Optimization of Dynamic Models for Mars Probes and Orbit Determination Research for Tianwen-1: A Postprint

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Abstract

The dynamic model of the Mars orbit determination software from Shanghai Astronomical Observatory has been optimized and upgraded, leading to the development of MarsODP V2.0, which is capable of processing multiple types of measurement data. High-precision dynamic modeling for Mars has been implemented, taking into account the intrinsic characteristics of Mars, with enhancements including three-body perturbations from Phobos and Deimos, Mars atmospheric drag perturbation, and optimizations of various Mars orientation models. Using actual MEX measurement data, a comparison of the orbit determination accuracy of the Mars orbit determination software before and after optimization was conducted. The computational results indicate that following dynamic model optimization, orbit determination accuracy is significantly improved, with reductions in both position and velocity differences relative to the reference orbit. Additionally, actual measurement data were employed to perform precise orbit determination for the US MRO spacecraft, thereby verifying the orbit determination accuracy after software optimization. Furthermore, orbit determination calculations and analysis were performed for the Tianwen-1 spacecraft's Mars orbit.

Full Text

Preamble

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Dynamic Model Optimization of Mars Probe and Orbit Determination Analysis of “Tianwen-1”

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Abstract

The dynamic model of the Mars probe orbit determination software at Shanghai Astronomical Observatory has been optimized and upgraded, resulting in the development of MarsODP V2.0, which can process multiple types of measurement data. High-precision dynamic modeling of Mars was performed, considering the intrinsic characteristics of Mars itself, with additions including three-body perturbations from Phobos and Deimos, Mars atmospheric drag perturbation, and optimization of various Mars orientation models. Using actual MEX measurement data, the orbit determination accuracy of the Mars orbit determination software before and after optimization was compared. The results demonstrate that after dynamic model optimization, the orbit determination accuracy improved significantly, with position and velocity differences relative to the reference orbit decreasing substantially. The optimized software was used for precise orbit determination of the American MRO probe to verify its accuracy. Finally, orbit determination calculations were performed for the “Tianwen-1” probe’s Mars-orbiting trajectory, and the orbit accuracy was evaluated.

Keywords: Mars probe; precise orbit determination; MRO; MEX; Tianwen-1

1 Introduction

Mars exploration represents humanity’s scientific investigation of Mars through the deployment of space probes, which holds immense significance for understanding Martian geology, landforms, hydrology, atmosphere, and resource utilization. In modern times, deep space exploration of Mars using spacecraft has permeated nearly the entire history of spaceflight.

The United States possesses extensive experience in Mars exploration, with orbit determination accuracy for probes such as Mars Global Surveyor (MGS) and Mars Reconnaissance Orbiter (MRO) reaching approximately 10 meters [1,2]. Although China has accumulated substantial experience in deep space probe orbit determination through lunar exploration projects [3-5], Mars exploration missions present new challenges for Chinese astronomers. Compared to lunar exploration, Mars is far more distant, and its space environment differs from that of the Moon. For instance, Mars has two natural satellites—Phobos and Deimos—possesses a thin atmosphere [6], and its gravity field files must be matched with corresponding orientation models [7]. Consequently, the development of high-precision Mars probe orbit determination software requires sufficiently accurate dynamic models.

Currently, eight probes are operating in Mars orbit: the American Mars Odyssey (ODY), Mars Reconnaissance Orbiter, MAVEN orbiter, ESA's Mars Express (MEX), India's Mars Orbiter Mission (MOM), the ESA-Russian ExoMars Trace Gas Orbiter (TGO), the UAE's Hope, and China's "Tianwen-1." Numerous rovers and landers are also active on the Martian surface, including American rovers Curiosity and Perseverance, and China's Zhurong rover, along with the InSight lander. The surface also hosts completed missions such as Sojourner, Spirit (MER-A), Opportunity (MER-B), and Phoenix, plus the first Mars helicopter Ingenuity.

On July 23, 2020, China's "Tianwen-1" Mars probe was launched from the Wenchang Space Launch Site in Hainan aboard a Long March 5 Y4 rocket, marking China's first independent Mars exploration mission. After two mid-course corrections, "Tianwen-1" successfully completed a deep space maneuver on October 9, 2020. Following two additional corrections, it was captured by Mars on February 10, 2021, through near-Mars braking, becoming a Mars-orbiting probe. After apoapsis orbit adjustment and two periapsis braking maneuvers, the probe successfully entered a large elliptical parking orbit on February 24 with a periapsis altitude of 280 km and an orbital period of 50 hours. The probe maintained flight in this orbit for 80 days to image the rover landing area and select an appropriate landing site. At 7:18 AM on May 15, the rover separated from the orbiter and successfully landed on the Martian surface, becoming China's first Mars lander. On May 17, the orbiter maneuvered into a relay communication maintenance orbit with an 8.2-hour period (250 km × 12,000 km) to maintain communication between the rover and Earth. On November 8, the orbiter transitioned to the remote sensing mission orbit to conduct global remote sensing of Mars.

This paper first optimizes and upgrades the existing Mars probe orbit determination software MarsODP [8] at Shanghai Astronomical Observatory, developing MarsODP V2.0, and compares the orbit determination accuracy before and after optimization using MEX data. Second, precise orbit determination of the American MRO probe verifies the software's correctness and accuracy. Finally, orbit determination calculations are performed for "Tianwen-1's" orbiting phase, and orbit accuracy is evaluated.

2 Mars Probe Dynamic Model Optimization

For China's independent Mars exploration mission "Tianwen-1," the Shanghai Astronomical Observatory orbit calculation team (hereafter referred to as the SHAO team) optimized and upgraded the existing Mars probe orbit determination software MarsODP [8], developing MarsODP V2.0. The specific new functions and models are shown in Table 1 .

MarsODP V2.0 performs high-precision modeling of observations within the solar system barycenter general relativistic reference frame. Building upon the original processing of range, Doppler, and VLBI delay/delay rate measurements, it adds optical angle measurement, same-beam VLBI, and other measurement types, enabling three calculation modes: dynamic statistical orbit determination, kinematic statistical orbit determination, and kinematic statistical positioning. The software can process not only single-target orbiting probes but also perform positioning for surface targets on Mars.

To improve orbit determination accuracy, the SHAO team conducted high-precision dynamic modeling for Mars probes, adding Mars satellite perturbations, Mars atmospheric drag perturbation, and Mars orientation model optimization to the original dynamic model.

2.1 Perturbation Force Magnitude Estimation for Mars Probes

The magnitude of a perturbation force acting on a satellite is typically estimated by the ratio of perturbation acceleration to central gravitational acceleration, representing the degree to which the actual orbit deviates from the two-body problem due to perturbations.

Let P be the perturbation acceleration and be the perturbation magnitude, then:

$$= |P| / (GM/R^2)$$

where GM is the gravitational constant of the central body (Mars) and R represents the position vector of the probe in the inertial frame. For Mars-orbiting satellites at altitudes of 100–1,000 km, the primary perturbation source is non-spherical gravity, followed by third-body gravitational perturbations. For low-orbit satellites, atmospheric drag effects must also be considered. Table 2 provides the magnitude of various perturbation forces acting on Mars probes [9-11].

Based on this analysis, this paper uses the Shanghai Astronomical Observatory Mars orbit determination software with the dynamic models and partial orbit determination strategies shown in Table 3 . The model includes Mars central gravity, non-spherical gravity perturbations, N-body perturbations, and Mars atmospheric perturbation. For N-body perturbations, besides the major planets and solar/lunar gravitational perturbations in the solar system, the three-body perturbations from Phobos and Deimos must be considered for Mars to improve orbit determination accuracy.

2.2 Refinement of Mars Probe Dynamic Model

Compared to lunar exploration, Mars exploration requires consideration of Mars satellite perturbations, Mars atmospheric drag perturbation, and the selection of different Mars orientation models. This section elaborates on software model optimization from three aspects.

2.2.1 Mars Satellite Perturbation Model Establishment Previously, the N-body perturbation in the SHAO comprehensive orbit determination software only considered major planets and solar/lunar gravitational perturbations. For precise orbit determination of Mars probes, the third-body gravitational perturbations from Mars' two natural satellites must be considered.

Phobos and Deimos have irregular shapes covered with impact craters, orbiting near the Martian equatorial plane in near-circular orbits. Phobos is the larger satellite, with triaxial radii of $(13.3 \times 11.1 \times 9.3) \pm 0.3$ km, mass of 10.6×10^{15} kg, and density of 1,900 kg/m^3. Deimos has triaxial radii of $(7.6 \times 6.2 \times 5.4) \pm 0.5$ km, mass of 2.4×10^{15} kg, and density of 1,750 kg/m^3. Although small, the Martian satellites are very close to Mars: Phobos orbits at a semi-major axis of 9,376 km with a period of only 0.31891 days, while Deimos orbits at 23,459 km with a period of 1.26244 days [6,12].

The Mars satellite ephemeris used in this paper is MAR097, released by JPL in 2014 [13]. This ephemeris integrates all Earth- and spacecraft-based observations from 1877–2011, including MEX's three Phobos flybys with Doppler tracking data and additional imaging observations of Deimos before 2011, and represents the primary Mars satellite ephemeris currently in use.

The MEX probe has a semi-major axis of approximately 8,700 km (Mars radius 3,397 km) and eccentricity e 0.57, making it a highly eccentric orbit satellite. Its periapsis altitude is about 300 km, apoapsis altitude exceeds 1×10^4 km, and orbital period is approximately 6.9 hours. Using MEX initial orbit time of 2009-08-07T20:00:00 (UTC), Figure 1 [Figure 1: see original paper] shows the altitude of Phobos and Deimos above the Martian surface during one satellite period, while Figure 2 [Figure 2: see original paper] shows the distance from MEX to the two satellites. The closest approach to Phobos is 4,538 km. Due to MEX's large elliptical orbit, it flies past Phobos and experiences significant gravitational perturbation from the Martian satellites [14].

2.2.2 Mars Orientation Model Optimization The definition of the Mars-fixed coordinate system must match the gravity field model used for orbit determination. Currently, the Mars probe orbit determination software uses the IAU Mars orientation model, while orbit determination employs gravity field models corresponding to the Pathfinder orientation model, such as MGS95J and MRO120D. Model mismatch introduces certain errors into orbit determination and prediction. The transformation relationship between the Mars body-fixed coordinate system and the Mars celestial reference system is shown in Figure 3

[Figure 3: see original paper].

Mars gravity field models can be downloaded from the Planetary Data System (PDS) website (https://pds.nasa.gov/). The documentation accompanying the gravity field model provides parameters required for the Pathfinder orientation model. In MGS95J, the prime meridian plane is selected to be consistent with the IAU 2000 standard at the J2000.0 epoch [1]. Comparing the Pathfinder orientation parameter model corresponding to the MGS95 gravity field model with the IAU 2009 orientation model, we calculated the coordinates of the rotation axis z-axis (0 km, 0 km, 3,396 km) and x-axis (3,396 km, 0 km, 0 km) transformed to the Mars celestial reference system, and compared the differences. Figure 4 [Figure 4: see original paper] shows the differences after coordinate transformation: from 2010–2030, the x-axis transformation difference approaches 100 m, while the z-axis difference approaches 50 m, with annual and semi-annual variation terms.

2.2.3 Mars Atmospheric Density Model Establishment Unlike the vacuum conditions on the Moon's surface, Mars has a thin atmosphere. The Martian atmosphere is primarily composed of CO_2 (95.32%), followed by N, Ar, and small amounts of O and water vapor, with an average pressure of only 700 Pa. The Martian atmosphere is divided into the exosphere, upper atmosphere (thermosphere), middle atmosphere, and lower atmosphere. The exosphere typically refers to the region at and above 200 km, where atmospheric boundaries are indistinct and gradually fade [6].

Due to the Martian atmosphere, atmospheric drag perturbation on low-orbit Mars satellites and landers during descent cannot be ignored. Atmospheric drag perturbation acceleration is a function of Martian atmospheric density, which can be obtained from Mars atmospheric databases or simplified Mars atmospheric models. Mars atmospheric databases comprehensively reflect atmospheric characteristics [15-18], containing not only atmospheric density but also temperature, pressure, and other information. However, the massive data volume makes database access time-consuming and unsuitable for real-time simulation.

Simplified Mars atmospheric models use curve fitting to reduce actual mission data or database-provided data to exponential models. These models typically provide relatively simple atmospheric information, often one-dimensional exponential models of atmospheric density varying with altitude, or layered exponential models, as well as models of temperature, pressure, and density varying with altitude, derived from fitting actual mission data or database information [19,20]. Simplified Mars atmospheric models offer the advantages of minimal computation and fast speed, making them suitable for real-time orbit determination and simulation analysis.

Equation (2) presents the Mars atmospheric model used in this paper, which selects a simple one-dimensional exponential model [21]:

$= {}_0 \times \exp(-h/hs)$

where ${}_0$ is the atmospheric density at the Martian surface (1.58×10^{-2} kg/m^3), hs is the scale height (9,354.5 m), h is the probe's altitude, and is the atmospheric density at the desired altitude.

The one-dimensional exponential model from Joel Benito (2008) considers only the distance from the probe to the Martian surface. Based on the probe's position, the current density can be calculated, and the atmospheric drag acceleration experienced by the probe can then be determined through formulaic calculation.

3 MRO Probe Orbit Determination Analysis Using Actual Data

From August 7, 2009, 20:00 to August 8, 04:13 (UTC), domestic VLBI stations in Shanghai, Kunming, and Urumqi, along with ESA's New Norcia deep space station, organized observation experiments of MEX. The New Norcia deep space station transmitted uplink signals in the X-band, which were transponded by the satellite and transmitted as downlink signals in both S and X bands. Domestic stations received these downlink signals, acquiring three-way Doppler data (5-second integration time) in both S and X bands, while also obtaining two-way Doppler data (1-second integration time) from ESA's NNO station tracking MEX.

Using MEX data from this arc, different software versions were selected for orbit determination calculations. The results were compared with the reference orbit provided by ESA (precise orbit from the Royal Observatory of Belgium, with accuracy of approximately 20–25 m [22]). The orbital differences are shown in Table 4 .

The results indicate that the optimized software significantly improves orbit determination accuracy. Position differences relative to the reference orbit decreased from over 300 meters to approximately 10 meters, while velocity differences decreased from about 0.1 m/s to 0.001 m/s. The substantial improvement is primarily because MEX's large elliptical orbit brings it close to Mars' satellites, making it significantly affected by their gravitational perturbations. After incorporating Mars satellite perturbations, Doppler measurements provide better orbital constraints for MEX, markedly improving orbit determination accuracy.

3 MRO Probe Orbit Determination Analysis Using Actual Data

NASA's Mars Reconnaissance Orbiter (MRO) was launched on August 12, 2005, and entered Mars orbit in March 2006. MRO carries a suite of advanced instruments to study Martian topography and surface conditions, detect subsurface water ice and minerals, monitor daily weather, and identify landing sites for future missions. The orbiter also tested a new communication system capable of transmitting data to spacecraft at higher speeds, enabling MRO to serve as an important relay satellite for other missions. MRO flies in a circular orbit at 255–320 km altitude with a period of approximately 2 hours [23].

This section analyzes MRO orbit determination accuracy using two-way Doppler measurement data from NASA's DSN deep space stations. The DSN stations are globally distributed with uniform coverage. During January–April 2017, multiple DSN stations conducted continuous tracking observations of MRO for 12–24 hours daily, with uniformly distributed observation arcs.

Using two-way Doppler measurement data from January–April 2017 for orbit determination analysis, the dynamic model parameter settings are shown in Table 3, with an orbit determination arc length of 24 hours, integration step of 10 seconds, and simultaneous solution for solar radiation pressure and atmospheric drag coefficients. The orbit determination results for four arcs were compared with NASA's reconstructed orbit. Figure 5 [Figure 5: see original paper] shows larger orbital differences in January, gradually decreasing thereafter, with position differences less than 30 m and velocity differences less than 0.03 m/s.

Figure 6 [Figure 6: see original paper] presents the cumulative days corresponding to different orbit determination accuracies from January–April 2017: 36 days with position differences less than 10 m, 38 days between 10–20 m, and 23 days between 20–30 m relative to NASA's MRO reconstructed orbit; velocity differences were less than 0.01 m/s for 40 days, 0.01–0.02 m/s for 39 days, and 0.02–0.03 m/s for 20 days.

From the January–April 2017 orbit determination results, one daily arc was selected from each month for ephemeris comparison, with results shown in Table 5 . The results indicate that position errors are primarily in the T and N directions, with R-direction errors less than 0.7 m, and Doppler measurement residuals around 0.6 mm/s.

In summary, using single-day two-way Doppler data from January–April 2017 for orbit determination, the post-fit residuals of 1-second integration time ramped two-way Doppler data are approximately 0.5–0.6 mm/s. The solved orbit shows position differences less than 30 m and velocity differences less than 0.03 m/s compared to NASA's reconstructed orbit.

4 "Tianwen-1" Mars Orbit Precision Analysis

This chapter focuses on precise orbit determination of the Mars orbiter, analyzing orbit determination accuracy through overlapping arc comparisons.

4.1 "Tianwen-1" Parking Orbit

After the third near-Mars braking on February 24, 2021, the Mars probe entered a 50-hour period large elliptical parking orbit with a periapsis altitude of 280 km and apoapsis altitude of approximately 59,000 km. The orbiter used multiple scientific payloads to conduct scientific investigations of the Martian space environment, atmosphere, and topography, mapping high-precision surface terrain of the landing area to select an appropriate landing site. To ensure successful scientific research, particularly high-resolution imaging of the landing area, high-precision orbital information is required. During parking orbit flight, three deep space stations conducted relay observations daily, with the VLBI network observing every two days to meet high-precision orbit determination requirements.

Using observation data from February 24–May 15, 2021, for precise orbit determination, the orbit determination arc length was approximately 3–5 revolutions (6–10 days) with an overlapping arc length of 1 revolution (2 days). Measurements used combined range and VLBI data, with overlapping precision analysis results shown in Figure 7 [Figure 7: see original paper]. Post-fit residuals were approximately 0.5 m for range, 0.16 mm/s for Doppler, 0.1 ns for delay, and 0.33 ps/s for delay rate.

The results indicate that for the Mars parking orbit, using combined range, Doppler, and VLBI data yields a 10-day arc orbit determination position accuracy better than 300 m (with R-direction better than 120 m), with errors primarily in the T-direction, and velocity accuracy better than 0.035 m/s.

4.2 “Tianwen-1” Maintenance Orbit

At 7:18 AM on May 15, 2021, the rover successfully landed in Utopia Planitia on Mars. Forty-nine hours later, the orbiter performed an orbital maneuver to enter an 8.2-hour period relay communication maintenance orbit with a periapsis altitude of 250 km and apoapsis altitude of approximately 12,000 km. The relay orbit is a recurring orbit operating three revolutions per Martian sol, passing over the landing site each sol to enable two relay communications with the rover per Martian day (near-periapsis and near-apoapsis). On November 8, the orbiter maneuvered to the remote sensing mission orbit to conduct global remote sensing of Mars.

After May 30, routine observations were conducted with two stations in relay mode, shortening daily observation arcs, with VLBI observations three times per week for 1 hour each. Intensive observations were conducted for a period before and after each orbital maneuver.

This section selected actual data from the “Tianwen-1” orbiter during May 24–June 11, 2021, for orbit determination accuracy analysis. This arc featured frequent small orbital maneuvers (every 8–12 hours), requiring solution of multiple empirical force parameters. Measurements used combined range and VLBI data, with range system errors solved by arc segment.

Overlapping arc analysis used approximately 48-hour arcs (5–7 revolutions) with 1 revolution (8 hours) overlap, as shown in Figure 8 [Figure 8: see original paper]. Post-fit range residuals were better than 0.4 m, and VLBI delay residuals were better than 0.2 ns.

Orbital overlapping arc comparison results are shown in Figure 9 [Figure 9: see original paper]. The results indicate that the maintenance orbit segment achieves average orbit determination position accuracy better than 180 m (with radial direction better than 3 m), with errors primarily in T and N directions, and velocity accuracy better than 0.035 m/s. Intensive observation arcs achieve higher orbit determination accuracy, with average position accuracy better than 100 m, while routine observation arcs show poorer accuracy with average position accuracy of approximately 350 m.

5 Summary

This paper refined and optimized the existing Shanghai Astronomical Observatory Mars probe orbit determination software for China's Mars exploration mission, adding Mars satellite perturbations, Mars orientation model optimization, and Mars atmospheric perturbations.

Using the optimized high-precision Mars orbit determination software for MRO probe orbit determination, comparison with the reference orbit shows average position accuracy better than 30 m and velocity accuracy better than 0.03 m/s, verifying the software's correctness and reliability.

Orbit determination of "Tianwen-1's" Mars parking orbit shows that 3–5 revolution arc lengths achieve position accuracy better than 300 m (with R-direction better than 120 m), with errors primarily in the T-direction, and velocity accuracy better than 0.035 m/s. Maintenance orbit determination achieves average position accuracy better than 180 m (with radial direction better than 3 m) and velocity accuracy better than 0.035 m/s. During intensive observations, orbit determination accuracy is higher, with average position accuracy better than 100 m.

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Note: Figure translations are in progress. See original paper for figures.