Derivation Principle and Calculation Method of Generalized Short-Circuit Ratio Based on Characteristic Subsystem

Authors: Xin Huanhai, Liu Chenxi, Huang Linbin, Huisheng Gao, Ju Ping, Huanhai Xin

Date: 2024-06-11T00:00:00+00:00

Abstract

With the large-scale integration of grid-following renewable energy and power electronic devices, the system voltage support strength decreases, increasing the security and stability risks of modern power systems. In scenarios with homogeneous integration of renewable energy and power electronic devices, a system voltage support strength quantification method strongly correlated with stability can be established using the grid generalized short-circuit ratio and device/station critical short-circuit ratio. In weakly heterogeneous scenarios, strength quantification can be achieved through first-order approximations of the generalized short-circuit ratio and device/station critical short-circuit ratio based on the special dynamic characteristics of devices and the grid, but a unified derivation principle and calculation method are lacking. To address this, this paper focuses on the voltage support strength quantification problem under small-disturbance conditions. First, by leveraging the principle that multi-infeed systems can be approximately decoupled into multiple low-dimensional subsystems, the concept and calculation method of characteristic subsystems are proposed, and their physical meaning is interpreted. Second, based on characteristic subsystems, the derivation principle and calculation method for the generalized short-circuit ratio and device/station critical short-circuit ratio are proposed. Additionally, specific calculation formulas for the generalized short-circuit ratio are provided for scenarios where power electronic devices operate at non-rated operating points, some devices have reverse active power, and grid-forming devices are included. Finally, case studies verify the effectiveness of the proposed principle and methods.

Full Text

Abstract

With the large-scale integration of renewable energy and grid-following power electronic devices, system voltage support strength decreases, thus threatening the system's safe and stable operation. In the homogeneous scenario where power electronic devices integrated into the system have the same dynamics, a theoretically rigorous and highly stable quantitative method for quantifying system voltage support strength can be formed based on the generalized short-circuit ratio (gSCR) and the device or station critical short-circuit ratio (SCR_0). In the heterogeneous scenario where power electronic devices integrated into the system have weakly different dynamics, system voltage support strength can be quantified by the first-order approximation of gSCR and SCR_0, based on some special characteristics of devices and power grid. However, there is a lack of unified derivation principles and calculation methods. To this end, this paper focuses on quantifying voltage support strength under small-signal stability and discovers that the multi-infeed system can be approximately decoupled into multiple low-dimensional systems. On this basis, this paper proposes the concept of eigen-subsystem and its calculation method and interprets their physical significance. Based on the concept of eigen-subsystems, the derivation principle and calculation method of gSCR and SCR_0 are given. Additionally, the specific calculation methods for indices are provided when considering grid-following converters under non-rated operating conditions, reverse active power output, and grid-forming devices. Finally, the effectiveness of the principles and methods is verified in several cases.

KEY WORDS: generalized short-circuit ratio; device or station critical short-circuit ratio; eigen-subsystems

1. Introduction

With the rapid development of renewable energy, power systems have gradually evolved into new power systems with high proportions of renewable energy and power electronic equipment (hereinafter referred to as "equipment"). Voltage support strength characterizes the ability of system voltage to resist deviation and instability under disturbances. High strength indicates strong resistance to external disturbances and good voltage response performance after being disturbed. However, the proportion of synchronous machines with strong supporting effects is decreasing, while the capacity of centralized renewable energy grid integration is large, the electrical distance from the main grid is far, and grid-following control strategies are often adopted, resulting in weak grid support strength. This makes the voltage support strength of new power systems lower and increases the safety and stability risks after disturbances, necessitating scientific strength quantification and calculation methods (considering that strength issues in the low-inertia dimension can be considered separately, for convenience of expression, system voltage support strength is hereinafter abbreviated as "system strength").

After large-scale integration of grid-following equipment (hereinafter referred to as "equipment" unless otherwise specified), the analysis and calculation of system strength face the following difficulties: the large number of equipment and grid buses, complex dynamic interactions between equipment and networks, and difficulty in simplifying high-dimensional dynamic characteristics; diverse equipment types, such as grid-following converters with different control strategies and parameters or grid-forming converters, further increase system complexity; and multiple operating conditions of renewable energy also pose significant challenges to strength quantification.

To analyze system strength and facilitate engineering applications, the industry commonly uses grid strength to approximately characterize system strength and introduces single-infeed and multi-infeed short-circuit ratios as quantitative indicators. Grid strength characterizes the ability of bus voltage to resist deviation from normal operating points under disturbances without considering the dynamics of grid-following equipment. Examples include the CIGRE multi-infeed short-circuit ratio (MSCR), site-dependent SCR (SDSCR), multi-renewable energy station SCR (MRSCR), equivalent circuit-based SCR (ESCR), and the SCR with interaction factors (SCRIF). While these indicators can reflect the steady-state voltage deviation before and after equipment integration, they have weak correlation with system stability due to not considering the wide-frequency coupling characteristics between equipment and the grid, lacking mechanism support when used to reflect stability characteristics.

Literature [7], [16]-[18] reshaped the connotation of system strength integrating equipment and grid dynamics from the perspective of dynamic interaction between equipment and grid, and proposed grid generalized short-circuit ratio (gSCR) indicators and equipment/station critical values based on voltage-to-current (or voltage-to-power) sensitivity equations, theoretically achieving correlation between system strength and small-disturbance stability. In homogeneous systems (where the dynamic characteristics of integrated equipment are similar, with specific definitions in literature [7]), multi-infeed power electronic equipment systems (hereinafter referred to as "multi-infeed systems") can be decoupled into multiple equivalent single-infeed systems while maintaining the overall equipment dynamics. Based on these equivalent single-infeed systems and analogizing the use of traditional short-circuit ratios, the grid generalized short-circuit ratio and equipment/station critical short-circuit ratio are defined, and it is demonstrated that the equipment/station critical short-circuit ratio equals the critical value of the generalized short-circuit ratio, thereby achieving system strength quantification considering equipment dynamic characteristics. In weakly heterogeneous scenarios, such as integration of externally different direct-drive wind turbines, photovoltaics, grid-forming equipment, and energy storage systems, or equipment operating at non-rated operating points, heterogeneous equipment dynamics can be regarded as perturbations to equipment or network dynamics in homogeneous systems, and strength quantification can be achieved using first-order approximations of the generalized short-circuit ratio and equipment/station critical short-circuit ratio. The key to calculating the above generalized short-circuit ratio and its critical value is solving the decoupled single-infeed system that can approximate the stability of critical modes of the original system. However, in these non-ideal scenarios, the theory and methods for finding equivalent decoupled single-infeed systems lack universality, which is not conducive to the application of generalized short-circuit ratios in complex scenarios.

Therefore, this paper aims to unify the derivation principles and calculation methods of generalized short-circuit ratios in multiple scenarios. First, the concept of eigen-subsystem is introduced and its solution method is proposed. Second, based on eigen-subsystems, the general derivation principle and calculation method of generalized short-circuit ratio and equipment/station critical short-circuit ratio are proposed. Finally, case studies verify the effectiveness of the proposed principles and methods.

2. System Modeling and Problem Description

[Figure 1: see original paper] shows a new power system containing n+m equipment units and their connected buses, r intermediate passive buses, and 1 infinite bus (if grid-forming equipment exists, the infinite bus may not be needed). Among them, n units are grid-following converters for renewable energy or energy storage, and m units are grid-forming equipment such as grid-forming wind turbines, energy storage, or synchronous condensers.

2.1 System Dynamic Modeling

First, considering that all line impedance ratios are the same, the sensitivity equation between grid port voltage micro-increment ΔU_{xy}(s) and current micro-increment ΔI_{xy}(s) in the global synchronous coordinate system, viewed from the equipment port toward the AC grid, can be expressed as [16]:

$$\Delta U_{xy}(s) = \left(\frac{1}{\omega_0} \begin{bmatrix} s & \tau\omega_0 \\ -\tau\omega_0 & s \end{bmatrix} \otimes B \right) \Delta I_{xy}(s) \tag{1}$$

where B is the equivalent admittance matrix or sensitivity matrix containing all nodes; τ = R/X is the line impedance ratio; Δ denotes variable increment; represents the Kronecker product; and ω_0 is the synchronous speed of the AC grid. When the network side contains power electronic reactive power compensation, grid-forming equipment, or loads, modifying the sensitivity matrix of AC network port voltage to current can account for the impact of these elements, and the subsequent strength quantification method remains applicable, though detailed expansion is omitted due to space limitations.

Second, the sensitivity equation between equipment port voltage micro-increment ΔU_{xy}(s) and current micro-increment ΔI_{xy}(s) can be expressed as [16]:

$$\Delta I_{xy}(s) = Y_{IBR}(s)\Delta U_{xy}(s) \tag{2}$$

where Y_{IBR1}(s) to Y_{IBR}(n+m)(s) are the admittance transfer function matrices of the n+m equipment units respectively; 0_{2r} ^(2r$×$2r) is a zero matrix.

Finally, combining equations (8) and (9), the closed-loop characteristic equation of the system shown in [Figure 1: see original paper] can be expressed using the following determinant:

$$\det(T_{het}(s)) = \det(Y_{IBR}(s) + Y_{net}(s)) = 0 \tag{3}$$

where T_{het}(s) is the inverse matrix of the closed-loop transfer function of the original system, i.e., the original system matrix, and det(·) denotes the determinant operation.

2.2 Review of System Strength Quantification Based on Generalized Short-Circuit Ratio

The closed-loop characteristic equation shown in (10) determines system stability. However, due to the high dimension of T_{het}(s), finding an approximate low-dimensional system is necessary to simplify analysis.

For homogeneous systems, the multi-infeed system can be decoupled into multiple equivalent single-infeed systems. Based on the equivalent single-infeed system with the highest instability risk and analogizing the definition of traditional short-circuit ratio, the generalized short-circuit ratio gSCR and equipment/station critical short-circuit ratio SCR_0 of the original system can be defined, forming a system strength analysis method based on generalized short-circuit ratio with the characteristic of separating grid dynamics from equipment dynamics. The calculation formulas are as follows [7]:

$$gSCR = \frac{\sigma_{\max}(S_B^{(n+m)})}{\lambda_{\min}(B_{red}Z_{red})} \tag{4}$$

$$SCR_0 = C_{gSCR0} \tag{5}$$

$$\beta_0\% = \frac{gSCR - SCR_0}{SCR_0} \times 100\% \geq \beta_{req}\% \tag{6}$$

where C_{gSCR0} is the critical value of generalized short-circuit ratio; $\beta_0\%$ is a threshold set to ensure sufficient system margin; the expression for gSCR is:

$$gSCR = \lambda_{\min}\left(\left(S_B^{(n+m)}\right)^{-1/2} B_{red}\left(S_B^{(n+m)}\right)^{-1/2}\right)$$

where S_B(n+m) = diag(S_{B1}, …, S_B(n+m)) is a diagonal matrix with S_{B1}, …, S_B(n+m) being the rated capacities of the n+m equipment units; B_{red} is the network admittance matrix or voltage-current sensitivity matrix retaining equipment buses; Z_{red} = B_{red}$^{-1}$; σ{max}(·) and λ{min}(·) represent solving for the maximum singular value and minimum eigenvalue of matrices respectively.

The generalized short-circuit ratio is the maximum singular value of the sensitivity matrix, reflecting the maximum sensitivity of voltage disturbance to current disturbance in multi-port AC networks [7].

For heterogeneous systems, gSCR and SCR_0 can be approximated using matrix perturbation theory. Specifically, according to the perturbation source and dynamic characteristics of equipment and grid, they are divided into two categories: 1) Grid-following equipment dynamics: Differences in external characteristics of grid-following converters are considered as perturbations to equipment dynamics in homogeneous systems. In this case, the boundary conditions for calculating gSCR remain unchanged, while SCR_0 depends on the weighted average of equipment dynamic characteristics (see literature [7][17][19][20]); 2) Equivalent AC network dynamics: When considering grid-forming converters or synchronous condensers with dominant voltage source characteristics, they can be considered as network modifications. In this case, the calculation method for SCR_0 remains unchanged, and modifying the boundary conditions for gSCR calculation can account for the impact of these grid-forming equipment (see literature [9][10][11]).

2.3 Problems to be Solved

In summary, the definition and calculation of gSCR indicators and their critical values rely on finding equivalent low-dimensional single-infeed systems, i.e., eigen-subsystems. However, existing research is only applicable to single typical scenarios, such as scenarios with only externally heterogeneous grid-following converters [6], grid-forming equipment [10][11], or energy storage systems [19]. The methods and theories lack universality, leading to relatively complex understanding of generalized short-circuit ratios. Therefore, this paper utilizes the more physically meaningful concept of eigen-subsystems to unify the derivation principles and calculation methods of generalized short-circuit ratios, focusing on two main problems:

Problem 1: The theoretical basis for low-dimensional decoupled subsystems (i.e., eigen-subsystems) that approximate the modes of the original large-scale system.

Problem 2: The unified derivation principle and calculation method for generalized short-circuit ratio and its critical values based on eigen-subsystems.

3. Eigen-Subsystems and Their Approximation Methods

Based on Definition 2, for a given dynamic system and its system matrix, there exist low-dimensional eigen-subsystems and generalized eigen-matrices. Eigen-subsystems can approximate the stability of the original dynamic system in modes of interest with lower dimensions and simpler analysis.

The key to the above analysis is obtaining eigen-subsystems, i.e., solving for generalized eigen-matrices. Therefore, this section first uses Schur complement transformation of matrices to simplify the original system matrix. Second, based on Lemma 1, the approximate eigen-subspace of the system matrix is obtained, and the corresponding generalized eigen-matrix and low-dimensional eigen-subsystem are derived.

3.1 System Matrix and Its Simplification

Considering the power system shown in [Figure 1: see original paper], the following provides an approximate solution method for its eigen-subsystems from the small-disturbance synchronization perspective. The solution methods for eigen-subsystems under other modes are similar.

The system is viewed as consisting of two parts: one part is the equivalent network, including the AC network and equivalent admittance of m grid-forming equipment (in the analysis frequency band, their external characteristics are dominated by Thevenin equivalent circuits, with equivalent admittance B_{IBRj} (j = 1,…,m)); the other part includes the dynamic models of n grid-following equipment. The original system matrix shown in (10) can be transformed as:

$$T_{het}(s) = \begin{bmatrix} T_1(s) & T_2(s) \\ T_3(s) & T_4(s) \end{bmatrix} \tag{7}$$

where B_{IBR} is the correction of m grid-forming equipment to the network sensitivity matrix; T_1(s), T_2(s), T_3(s), and T_4(s) are submatrices with appropriate dimensions; I_2 is a 2-dimensional identity matrix; 0_n and 0_r are n-dimensional and r-dimensional zero matrices respectively.

Assuming the grid is stable without equipment connection, according to the Schur complement theorem [22], the zeros of det(T_{het}(s)) = 0 are determined by det(T_1(s) - T_2(s)T_4^{-1}(s)T_3(s)) = 0. Therefore, det(T_1(s) - T_2(s)T_4^{-1}(s)T_3(s)) = 0 determines the stability of the system shown in [Figure 1: see original paper], where (derivation see Appendix B):

$$T_1(s) - T_2(s)T_4^{-1}(s)T_3(s) = Y_{IBR}(s) + \left(\frac{1}{\omega_0} \begin{bmatrix} s & \tau\omega_0 \\ -\tau\omega_0 & s \end{bmatrix} \otimes B'_{red} \right) \tag{8}$$

where B_1 ^(n×n), B_2 ^(n×(m+r)), B_3 ^((m+r)×n), B_4 ^((m+r)×(m+r)).

Further considering the impact of operating points, the actual operating conditions of the remaining n grid-following equipment are decoupled from their dynamics [20]. Specifically, the matrix (U^2/P) I_2 containing the actual operating condition information of the equipment is left-multiplied to T_{het}(s). Let T_{het}(s) be transformed into T_{het}(s) as shown below. The characteristic function of T_{het}(s) determines the stability of the system in [Figure 1: see original paper].

$$T_{het}(s) = (U^2/P) \otimes I_2 \cdot T_{het}(s) \tag{9}$$

where U^2/P = diag(U_i^2/P_i), with U_i and P_i being the terminal voltage and output active power of equipment i respectively. When considering reverse power flow of equipment, the active power is negative.

Note 1: When equipment operates at rated operating points, its terminal voltage is 1.0 p.u., and output active power equals its rated capacity S_{Ni} (i = 1,…,n). In this case, U^2/P = diag(U_i^2/P_i) = diag(S_{Ni}$^{-1}$). Therefore, the analysis method for rated operating points is a special case of actual operating conditions.

3.2 Eigen-Subsystem Approximation Method

When the dynamic models of n equivalent equipment are homogeneous, the eigen-subspace of the equivalent admittance matrix (U^2/P)B'*{red}* I_2 *with respect to its own double eigenvalues is a set of eigen-subspaces of the system matrix. When q (q < n) equipment units in the system have reverse active power (negative values), (U^2/P)B'*{red} I_2 has q negative eigenvalues. Sorting the eigenvalues as λ-*q* ≤ … ≤ λ-1 ≤ 0 ≤ λ_1 ≤ λ_2 ≤ … ≤ λ_(n-q), λ_1 is the smallest positive eigenvalue. The eigen-subspaces (j = -q,…,-1,1,…,n-q) correspond to eigenvalue λ*j of* (U^2/P)*B'*{red} I_2. Further, according to Definition 1 and 2 and analogizing (3), multiple eigen-subsystems of the original system can be solved using (k   {-q,…,-1,1,…,n-q}) to approximate the stability of the original system under different modes, summarized as Lemma 2 (specific proof see Appendix C).

Lemma 2: The generalized eigen-matrix satisfies:

$$L_k = Y_k^H M X_k = \lambda_k I_2 + Y_{IBReq}^H(s) \cdot \zeta(s) \tag{10}$$

where Y_{IBReq}(s) is the dynamic model of a similar single equivalent equipment; X_k = x_k I_2, Y_k = y_k I_2, the column vectors of X_k and Y_k are the right and left eigenvectors of (U^2/P)B'_{red} I_2 with respect to λ*k*, *x_k, y_k* $^{(n\$\times 1), and x_k, y_k are the right and left eigenvectors of (U}$ *{2}$/P)B'*{red} with respect to λ_k.

When equipment dynamic models differ, an equivalent homogeneous system is constructed whose system matrix's eigen-subspace can approximate that of the original system matrix. Then, the approximation of the generalized eigen-matrix can be obtained from Lemma 1. Specifically, first using the eigen-subspace, the homogeneous system matrix M_k is:

$$M_k = T_{hom\_k}(s) = Y_{IBR\_k}(s) + \left(\frac{1}{\omega_0}\begin{bmatrix} s & \tau\omega_0 \\ -\tau\omega_0 & s \end{bmatrix} \otimes B'_{red}\right) \tag{11}$$

where T_{hom}k(s) is the equivalent homogeneous system matrix corresponding to λk*; G_k(s) is the equivalent weighted equipment model;* p{ki} is the participation factor of the i-th equipment; x_{ki} and y_{ki} are the i-th elements of x_k and y_k respectively.

Second, observing the constructed homogeneous system M_k, the equipment dynamics are weighted averages of all grid-following equipment dynamics in the original system. According to Lemmas 1 and 2, remains the eigen-subspace of M_k. Using this eigen-subspace, the generalized eigen-matrix of can be solved as:

$$L_k \approx Y_k^H M X_k = \lambda_k I_2 + Y_k^H G_k(s) X_k \cdot \zeta(s) \tag{12}$$

Finally, viewing as the system matrix of a low-dimensional system, this low-dimensional system is an eigen-subsystem of the original system that can approximate the stability of the original system in the corresponding mode (since retains the zeros and poles of the original system matrix, it can also reflect the robustness of the original system).

This conclusion is summarized as Theorem 1 (specific proof see Appendix).

Theorem 1: Treating the original system matrix as a perturbation of the homogeneous system matrix M_k, the eigen-subspace of M_k is approximately equal to that of , and the generalized eigen-matrix of can be calculated by (19).

The obtained eigen-subsystem has lower dimensions and can approximately reflect the characteristics of the original system in the corresponding mode. Therefore, complex system analysis is simplified based on eigen-subsystems.

Note 2: When grid-following equipment dynamic models are homogeneous, the eigen-subspace of the equivalent homogeneous system matrix is identical to that of the original system matrix, and the stability of the eigen-subsystem is the same as that of the original system. Conversely, when differences exist between the equivalent homogeneous system matrix and the original system matrix, there is deviation between the approximate and true generalized eigen-matrices, and the eigen-subsystem approximately reflects system stability and robustness.

3.3 Physical Meaning of Eigen-Subsystems

From the previous section, using eigen-subspace perturbation can solve the generalized eigen-matrix of the original system matrix, and the corresponding eigen-subsystem can be used to analyze the stability of the original system. Physically, the eigen-subsystem is a low-dimensional single-infeed system containing equipment and network dynamics, essentially a decoupled subsystem of the original system in modal coordinates. The physical meaning and solution process of eigen-subsystems are illustrated in [Figure 2: see original paper] and [Figure 3: see original paper].

From (19) and [Figure 2: see original paper], the eigen-subsystem is a single-machine infinite-bus system composed of equivalent weighted grid-following equipment and equivalent lines. The characteristics of the integrated equipment are the weighted average of grid-following equipment in the original system, reflecting the overall dynamic characteristics of equipment in the system. The equivalent line dynamics account for the impact of grid-forming equipment equivalent admittance and other factors on network dynamics. Furthermore, from the perspective of traditional short-circuit ratio, the short-circuit ratio of the eigen-subsystem takes the value of the corresponding equivalent line admittance, which is the absolute value of eigenvalue λ_k and can be used to characterize the grid strength of the eigen-subsystem.

It is also worth noting that since the stability mechanisms of different modes differ, their corresponding eigen-subspaces and eigen-subsystems should also differ, corresponding to different values of k. Therefore, in practical problems, the appropriate eigen-subsystem should be selected according to specific modes and stability mechanisms to achieve simplified analysis of complex systems. Moreover, since this paper focuses on the dominant mode of grid-following equipment and equivalently treats grid-forming equipment as ground branches, the corresponding eigen-subsystem is only a single-machine infinite-bus system for grid-following equipment. However, under other modes, the corresponding eigen-subsystem forms are more complex, including but not limited to single-machine infinite-bus systems.

4. Derivation Principle of Generalized Short-Circuit Ratio Based on Eigen-Subsystems and Strength Quantification Method

This section proposes a unified derivation principle of generalized short-circuit ratio based on eigen-subsystems and provides the corresponding process and calculation methods for several typical scenarios.

4.1 Grid Strength and Generalized Short-Circuit Ratio Indicators in Eigen-Subsystems

According to Section 3.3, eigen-subsystems can approximately characterize the stability of the original system and are composed of equivalent weighted equipment dynamics and equivalent line dynamics. The short-circuit ratio of the grid equals the absolute value of eigenvalue λ_k. Therefore, the strength of the original system depends on the dynamics of the eigen-subsystem. When the equivalent weighted equipment dynamics are constant, the original system strength depends on the short-circuit ratio of the eigen-subsystem, i.e., the equivalent line admittance.

When grid voltage support strength is low, the risks of static voltage instability and small-disturbance instability caused by phase-locked loops are relatively high. Therefore, the eigen-subsystems corresponding to these two types of stability problems (i.e., k = 1) receive particular attention. Specifically, in this mode, the smaller the short-circuit ratio, the higher the system instability risk [7]. Thus, the eigen-subsystem corresponding to short-circuit ratio λ_1 (k = 1) is most prone to instability, and this short-circuit ratio is defined as the "generalized short-circuit ratio" as shown in (20). Therefore, the generalized short-circuit ratio is only a specific application based on eigen-subsystems. Other important eigen-subsystems can also reflect the characteristics of the original system under other modes and can propose corresponding indicators. However, generally speaking, short-circuit ratio-based analysis methods are suitable for eigen-subsystems corresponding to static stability and small-disturbance synchronization modes, characterized by strong correlation with grid electrical distance. As for eigen-subsystems corresponding to high-frequency oscillations with LC resonance stability mechanisms, short-circuit ratio-based analysis methods may no longer be applicable, as discussed in literature [7].

From (20), the generalized short-circuit ratio can be defined both through matrix eigenvalues and through closed-loop system eigenvectors. When the eigensubspace of the closed-loop system is the same as that of the open-loop system (i.e., considering only the grid admittance matrix), as in homogeneous systems, both approaches yield consistent conclusions. Conversely, the eigenvalue-based definition remains a calculation method with small error and convenient for practical operation.

Furthermore, combining (19) and (20), when the equivalent weighted equipment/station dynamics are constant, the minimum short-circuit ratio required for the equivalent weighted equipment/station to reach a critical stable state or given performance requirements (referred to as "equivalent weighted equipment/station critical short-circuit ratio SCR_0") can be determined based on the positive correlation between generalized short-circuit ratio and system strength. SCR_0 reflects the tolerance of equipment/station dynamics to grid strength and is precisely the critical value of the generalized short-circuit ratio C_{gSCR0}, expressed as:

$$SCR_0 = \arg\{\det(\lambda_1 I_2 + Y_{IBReq}^H(s_d)\cdot\zeta(s_d)) = 0\} \tag{13}$$

where arg{ • } denotes solving for the root of the equation, and s_d is the dominant eigenroot of the system solved under given performance requirements for the equivalent weighted equipment.

In summary, the unified derivation process for generalized short-circuit ratio of complex systems is: 1) Using the eigen-subspace of the equivalent homogeneous system, solve the approximate generalized eigen-matrix of the original system matrix to obtain eigen-subsystems that can approximate the dynamic characteristics of the original system; 2) Based on specific eigen-subsystems, calculate their short-circuit ratio indicators (i.e., generalized short-circuit ratio) and equipment/station critical values (equal to the critical value of generalized short-circuit ratio), thereby forming a quantification method integrating grid generalized short-circuit ratio and equipment/station critical short-circuit ratio. The generalized short-circuit ratio depends only on equivalent network dynamics and the capacity of integrated equipment, while the critical value depends only on the dynamics of equivalent weighted equipment/stations. Therefore, the generalized short-circuit ratio method retains the advantage of traditional short-circuit ratio methods in separating network dynamics from equipment dynamics quantification.

4.2 Calculation Methods for Generalized Short-Circuit Ratio in Typical Scenarios

Combining the system matrix simplification method in Section 3.1 (see equations (15)-(16)) and the generalized short-circuit ratio calculation formula in Section 4.1 (see (20)), this section provides the specific forms of system matrices under three typical scenarios, thereby enabling calculation of generalized short-circuit ratio.

4.2.1 Generalized Operational Short-Circuit Ratio Considering Equipment at Non-Rated Operating Points Consider the system shown in [Figure 1: see original paper] containing only n renewable energy converters with different control strategies and parameters, with m = 0. The renewable energy converters operate at different non-rated operating points, with output active power and terminal voltage ranging from 0.2-1 p.u. and 0.9-1.1 p.u. respectively [23]. According to (15)-(16) and (20), the simplified system matrix and generalized short-circuit ratio for this scenario can be obtained. The specific process is:

First, consider no correction of network dynamics by grid-forming equipment equivalent admittance, i.e., B_{IBR} = 0_(n+r). Substituting B_{IBR} = 0_(n+r) into (15) and setting m = 0 yields the network equivalent admittance matrix B'*{red} containing only n equipment connection buses. Second, consider decoupling the actual operating conditions of n equipment from their dynamics, as shown in (22). Finally, substituting B'*{red} and into (16) and (20) yields the system matrix and generalized short-circuit ratio for this scenario.

$$Y_{IBR}(s) = \mathrm{diag}(Y_{VSCi}(s)), \quad i = 1, \dots, n \tag{14}$$

$$U^2/P = \mathrm{diag}(U_i^2/P_i) \tag{15}$$

where Y_{VSCi}(s) is the admittance transfer function matrix of the i-th renewable energy converter, and U_i, P_i are the terminal voltage and output active power of equipment i respectively.

4.2.2 Generalized Short-Circuit Ratio Considering Reverse Active Power of Equipment According to the physical characteristics of equipment and system requirements, some equipment can absorb power from the grid in reverse, such as energy storage systems and flexible DC transmission. Without loss of generality, taking conventional energy storage systems as an example, consider the system shown in [Figure 1: see original paper] containing n renewable energy converters with different control strategies and parameters, and m conventional energy storage systems. All equipment operate at rated conditions; renewable energy converters mainly inject active power into the grid, while energy storage converters mainly inject (discharge) or absorb (charge) active power. According to (15)-(16) and (20), the simplified system matrix and generalized short-circuit ratio for this scenario can be obtained. The specific process is:

First, consider no correction of network dynamics by grid-forming equipment equivalent admittance, i.e., B_{IBR} = 0_(n+m+r). Substituting B_{IBR} = 0_(n+m+r) into (15) yields the network equivalent admittance matrix B'*{red} containing only n+m equipment connection buses. Second, consider decoupling the rated capacity of n+m equipment from their dynamics. When energy storage converters are charging, their rated capacity needs a negative sign. For example, considering m energy storage units all operating in charging state, as shown in (23). Finally, substituting B'*{red} and into (16) and (20) yields the system matrix and generalized short-circuit ratio for this scenario.

$$Y_{IBR}(s) = \mathrm{diag}(Y_{VSCi}(s), Y_{PCSj}(s)) \tag{16}$$

$$U^2/P = \mathrm{diag}(S_{Ni}^{-1}, -S_{N(n+j)}^{-1}) \tag{17}$$

where Y_{PCSj}(s) is the admittance transfer function matrix of the j-th energy storage converter, and S_{Ni}, S_N(n+j) are the rated capacities of the i-th renewable energy converter and j-th energy storage converter respectively.

4.2.3 Generalized Short-Circuit Ratio Considering Impact of Grid-Forming Equipment Consider the system shown in [Figure 1: see original paper] containing n renewable energy converters with different control strategies and parameters, and m grid-forming energy storage systems or synchronous condensers. All equipment operate at rated conditions; synchronous condensers and grid-forming converters with large inertia coefficients and approximately voltage source characteristics can provide voltage support capability for the system and can be equivalent to voltage sources with internal reactance in small-disturbance synchronization stability problems dominated by phase-locked loops. According to (15)-(16) and (20), the simplified system matrix and generalized short-circuit ratio for this scenario can be obtained. The specific process is:

First, consider the correction of network dynamics by m grid-forming equipment equivalent admittance, i.e., B_{IBR} = diag(0_n, diag(B_{IBRj}), 0_r). Substituting into (15) yields the network equivalent admittance matrix B'*{red} containing only n equipment connection buses. Second, consider decoupling the rated capacity of n equipment from their dynamics, as shown in (24). Finally, substituting B'*{red} and into (16) and (20) yields the system matrix and generalized short-circuit ratio for this scenario.

$$Y_{IBR}(s) = \mathrm{diag}(Y_{VSCi}(s)) \tag{18}$$

$$U^2/P = \mathrm{diag}(S_{Ni}^{-1}) \tag{19}$$

where B_{IBRj} is the equivalent admittance of the j-th grid-forming converter or synchronous condenser (j = 1,…,m), with B_{IBRj} = diag(1/x_{dj}'') or diag(1/S_{VSMj} B_p). Here, x_{dj}" is the equivalent internal reactance (subtransient reactance) of the j-th synchronous condenser, and 1/S_{VSMj} B_p is the equivalent internal reactance of the j-th grid-forming converter, where S_{VSMj} is the rated capacity of the j-th grid-forming converter and B_p is the equivalent admittance per unit capacity of grid-forming converters.

To intuitively compare the composition of original system matrices under the above three typical scenarios, summarizes the above formulas. Additionally, provides the mechanism of action of the above scenarios on system strength, with detailed discussions available in references [9][10][11][19][20].

4.3 System Strength Quantification Steps

For practical application, this section provides the system strength quantification steps based on generalized short-circuit ratio as follows:

1) According to equipment physical characteristics, treat equipment dynamics as either grid-following equipment dynamics or corrections to network dynamics;
2) Based on network parameters and equipment characteristics, obtain the equivalent network dynamics B + B_{IBR} using (15); based on actual operating conditions, equipment numbers, and B + B_{IBR}, obtain the equivalent admittance matrix (U^2/P)B'_{red} using (15);
3) According to (20), solve for the eigenvalues or singular values of the equivalent admittance or sensitivity matrix, i.e., the generalized short-circuit ratio;
4) Solve for SCR_0 using (21) or (25). Specifically, the former can be obtained through analytical calculation or simulation testing with higher accuracy

[7]; the latter requires combining practical engineering experience and using participation factors (see (18)) for calculation.

$$SCR_0 = \sum_{i=1}^{n} p_i \cdot SCR_{0,i} \tag{20}$$

where SCR_0_i is the minimum short-circuit ratio required for the i-th equipment to reach critical stability.

5) Solve for $\beta\%$ using (12) to quantify strength.

It is worth noting that when the system is a black-box model, equivalent network dynamics and actual operating condition matrices can be obtained through disturbance identification methods [24] and PMU measurement methods.

In summary, by introducing eigen-subsystems, the derivation principles and calculation methods of generalized short-circuit ratio and its critical values are unified across multiple scenarios, with advantages: 1) Physical characteristics are retained—eigen-subsystems preserve the dynamic characteristics of equipment and networks; 2) The method is universal and scalable—not only applicable to complex scenarios of heterogeneous multi-infeed systems but also potentially extendable to stability problems beyond small-disturbance synchronization stability and static voltage stability.

5. Case Studies and Verification

First, a 4-infeed system is used to verify the eigen-subsystem solution method and its effectiveness in describing the stability of the original system. Second, an electromagnetic transient model of an actual renewable energy base is used to verify the effectiveness of the generalized short-circuit ratio indicator in new power systems and to analyze the impact patterns of equipment at non-rated operating points, reverse active power, and inclusion of grid-forming equipment on indicator calculation and system strength.

5.1 Validity of Eigen-Subsystems and Their Solution Method

To verify the effectiveness of eigen-subsystems in describing the stability of the original system, an electromagnetic transient simulation model of a 4-infeed system is built, with topology shown in [Figure 4: see original paper]. Nodes 1 and 4 are connection nodes for conventional and grid-forming energy storage systems, nodes 2 and 3 are connection nodes for renewable energy converters. Equipment control strategies, parameters, and network parameters are shown in Appendices E1-E3.

Specifically, consider proportionally increasing all line inductances shown in Appendix E3 with scaling parameter k = 1, 1.5, 2, 3, 4. Conventional energy storage converters absorb power from the grid, while grid-forming converters inject power. Actual equipment operating conditions are shown in . Based on this, first solve for eigen-subsystems based on eigen-subspace perturbation theory; second, compare the dominant eigenroots of the original 4-infeed system and the eigen-subsystem to verify the effectiveness of eigen-subsystems.

Further, combining the analysis in Section 4.1, theoretically the eigen-subsystem corresponding to the smallest positive eigenvalue λ_1 can approximate the small-disturbance synchronization characteristics of the original system. To verify this conclusion, [Figure 5: see original paper] shows the dominant eigenroots of the 4-infeed system and the eigen-subsystem corresponding to λ_1. It can be seen that as grid strength changes, the eigenroots of the original system and this eigen-subsystem are basically consistent, verifying that this eigen-subsystem can approximate the small-disturbance synchronization characteristics of the original system.

5.2 System Strength Quantification Based on Generalized Short-Circuit Ratio

An electromagnetic transient simulation model of an actual renewable energy base is built, as shown in [Figure 6: see original paper]. The system includes a total of 27 wind turbine units, 2 conventional energy storage systems, and 2 grid-forming energy storage systems across 4 areas. The subscript numbers of wind turbine units or energy storage systems represent the converter numbers used internally. Nodes 1-54 in the figure are equipment connection nodes, where nodes 43 and 54 are grid-forming converter connection nodes, nodes 14, 15, 28, and 29 are conventional energy storage converter connection nodes, and the remaining nodes are renewable energy converter connection nodes. Nodes 55-92 are intermediate nodes, and node 93 is the equivalent node of the external grid outside the plant. Specifically, four case scenarios are considered, with consistent equipment control strategies, parameters, and network parameters as shown in Appendices E1-E3, and actual equipment operating conditions as shown in Appendix E4.

Case 1: All equipment in the system operate under actual conditions, where output active power and terminal voltage often deviate from rated values. Conventional energy storage absorbs power from the grid, while other equipment inject power into the grid.

Case 2: Based on Case 1, all equipment operating conditions are adjusted to rated conditions. Conventional energy storage still absorbs power from the grid, while other equipment inject power.

Case 3: Based on Case 1, the operating mode of conventional energy storage converters is changed to power injection.

Case 4: Based on Case 1, grid-forming converters are removed, i.e., the grid-forming converter connection nodes in the system are left floating.

First, taking Case 1 as an example, the specific solution process for eigen-subsystems is given. The steps are: build the system matrix considering the correction of grid-forming converters to the network; reduce the order of the system matrix and left-multiply it by (U^2/P) I_2; finally, the system matrix is transformed into:

$$T_{het}(s) = Y_{IBR}(s) + \left(\frac{1}{\omega_0}\begin{bmatrix} s & \tau\omega_0 \\ -\tau\omega_0 & s \end{bmatrix} \otimes B'_{red}\right) \tag{21}$$

where Y_{VSC1}(s), Y_{VSC2}(s), Y_{PCS}(s) are the admittance transfer function matrices of renewable energy converters and conventional energy storage converters respectively; $U^2/P = \mathrm{diag}(U_1{}^2/P_1, U_2{}^2/P_2, U_3{}^2/P_3)$.

The matrix (U^2/P)B'*{red} I_2 has three double eigenvalues λ_1 = 5.19, λ_2 = 17.10, λ $_1$ = -53.62, with eigen-subspaces (j = -1,1,2) satisfying the following, where X_j = x_j I_2, Y_j = y_j I_2, and x_j, y_j are the right and left eigenvectors of (U^2/P)B'*{red} with respect to λ_j.

The system matrix is decoupled into three generalized eigen-matrices, i.e., the system matrices of three eigen-subsystems, expressed as:

$$L_j \approx Y_j^H T_{het}(s) X_j = \lambda_j I_2 + Y_j^H G_j(s) X_j \cdot \zeta(s) \tag{22}$$

where i = 1,2,3, and x_{ji}, y_{ji} are the i-th elements of x_j and y_j respectively.

First, the generalized short-circuit ratio of the system in the four cases is analytically calculated according to the above indicator calculation process. Second, SCR_0 for the four cases is obtained based on (21) through building equivalent weighted single-machine simulations or analytical methods. Finally, based on (11), system stability is determined by comparing gSCR with SCR_0, and β% is calculated using (12) to evaluate system strength.

shows the evaluation results. It can be seen that in Cases 1, 3, and 4, gSCR > SCR_0. In Case 1, $\beta\% > 20\%$, meeting small-disturbance stability margin requirements. In Case 2, gSCR = 2.39 < 2.63 = SCR_0, indicating system instability. This stability judgment result is consistent with the time-domain simulation trends of equipment terminal voltages in Cases 1 and 2 shown in Figure 7: see original paper(b), verifying the effectiveness of the quantification method based on generalized short-circuit ratio.

Additionally, Figure 7: see original paper shows the equal-amplitude oscillation waveform of equipment terminal voltages in Case 1 when gSCR = SCR_0, verifying the effectiveness of gSCR = SCR_0 in describing the system stability boundary. The network parameters of the system at this time are shown in Appendix E3.

5.3.1 Impact of Non-Rated Operating Points

Consider comparative analysis between equipment operating under actual conditions (Case 1) and rated conditions (Case 2).

First, in the indicator calculation process, the only difference between Cases 1 and 2 lies in the matrix U^2/P. Specifically: under rated conditions, $U^2/P =$ S_{N52}$^{-1}$, where S_{N52} is a 52$\times 52 diagonalmatrixwithelementsbeingequipmentratedcapacities(negativ$ $absorbingequipment); underactualconditions, U^{\{2\}/P = diag(U_i}\{2\}/P_i)_{52}, a52\times$52 diagonal matrix.

Second, as shown in , both gSCR and SCR_0 in Cases 1 and 2 change with operating conditions, demonstrating the impact of actual operating conditions on system strength.

5.3.2 Impact of Reverse Active Power

Consider comparative analysis between conventional energy storage converters absorbing active power (Case 1) and injecting active power (Case 3).

First, according to the physical characteristics of conventional energy storage converters in charging or discharging modes, the main difference between Cases 1 and 3 in the indicator calculation process lies in matrix U^2/P. Specifically, when conventional energy storage converters absorb power, the diagonal element corresponding to their connection node in U^2/P is negative, making the equivalent admittance matrix have corresponding negative eigenvalues. When injecting power, the corresponding diagonal element is positive, making all eigenvalues positive.

Second, comparing the indicators and critical values in the two cases shows that when energy storage converters operate in charging mode, the system's gSCR is improved to some extent and SCR_0 changes slightly, indicating that energy storage converters in charging mode can improve the generalized short-circuit ratio and change SCR_0.

5.3.3 Impact of Grid-Forming Equipment

Consider scenarios with (Case 1) and without (Case 4) grid-forming converters.

First, according to the physical characteristics of grid-forming equipment, the main difference between Cases 1 and 4 in the indicator calculation process lies in the correction of equivalent network dynamics B_{IBR}. Specifically, when considering grid-forming converters, B_{IBR} contains the equivalent admittance of grid-forming converters, which modifies network dynamics. When not connected, all elements in B_{IBR} are zero.

Second, comparing gSCR and SCR_0 in Cases 1 and 4 shows that after connecting grid-forming converters, the system's generalized short-circuit ratio is improved to some extent and SCR_0 changes slightly, indicating that grid-forming converters can improve the generalized short-circuit ratio and change SCR_0.

6. Conclusions and Outlook

This paper provides the definition, physical meaning, and calculation method of eigen-subsystems for multi-infeed systems, and proposes a general derivation principle and calculation method for generalized short-circuit ratio and its critical values in complex scenarios, including multiple scenarios such as non-rated operating points, reverse active power, and inclusion of grid-forming equipment. Eigen-subsystems retain the dynamic characteristics of AC networks and grid-following equipment and have corresponding low-dimensional physical systems. Therefore, the generalized short-circuit ratio calculation and system strength quantification methods derived from this have relatively rigorous theoretical foundations and clear physical meaning.

This paper focuses on the dominant stability modes of grid-following equipment, where the eigen-subsystem is a single-machine infinite-bus system for grid-following equipment. Under other modes, such as when the control characteristics of grid-forming equipment cannot be ignored, eigen-subsystems need to retain grid-forming equipment, and their characteristics require in-depth research. Furthermore, eigen-subsystems may be basic units of power systems, including but not limited to single-machine infinite-bus systems, and their basic theories and methods need further improvement.

References

[1] ZHOU Xiaoxin, CHEN Shuyong, LU Zongxiang, et al. Technology features of the new generation power system in China[J]. Proceedings of the CESS, 2018, 38(7): 1893-1904 (in Chinese).

[2] JIANG Qirong, WANG Yuzhi. Overview of the Analysis and Mitigation Methods of Electromagnetic Oscillations in Power Systems With High Proportion of Power Electronic Equipment[J]. Proceedings of the CSEE, 2020, 40(22).

[3] Yao Zhiqing, Yu Fei, Qian Zhao, et al. Simulation Research on Large-scale PV Grid-connected Systems Based on MMC[J]. Proceedings of the CSEE, 2013, 33(36): 27-33(in Chinese).

[4] MA Ningning, XIE Xiaorong, HE Jingbo, et al. Review of wide-band oscillation in renewable and power electronics highly integrated power systems[J]. Proceedings of the CSEE, 2020, 40(15): 12 (in Chinese).

[5] IEEE. IEEE Std 2800 — 2022 IEEE standard for interconnection and interoperability of inverter-based resources(IBRs) interconnecting with associated transmission electric power systems[S]. New York: IEEE, 2022.

[6] Li Jianlin, Gao Zhigang, Hu Shuju, et al. Application of Parallel Back-to-back PWM Coverter on the Direct-drive Wind Power System[J]. Automation of Electric Power System, 2008, 32(5): 59-62(in Chinese).

[7] ZHOU Yuhan, XIN Huanhai*, JU Ping. System Strength Quantification Principle and Method of Multi-infeed Systems Based on Generalized Short-

circuit Ratio: Reviews, Discussions and Outlooks[J]. Proceedings of the CSEE, 2023, 43(10): 3794-3811.

[8] YUAN Hui, XIN Huanhai, WANG Guanzhong. et al. Analysis on Oscillation of Multi-infeed System with Renewable Energy and Static Var Generator and Calculation of Its Generalized Short-circuit Ratio[J]. Automation of Electric Power System, 2021, 45(14): 38-46.

[9] WANG Kang, LI Ziheng, YANG Chaoran. et al. Siting Method of Synchronous Condenser for Small-signal Stability Improvement of Large-scale Renewable Energy Base[J]. Automation of Electric Power System, 2022, 46(04): 66-74.

[10] YANG C R, HUANG L B, XIN H H, et al. Placing grid-forming converters to enhance small signal stability of PLL integrated power systems[J]. IEEE Transactions on Power Systems, 2021, 36(4): 3563-3573.

[11] WU Chen, LIU Chenxi, HUANG Wei, et al. Siting Selection and Capacity Determination Method of Grid-forming Converters for Improving Stability of Renewable Energy Power System[J]. Automation of Electric Power System, 2023, 47(12): 130-136.

[12] C.W.G. B4. 62, Connection of wind farms to weak ac networks. CIGRE Tech. Brochure 671, pp. 89–120, 2016.

[13] LIN Weifang, TANG Yong, PU Guangquan. Definition and application of short circuit ratio for multi-infeed AC/DC power systems[J]. Proceedings of the CSEE, 2008(31): 1-8. (in Chinese).

[14] WU Di, LI Gangan, JAVADI Milad, et al. Assessing impact of renewable energy integration on system strength using site-dependent short circuit ratio[J]. IEEE Transactions on Sustainable Energy, vol. 9, no. 3, pp. 1072-1080, July 2018.

[15] SUN Huadong, XU Shiyun, XU Tao, et al. Definition and index of short circuit ratio for multiple renewable energy stations[J]. Proceedings of the CSEE, 2021, 41(2): 497-506 (in Chinese).

[16] DONG Wei, XIN Huanhai, WU Di, et al. Small signal stability analysis of multi-infeed power electronic systems based on grid strength assessment[J]. IEEE Transactions on Power Systems, vol. 34, no. 2, pp. 1393-1403, March 2019.

[17] XIN Huanhai, GAN Deqiang, JU Ping. Generalized short circuit ratio of power systems with multiple power electronic devices: analysis for various renewable power generations[J]. Proceedings of the CSEE, 2020, 40(17): 5516-5526 (in Chinese).

[18] XIN Huanhai, ZHANG Feng, YU Yang, et al. Generalized short circuit ratio for multi-infeed DC systems: definition and theoretical analysis[J]. Proceedings of the CSEE, 2016, 36(03): 633-647 (in Chinese).

[19] Hao Yi, Zhou Yuhan, Liu Chenxi, et al. Small-disturbance Voltage Support Strength Analysis for Renewable Multi-infeed System with Grid-following Energy Storage[J/OL]. Transactions of China Electrotechnical Society: 1-12[2023-08-07].

[20] C. Liu, H. Xin, D. Wu, H. Gao, H. Yuan and Y. Zhou, "Generalized Operational Short-Circuit Ratio for Grid Strength Assessment in Power Systems With High Renewable Penetration," IEEE Transactions on Power Systems, doi: 10.1109/TPWRS.2023.3340158.

[21] G. W. Stewart. Matrix perturbation theory[M]. 1990.

[22] Zhang Xianda. Matrix Analysis and Applications[M]. Tsinghua University Press. 2004.

[23] GB/T 19963.1-2021, Technical specification for connecting wind farm to power system—Part 1: On shore wind power[S].

[24] HUANG Linbin, XIN Huanhai, et al. Identification of Generalized Short-Circuit Ratio for online Stability Monitoring of Wind Farms[J]. IEEE Transactions on Power Systems, vol. 35, no. 4, pp. 3282-3285, July 2020.

Appendix A Example for Lemma 1

Taking a two-dimensional matrix as an example to illustrate Lemma 1. Let M and be:

$$M = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}, \quad \tilde{M} = M + E = \begin{bmatrix} 3.4 & 4.3 \\ 4.2 & 3.1 \end{bmatrix}$$

The generalized eigen-matrix and its approximation are:

$$L_1 = \begin{bmatrix} -0.37 & 0 \\ 0 & -0.37 \end{bmatrix}, \quad \tilde{L}_1 \approx \begin{bmatrix} -0.37 & 0.12 \\ 0.12 & -0.37 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 5.37 & 0 \\ 0 & 5.37 \end{bmatrix}, \quad \tilde{L}_2 \approx \begin{bmatrix} 5.37 & 0.12 \\ 0.12 & 5.37 \end{bmatrix}$$

If $_1$, $_2$, $_3$, $_4$ are 0.4, 0.3, 0.2, 0.1 respectively, the eigen-subspaces of M and are close.

Appendix B Detailed Derivation of Equation (15)

The original system matrix is:

$$T_{het}(s) = \begin{bmatrix} Y_{IBR}(s) + \zeta(s) \otimes B_{11} & \zeta(s) \otimes B_{12} \\ \zeta(s) \otimes B_{21} & \zeta(s) \otimes B_{22} \end{bmatrix}$$

Substituting the submatrix expressions and simplifying yields:

$$T_1(s) - T_2(s)T_4^{-1}(s)T_3(s) = Y_{IBR}(s) + \zeta(s) \otimes (B_{11} - B_{12}B_{22}^{-1}B_{21}) = Y_{IBR}(s) + \zeta(s) \otimes B'_{red}$$

Appendix C Proof of Lemma 2

According to the definitions of eigenvalues and eigen-subspaces, (U^2/P)B'_{red} I_2 has n double eigenvalues λj *with a set of eigen-subspaces , where X_j = x_j* I_2*, Y_j = y_j* I_2*, and the column vectors of X_j and Y_j are the right and left eigenvectors of (*U^2*/P)B'*{red} I_2 with respect to λj*. Therefore, according to Definition 1 (3), the n generalized eigen-matrices of (*U^2*/P)B'*{red} I_2 can be obtained using :

$$L_j = Y_j^H \left((U^2/P)B'_{red} \otimes I_2\right) X_j = \lambda_j I_2$$

Further, when the equivalent equipment models in the original system matrix are similar, , X_j, and Y_j satisfy:

$$MX_j = X_j L_j + O(\epsilon), \quad Y_j^H M = L_j Y_j^H + O(\epsilon)$$

where _i are small values, i = 1,2,3,4.

According to (29), using two eigen-subspaces of M, the approximation of the generalized eigen-matrix can be solved:

Comparing (1) and (37) and combining the definition of eigen-subspace (Definition 1), let X = (X_-q … X_n-q), Y = (Y_-q … Y_n-q). Using X and Y, the original system matrix can be transformed into:

Since the column vectors of X and Y are linearly independent, is equivalent to , i.e., equivalent to . Therefore, the corresponding eigen-subsystem (as the eigen-subsystem matrix) can approximate the stability of the original system. QED.

Appendix D Proof of Theorem 1

Proof: Using can solve the generalized eigen-matrix of the equivalent homogeneous system matrix M_k, satisfying:

$$Y_k^H M_k X_k = L_k$$

Combining (39) and (19), the generalized eigen-matrix of the equivalent homogeneous system is an approximation of that of the original system. According to Lemma 1, using yields the approximation of the generalized eigen-matrix. QED.

Appendix E

Table E1 Control parameters of grid-following converters

Parameter	Renewable Energy Converter	Energy Storage Converter
Equipment rated capacity	1.5 MVA	0.68 MVA (14,15), 0.72 MVA (28,29)
AC system rated capacity	1500 kVA, 620 V	-
DC side capacitance, voltage	0.1089 p.u., 1.1 kV	0.1089 p.u., 1.5 kV
Outer loop PI	0.5, 40 (DC voltage)	0.5, 20 (power)
Filter inductance, capacitance	0.05, 0.0006	0.05, 0.0006
Current inner loop PI	0.2, 60	0.5, 15 (14,15), 0.3, 10 (28,29)
PLL PI	14, 10400 (4-infeed system); 10, 10500 (1-13), 21, 10300 (16-27), 26, 10400 (30-38), 16, 10200 (39-42,44-48), 31, 10200 (49-53) (actual plant)	40, 24000
Voltage feedforward time constant	-	-

Table E2 Control parameters of grid-forming converter

Parameter	Value
Equipment rated capacity	1.5 MVA
Filter inductance, capacitance, resistance	0.05, 0.0005, 0.02
Voltage outer loop, current inner loop PI parameters	0.8, 12; 6, 10
Grid-forming converter inertia coefficient, damping coefficient	-
Voltage feedforward filter time constant	-

Table E3 Parameters of Simulations

4-infeed system grid parameters (p.u.): $X_{1,5} = 0.20$, $X_{1,3} = 0.40$, $X_{2,6} = 0.40$, $X_{3,4} = 0.20$, $X_{3,7} = 0.60$, $X_{1,2} = 0.40$, $X_{23} = 0.40$

Actual renewable energy plant grid parameters ($×10^{-3}$ p.u.): $X_{55,86} = 0.24$, $X_{56,86} = 1.00$, $X_{57,86} = 2.40$, $X_{58,86} = 3.30$, $X_{59,86} = 2.90$, $X_{60,86} = 2.00$, $X_{61,86} = 1.70$, $X_{62,86} = 0.03$, $X_{63,87} = 1.10$, $X_{64,87} = 3.10$, $X_{65,87} = 2.30$, $X_{66,87} = 1.90$, $X_{67,87} = 1.40$, $X_{68,87} = 0.35$, $X_{69,87} = 0.16$, $X_{70,87} = 0.02$, $X_{71,88} = 2.50$, $X_{72,88} = 2.20$, $X_{73,88} = 1.50$, $X_{74,88} = 1.30$, $X_{75,88} = 0.81$, $X_{76,88} = 1.30$, $X_{77,88} = 0.81$, $X_{78,90} = 0.19$, $X_{79,90} = 2.20$, $X_{80,89} = 0.19$, $X_{81,89} = 2.20$, $X_{82,91} = 1.50$, $X_{83,91} = 1.30$, $X_{84,91} = 1.50$, $X_{85,91} = 1.30$, $X_{86,87} = 1.00$, $X_{87,92} = 3.10$, $X_{88,92} = 4.40$, $X_{89,90} = 7.00$, $X_{90,92} = 7.80$, $X_{91,92} = 0.87$, $X_{92,93} = 6.90$

Transformer reactance: 64.0

Note: All grid parameters in Case 1 when gSCR = SCR_0 are 1.1 times the above parameters.

Table E4 The operating conditions of devices in four examples of Practical Renewable Energy Plant

Case	Active Power (p.u.)	Voltage (p.u.)
Case 1	1-13: 0.7; 16-27: 0.8; 30-38: 0.6; 39-42: 0.75; 44-53: 0.9; 14-15: -0.45; 28-29: -0.48; 43,54: 1.00	1-13, 16-27, 30-41, 44-48: 1.01; 14-15, 28-29: 1.03; 42,43,54: 1.00; 49-53: 1.02
Case 2	Same as Case 1 but all at rated active power	Same as Case 1 but all at rated voltage
Case 3	14-15: 0.45; 28-29: 0.48; others same as Case 1	Same as Case 1
Case 4	Same as Case 1	Same as Case 1

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.