subsequent sections.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.