Postprint: Research Advances in Navigation Satellite Attitude and Solar Radiation Pressure Models

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Date: 2023-12-13T00:00:00+00:00

Abstract

Satellites across different navigation systems exhibit variations in shape, material, orbital design, and attitude control mode. In multi-system data joint processing, achieving high-precision satellite orbits necessitates differentiated treatment of attitude models and solar radiation pressure models for various satellite types. As the most significant non-conservative force acting on navigation satellites, extensive research both domestically and internationally has focused on the establishment and refinement of solar radiation pressure models to develop optimal models and thereby enhance orbit determination accuracy. Current studies demonstrate that new-generation Galileo and BeiDou satellites utilize attitude control modes distinct from their predecessors, consequently requiring different solar radiation pressure models. This work summarizes the currently prevalent navigation satellite attitude models, particularly the latest officially published attitude models for BeiDou satellites, and elaborates on the research progress of satellite solar radiation pressure models along with construction methodologies for solar radiation pressure models and their a priori models for recently launched satellites, while analyzing the merits, drawbacks, and applicability of each model. Through an analysis of existing navigation satellite attitude and solar radiation pressure models, this paper identifies issues warranting improvement for higher-precision orbit determination missions, offering references for the selection and development of solar radiation pressure and attitude models in precise orbit determination for navigation satellites, especially newly launched ones.

Full Text

Preamble

Vol. 41, No. 3

September 2023

PROGRESS IN ASTRONOMY Vol. 41, No. 3 Sept., 2023 doi: 10.3969/j.issn.1000-8349.2023.03.03

Research Progress of Attitude and Solar Radiation Pressure Model for Navigation Satellites

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Abstract

Satellites from different navigation systems vary in shape, material, orbit design, and attitude control mode. To obtain high-precision satellite orbits in multi-system data processing, different types of satellites require distinct treatment in terms of attitude models and solar radiation pressure (SRP) models. As the largest non-conservative force acting on navigation satellites, SRP has been the focus of extensive research worldwide on model establishment and refinement to achieve optimal models and higher orbit determination accuracy. Current studies indicate that new Galileo and BeiDou satellites employ different attitude control modes compared to earlier satellites, and consequently require different SRP models. This paper summarizes the currently used attitude models for navigation satellites, particularly the recently officially published attitude models for BeiDou satellites. It provides a detailed review of research progress on satellite SRP models, including models for newly launched satellites and methods for constructing their a priori models, and analyzes the advantages, disadvantages, and applicability of each model. Through analysis of current navigation satellite attitude and SRP models, we identify issues that need improvement for higher-precision orbit determination tasks, providing a reference for the selection and establishment of SRP and attitude models in precise orbit determination of navigation satellites, especially newly launched ones.

Key words: navigation satellite; solar radiation pressure; satellite attitude; precise orbit determination

CLC number: P128.15 **Document code:** A

1 Introduction

The European Union's Galileo system and China's BeiDou Navigation Satellite System (BDS), as new-generation global navigation satellite systems (GNSS), currently provide global services, while the earlier GPS and GLONASS systems continue to develop and modernize their satellites. The emergence of multiple satellite navigation systems aims to provide global coverage, all-weather, and high-precision navigation, positioning, and timing services. Precise satellite orbits are a prerequisite for achieving high-precision navigation and positioning services, and precise orbit determination and prediction require support from high-precision dynamic models.

The main non-conservative forces acting on navigation satellites include solar radiation pressure (SRP), Earth albedo radiation pressure, thermal radiation, and antenna electromagnetic radiation, which are primarily influenced by satellite attitude, geometric structure, and material properties [1,2]. SRP is the force generated when solar photons strike the satellite surface. As the largest non-conservative force on navigation satellites, SRP errors accumulate in the integrated satellite velocity and position, seriously affecting precise orbit determination accuracy [3]. Currently, SRP modeling and refinement are active research topics among many International GNSS Service (IGS) analysis centers. Studies show that different SRP modeling methods lead to differences in orbit products from different IGS analysis centers [4]. In addition to total solar irradiance (TSI) and the relative position between satellite and Sun, SRP is also affected by satellite attitude, area-to-mass ratio, shape, and materials, with attitude control precision directly impacting SRP model accuracy. Attitude control law forms the basis for establishing SRP models.

For navigation satellites, numerous SRP models are currently available, but the optimal model and corresponding a priori model for certain satellites are not entirely clear. For the same satellite type, applicable SRP models may differ during different orbital periods (eclipse vs. non-eclipse). The applicability of SRP models for different satellite types in BDS, which employs a heterogeneous constellation, requires further verification. The attitude and SRP models for newly launched GPS III, GLONASS-K, and BDS-3 satellites also need validation. This paper introduces the attitude models of various navigation systems and commonly used SRP models, analyzes their applicability, and provides references for SRP modeling and optimization of navigation satellites.

2.1 Satellite Attitude Control Law

Satellite attitude refers to the spatial orientation state of satellite axes in orbit. For normal operation, navigation satellites must point their antennas toward Earth's center and keep their solar panel planes perpendicular to the satellite-Sun line. Under nominal attitude, these two conditions are satisfied. To maintain nominal attitude, the satellite attitude control system requires continuous adjustment.

Satellite attitude affects precise orbit determination in two aspects: (1) correction of geometric errors such as satellite antenna phase center offset and variation; and (2) changes in illuminated surfaces caused by attitude variations, leading to changes in SRP and Earth albedo radiation pressure. Therefore, accurate satellite attitude must be obtained first to construct high-precision SRP models and achieve precise orbit determination [5]. In precise orbit determination, satellite attitude is primarily used to describe the transformation between the spacecraft-fixed coordinate system and inertial coordinate system. The GNSS satellite body-fixed coordinate system has its origin at the satellite's center of mass, with the Y-axis along the solar panel rotation axis, Z-axis along the L-band antenna signal transmission direction, and X-axis completing the right-handed coordinate system. [Figure 1: see original paper] shows the body-fixed coordinate system and the relative positions of satellite, Earth, and Sun [1]. Here, β is the Sun elevation angle representing the angle between the Sun-Earth line and the orbital plane; is the angular separation between Earth center and Sun center as viewed from the satellite; and u is the orbital angle representing the angle between the apogee-Earth line and satellite-Earth line within the orbital plane. The X-axis reflects the satellite roll direction, Y-axis the pitch direction, and Z-axis the yaw direction. The three-axis attitude stabilization module of navigation satellites can control the body-fixed Z-axis pointing error to within 0.1°, making roll and pitch effects negligible [2]. Navigation satellite attitude control primarily concerns yaw control. [Figure 2: see original paper] and [Figure 3: see original paper] show schematics of medium Earth orbit (MEO) satellites developed by the China Academy of Space Technology (CAST) and Shanghai Engineering Center for Microsatellites (SECM), respectively.

The nominal yaw angle is the angle between the positive X-axis of the body-fixed system and the satellite velocity direction, calculated as:

= ATAN2(($-\tan \beta$; sin u)) (1)
= ATAN2($\tan \beta$; ($-$ sin u)) (2)

where ATAN2 equals arctan(a/b) with return range [$-180°$, $+180°$], and β, u are consistent with those in [Figure 1: see original paper]. For different satellites, when the positive X-axis points toward the Sun, equation (1) applies, used by GPS Block IIF, GLONASS-M, BeiDou IGSO, and MEO satellites. When the positive X-axis points away from the Sun, equation (2) applies, used by GPS Block IIR, Galileo, etc. [5,6]. The difference between the two modes lies in the nominal X and Y axes. Due to limited momentum wheel adjustment rates, current GNSS satellite attitude control strategies mainly fall into two categories: orbital maneuvers when the Sun elevation angle is below a certain threshold, or maneuvers when the momentum wheel adjustment rate exceeds a certain value [5].

Taking partial derivatives of equations (1) and (2) with respect to time yields the yaw rate:

˙ = $\dot{u}$ tan β cos u / ($\sin^2$u + $\tan^2\beta$) (3)

where $\dot{u}$ is the satellite's mean motion angular rate. ˙ reaches its maximum value of $\dot{u}$ tan β at u = 0°, 180°, meaning the satellite yaw rate may exceed the momentum wheel adjustment rate limit near perigee and apogee, triggering orbital maneuvers. Since optical sensors require sufficient illumination, some satellites also perform eclipse maneuvers when the Sun elevation angle is small.

2.2 Existing Navigation Satellite Attitude Models

Current GNSS systems employ three main attitude control modes: nominal yaw (dynamic) mode, maneuver yaw control (continuous yaw control) mode, and yaw-steering mode. Nominal yaw mode maintains the satellite in nominal attitude as the yaw angle varies with the relative positions of Sun, Earth, and satellite. Yaw-steering mode fixes the yaw angle at 0° to ensure the angle between the solar panel normal and sunlight does not exceed a set value. For satellites during eclipse periods, maneuver yaw control mode implements designed attitude changes or maximum angular velocity changes near midnight and noon to achieve target yaw angles. Compared with yaw-steering mode, maneuver yaw control mode significantly mitigates service performance degradation [5,7].

GPS satellites have two maneuver patterns: one with noon and midnight maneuvers; the other with noon and eclipse maneuvers (with a large eclipse period range, thus no midnight maneuver). Block IIR satellites follow the first pattern, while Block IIF and Block II/IIA follow the second. Noon and midnight maneuver models are basically consistent across satellite types: when the Sun elevation angle is below the design value and reaches the orbital angle for initiating maneuvers, the momentum wheel rotates at maximum angular velocity until actual attitude matches nominal attitude. Block IIF and Block II/IIA satellites differ in eclipse maneuvers: for Block II/IIA, the momentum wheel rotates at maximum angular velocity until exiting eclipse, but since the attitude at eclipse exit is inconsistent with nominal attitude, an eclipse recovery period exists; for Block IIF, a fixed angular velocity is calculated based on the yaw angle at eclipse exit, resulting in a shorter eclipse recovery period (within 5 minutes) whose impact can be neglected [5,7,8].

Currently, most operational GLONASS satellites are GLONASS-M, while subsequently launched GLONASS-K1 and GLONASS-K2 satellites differ mainly in their adoption of a non-hermetic bus design based on honeycomb panels [9]. GLONASS-M satellites perform eclipse and noon maneuvers. When $|\beta| \leq 14.2°$, eclipse maneuvers begin, with the satellite adjusting at approximately 0.25°/s to reach the nominal attitude at eclipse exit as quickly as possible, after which the yaw angle remains constant until eclipse exit. During noon maneuvers, the satellite begins attitude control at maximum adjustment rate at a certain moment before noon, maintaining this until the satellite satisfies attitude control law after passing noon, ensuring symmetric attitude distribution about noon to minimize user impact [9]. Current research indicates GLONASS-K adopts a similar attitude control mode to GLONASS-M during eclipse periods [10].

For Galileo, in October 2017, the European GNSS Agency disclosed relevant satellite parameters including yaw attitude models to facilitate Galileo data processing. Galileo satellites adopt maneuver yaw control during orbital maneuvers. The IOV satellite attitude model during orbital maneuvers is:

= ATAN2(Shy; Sx) (4)

where Sx, Sy, Sz are Sun unit vectors calculable from Sun elevation angle β and orbital angle u (starting from perigee), expressed as:

$[\mathrm{Sx}\ \mathrm{Sy}\ \mathrm{Sz}] = ((-\sin u \cos\beta)\ (-\sin\beta)\ (-\cos u \cos\beta))$ (5)

Shy is the smoothed Sy. The FOC satellite attitude model during orbital maneuvers is:

$= 90° \cdot \mathrm{SIGN}(1;\ (us)) + [\ (us) - 90° \cdot \mathrm{SIGN}(1;\ (us))] \cdot \cos(\pi/2) \cdot (u - us) / (t\mathrm{max} \cdot \dot{u})$ (6)

where us is the orbital angle at maneuver start, (us) is the nominal attitude angle at maneuver start, SIGN(a; b) returns the sign of b with magnitude a, tmax is the maximum maneuver duration, and $\dot{u}$ is the average orbital angle change rate. The Galileo attitude model ensures yaw rate remains within 0.203°/s, with maneuver start and end positions symmetric about midnight (or noon), ensuring actual yaw angle matches nominal yaw angle ($±$90°) at noon (or midnight) [5].

For BDS-2, Guo Jing [5] established attitude models for IGSO and MEO satellites: yaw-steering mode when $|\beta| \le 4°$, nominal yaw mode when $|\beta| > 4°$. Based on this, Dai et al. [11] developed a more precise attitude model with switching conditions: nominal-to-yaw-steering when $|\beta| \le 4°$ and $|\ | \le 5°$, or $|\beta| \le 4°$ and $5° < |\ | < 20°$ with ˙ > 0; yaw-steering-to-nominal when $|\beta| > 4°$ and $|\ | \le 5°$, or $|\beta| > 4°$ and $5° < |\ | < 20°$ with ˙ > 0. The BDS-2 satellite IGSO-6 (C016, C13) launched in March 2016 adopted not nominal yaw and yaw-steering, but nominal yaw and maneuver yaw control, similar to Galileo FOC satellites. Additionally, MEO-6 (C015, C14) followed maneuver yaw control after October 2016, and IGSO-1 (C05, C06) after March 2017, indicating that attitude control modes of in-orbit satellites can be reconfigured. Currently, all BDS GEO satellites adopt yaw-steering attitude control mode [12].

For CAST's BDS-3 satellites, when the Sun elevation angle is between $±$3° and the angle between the Sun-Earth vector projection in the orbital plane and the Earth-satellite line is ≤ 30°, the yaw angle is approximately:

$= \mathrm{ATAN2}(\tan\delta;\ \mathrm{Sox})\ \beta > 0$
$= -\mathrm{ATAN2}(\tan\delta;\ \mathrm{Sox})\ \beta \le 0$
$\delta\mathrm{m} = 3/57.3$ (rad)
$\delta = \delta\mathrm{m} \cdot (\cos(0.5\mathrm{Soz}))^2$ (7)

where Sox is the X-component of the Sun vector in orbital coordinates, and Soz is the Z-component.

SECM-developed BDS-3 satellites adopt the following yaw attitude model [1]:

$= \mathrm{ATAN2}((-\tan\beta;\ \sin u))\ \mathrm{ATAN2}((-\tan\beta_0;\ \sin u))\ 0 < \beta < \beta_0$
$= \mathrm{ATAN2}(\tan\beta_0;\ \sin u)\ |\beta| > \beta_0$
$= -\mathrm{ATAN2}(\tan\beta_0;\ \sin u)\ (-\beta_0 < \beta < 0)$ (8)

where $\beta_0 = 3°$ is the set threshold. Current research shows that actual satellite attitudes are not completely consistent with currently published attitude models, and BDS-3 satellite attitude variations at low Sun elevation angles require further study. summarizes attitude control modes for currently used navigation system satellites [5,7,8,14,16].

3 Solar Radiation Pressure Models

Satellite attitude models affect non-conservative forces including SRP and are prerequisites for precise orbit determination and SRP model establishment. Besides satellite attitude, shape, and material properties, SRP is also affected by TSI and the area of satellite surfaces illuminated by the Sun, all of which vary continuously and should be considered in SRP modeling. Studies show that periodic variations in real solar activity have amplitude less than 0.1%, and GPS orbit determination accuracy based on variable TSI models improves slightly by 0.1–0.5 mm [17]. For BDS IGSO and MEO satellites, the improvement is within 0.5 mm, while for BDS GEO satellites it can reach 3 mm [18]. Additionally, since parts of navigation satellites are shaded by Earth (or Moon) during operation, the shadow factor (eclipse factor) represents the ratio of the Sun's apparent area when partially shaded to its full apparent area, requiring calculation for accurate SRP. Navigation satellite SRP models can be categorized into three types: analytical, empirical, and semi-analytical models. Below we introduce several commonly used SRP models.

3.1.1 ROCK Series Models

The ROCK model is an SRP model designed by Porter et al. (1976) at Rockwell Corporation, the GPS satellite manufacturer, based on satellite external shape and materials. Described in the spacecraft-fixed coordinate system, this model simplifies satellite structure and establishes SRP models based on optical parameters, shadow factors, and occlusion relationships between components [19]. Due to differences in shape and material optical properties between Block I and Block II satellites, ROCK4 (1985) and ROCK42 (1989) models were developed respectively [20,21].

For any plane, the model includes three force components generated by incident light, specular reflection, and diffuse reflection. The normal component is perpendicular to the surface, consisting of the normal component of incident light plus recoil from specular reflection:

F_1,N = $(-AE/c)(1 + \ ) \cos^2$ (9)

where is reflectivity, s is specular reflectivity (both ranging 0–1), A is plane area, and is the angle between incident sunlight and plane normal. The solar pressure constant is solar power constant E (1368 W/m^2) divided by speed of light.

The tangential component is tangent to the plane, directed away from the Sun, generated by the tangential component of incident light minus momentum carried away by the specular reflected beam:

$F_1,S = (-AE/c)(1 - \ s) \sin \ \ \cos$ (10)

The diffuse reflection component is in the same direction as the normal component:

$F_1,D = (-2/3)(AE/c)\ (1 - s) \cos$ (11)

For cylinders, the three components are:

$F_2,N = (-AE/c)(1 + \ s) \cos^2$
$F_2,S = (-AE/c)(1 - \ s) \sin \ \ \cos$
$F_2,D = (-\pi/2)(AE/c)\ (1 - s) \cos$ (12)

where A is the cross-sectional area through the cylinder axis.

Based on component structure and optical property parameters of each satellite, all auxiliary angles are expressed in terms of angle B between Sun and +Z direction, enabling calculation of SRP components in X and Z axes of the body-fixed coordinate system with B as the only variable. The original ROCK model is called the standard ROCK model (ROCK S model, S10 for Block I and S20 for Block II/IIA). The S10 model is (in units of 10^{-5} N):

$X = -4.34 \sin(B) + 0.10 \sin(2B + 1.1) - 0.05 \cos(4B) + 0.06$
$Z = -4.34 \cos(B) + 0.17 \sin(2B - 0.4) - 0.05 \sin(4B) - 0.06$ (13)

The S20 model is:

$X = -8.10 \sin(B) + 0.05 \cos(2B) - 0.056 \sin(4B + 1.4) + 0.07$
$Z = -7.80 \cos(B) + 0.024 \sin(2B - 0.8) - 0.047 \sin(4B + 0.9) - 0.02$ (14)

Subsequently, considering Block II satellite upgrades and satellite thermal radiation, new thermal ROCK models were developed (ROCK T model, T10 for Block I, T20 for Block II/IIA, T30 for Block IIR) [19,22]. The upgraded T10 model is:

$X = -4.55 \sin(B) + 0.08 \sin(2B + 0.9) - 0.06 \cos(4B + 0.08) + 0.08$
$Z = -4.54 \sin(B) + 0.20 \sin(2B - 0.43) - 0.03 \cos(4B)$ (15)

The T20 model is:

$X = -8.96 \sin(B) + 0.16 \sin(3B) + 0.10 \cos(5B) - 0.07 \sin(7B)$
$Z = -8.43 \cos(B)$ (16)

The T30 model is:

$X = -11.000 \sin(B) - 0.20 \sin(3B) + 0.20 \sin(5B)$
$Z = -11.30 \cos(B) + 0.10 \cos(3B) + 0.20 \cos(5B)$ (17)

3.1.2 Box-Wing Model

Given the complex shape of satellites, Marshall and Luthcke [23] (1992) developed a Box-Wing model for precise orbit determination of Topex/Poseidon satellites. With given geometric and physical parameters of each surface and satellite attitude information, the SRP perturbation force from each surface can be calculated, and the total SRP force is obtained by superposition [24]. This analytical model can reflect the SRP perturbation force on satellites. The model simplifies satellite components by treating the six faces of the satellite bus and two faces of solar panels separately. The SRP on one plane can be expressed as [25,26]:

a = $(-S_0/c)$(A/M) cos [(1 −)e_D + 2(_d/3 + cos)e_N] (18)

where is the fraction of reflected photons (specular reflection coefficient), _d is the fraction of scattered photons (diffuse reflection coefficient), and _a is the fraction of absorbed photons (absorption coefficient), with _a + + _d = 1, convertible from reflectivity and specular reflectivity in equation (9): _a = 1 − , = s , _d = (1 − s). A is the surface area, M is total mass, S_0 = 1367 W/m^2 represents solar TSI, c is speed of light, e_D and e_N are direction vectors pointing toward the Sun and plane normal respectively, and is the angle between e_D and e_N.

Solar panel SRP can be calculated using equation (18). For the satellite bus with insulated surfaces, energy absorbed by the surface is re-radiated as heat according to Lambert's law. Subtracting the reflected SRP in equation (18), the SRP on the bus can be calculated as:

a = $(-S_0/c)$(A/M) cos [(_a + _d) + 2 cos]e_N (19)

Analytical models have clear physical meaning and are easy to understand, but generally have relatively complex formulas and obvious disadvantages: (1) For satellites with complex structures and occlusions, accurate modeling is difficult, and analytical SRP models can only describe the main SRP components, not accurately calculate the forces; (2) With in-orbit service life typically over a decade, satellite surface material aging causes continuous changes in SRP during different operational phases [2].

3.2.1 Colombo Model

Regarding empirical models, Colombo (1989) discovered that GPS broadcast ephemeris orbit errors exhibit resonance phenomena that could be absorbed by an empirical model, effectively eliminating effects from various perturbing forces including Earth gravity field and SRP [27]. The model can be written as Fourier components with resonance frequencies of 0 and n_0:

Δx_i = A_{xi} cos n_0t + B_{xi} sin n_0t + C_{xi} (20)

where i = 1,2,3 correspond to orbit errors in three directions, with estimable parameters A_{xi}, B_{xi}, C_{xi}. However, when applied to GPS orbit improvement, this model considered Earth gravity field as the main error source without explicit correlation to satellite-Sun geometry, making it incomplete for SRP improvement.

3.2.2 ECOM Series Models

3.2.2.1 Original ECOM Model

In 1994, Beutler et al. at the University of Bern developed the Empirical CODE Orbit Model (ECOM) based on the Colombo model using CODE data. The model adopts a DYB coordinate system decomposing SRP into three directions suitable for modeling: D (satellite-to-Sun direction), Y (solar panel rotation axis), and B (orthogonal to both forming a right-handed system). ECOM expanded estimable parameters to nine (D_0, Y_0, B_0 and six periodic parameters) and combined with ROCK model a priori information to achieve superior orbit determination accuracy [28]. The specific model is:

$$a_{SRP} = a_{ROCK} + a_D \cdot e_D + a_Y \cdot e_Y + a_B \cdot e_B \quad (21)$$

where the first term is the a priori value from ROCK model, and a_D, a_Y, a_B are SRP in D, Y, B directions respectively:

$$a_D = D_0 + D_c \cos u + D_s \sin u$$
$$a_Y = Y_0 + Y_c \cos u + Y_s \sin u$$
$$a_B = B_0 + B_c \cos u + B_s \sin u \quad (22)$$

where u is the argument of latitude in [0°, 360°] (distinct from orbital angle u). Although the 9-parameter ECOM was developed before Block IIR satellite launch, subsequent applications showed it also applied to Block IIR satellites. The model has been used at CODE analysis center since 1996 [33]. Nine parameters increase computational degrees of freedom; for three-day solutions, not all nine parameters need estimation, and significant correlations exist between orbital parameters and others (e.g., UT1-UTC), which reduce to acceptable levels when arc length increases to 5 days. Further studies showed that ignoring B-direction parameters while adding five stochastic force parameters significantly improved orbit determination accuracy (by 5–15 cm) [34].

3.2.2.2 ECOM1 Model

Springer et al. (1999) found that too many SRP parameters introduced biases in other estimates due to parameter correlations, and thus proposed a simplified ECOM model (ECOM1) retaining only one parameter each in D and Y directions. ECOM1 yields better orbit determination results for GPS satellites by avoiding correlation between D and Y direction parameters [29,30]. This currently most widely used SRP model has five parameters:

$$a_D = D_0$$
$$a_Y = Y_0$$
$$a_B = B_0 + B_c \cos u + B_s \sin u \quad (23)$$

Studies show that periodic terms in B direction significantly optimize model deficiencies compared to other periodic terms. Orbit determination accuracy improved by 2–3 times compared to earlier ECOM. New a priori models were developed for Block II and Block IIA satellites based on ECOM, determining six parameters (D, Y, B, Z_1, X_1, X_3) and their relationships with β, fitting model coefficients using 5.5 years of orbital data [29]. This model improved accuracy by an order of magnitude over ROCK models and replaced them at CODE from 2005 to 2013. Subsequent studies showed that using ECOM1 alone for GPS and GLONASS precise orbit determination also achieved high accuracy, and CODE used ECOM1 exclusively for some time after 2014 [33].

ECOM1 applies to various satellites including GPS, GLONASS, and BDS-2 for SRP calculation. However, for Galileo and QZSS satellites, laser validation results show systematic errors correlated with β angle, primarily because their rectangular bus structures prevent complete SRP absorption or compensation by ECOM1 [29,32]. BDS-3 satellites also have rectangular bus structures, making ECOM1 unsuitable.

3.2.2.3 ECOM2 Model

For a long time, analysis of Earth rotation parameters and station coordinate time series power spectra revealed spurious spectral lines that became more pronounced after GLONASS inclusion, later identified as primarily caused by ECOM1 [36]. Arnold et al. [33] at CODE (2015) further studied GPS and GLONASS satellites regarding ECOM1 issues, showing that SRP produces only even-order perturbations in D direction and odd-order perturbations in B direction. The final model expression depends on the number of periodic terms in D and B directions. Through comparative analysis, two periodic terms in D direction and one in B direction were selected, creating a 9-parameter model called ECOM2:

$$a_D = D_0 + D_{2c} \cos(2\Delta u) + D_{2s} \sin(2\Delta u) + D_{4c} \cos(4\Delta u) + D_{4s} \sin(4\Delta u)$$
$$a_Y = Y_0$$
$$a_B = B_0 + B_c \cos(\Delta u) + B_s \sin(\Delta u) \quad (24)$$

Unlike previous ECOM series, this model adds even-order periodic parameters in D direction. Additionally, the independent variable changes from u to $\Delta u = u - u_s$, the orbital angle between Sun and satellite in the orbital plane, where u_s is the angle between ascending node and Sun. Δu maintains stable phase reference for periodic parameters, independent of Earth's revolution, facilitating parameter analysis.

Results from 2009–2011 GPS and GLONASS combined orbit determination show that ECOM2 significantly reduces periodic errors in geocenter coordinate Z component compared to ECOM1, while improving day-boundary orbit discontinuities, Earth rotation parameter consistency, and orbit internal accuracy. Since January 4, 2015, CODE has used ECOM2 for analysis center product calculations.

However, ECOM2 is not optimal for all satellite types. Compared to ECOM1, ECOM2 reduces precise orbit determination accuracy for BDS-2 IGSO and MEO satellites [36]. For GLONASS satellites, ECOM1 yields higher accuracy during non-eclipse periods, while ECOM2 performs better during eclipse periods, because radiators on the (−X) face produce disturbing accelerations during eclipse that can be significantly improved by establishing an effective a priori model [10]. For BDS-3 SECM satellites, studies show that ECOM2 yields smaller mean laser validation residuals than ECOM1, indicating that some radial errors can be compensated by ECOM2's added periodic terms in D direction, making ECOM2 more suitable for calculating SRP for this satellite type [2].

3.2.3 GSPM Series Models

The GPS Solar Pressure Model (GSPM) developed by NASA's Jet Propulsion Laboratory (JPL) calculates SRP perturbation forces using Fourier series in (see [Figure 1: see original paper]), representing another high-precision SRP model.

Bar-Sever (1997) developed the GSPM.97 model expressed in DYB coordinates:

$$D = f(m; r)s[D_0 + D_{c1} \cos \ + D_{s1} \sin \ + D_{c2} \cos 2 \ + D_{s2} \sin 2 \ ]$$
$$Y = Y_0 + Y_{c1} \cos \ + Y_{s1} \sin$$
$$B = f(m; r)s[B_{c1} \cos \ + B_{s1} \sin \ + B_{c2} \cos 2 \ + B_{s2} \sin 2 \ ] \quad (25)$$

where f(m; r) is a function of satellite mass m and satellite-Sun distance r, and s is a scale factor absorbing errors from satellite materials and environmental changes. GSPM.97 estimable parameters are s, D_0, and Y_0.

In 2004, Bar-Sever and Kuang [37] further developed GSPM.04 in spacecraft-fixed XYZ coordinates, creating GSPM.IIA.04 for Block IIA and GSPM.IIR.04 for Block IIR satellites:

$$X = s \cdot 10^{-5}(AU/r)^2/m[X_{s1} \sin \ + X_{s2} \sin 2 \ + X_{s3} \sin 3 \ + X_{s5} \sin 5 \ + X_{s7} \sin 7 \ ]$$
$$Y = Y_0 + 10^{-5}(AU/r)^2/m[Y_{c1} \cos \ + Y_{c2} \cos 2 \ ]$$
$$Z = s \cdot 10^{-5}(AU/r)^2/m[Z_{c1} \cos \ + Z_{c3} \cos 3 \ + Z_{c5} \cos 5 \ ] \quad (26)$$

where AU is astronomical unit, with estimable parameters s and Y_0. The new model improved orbit fitting accuracy by 24% and 80% for Block IIA and Block IIR satellites respectively, and orbit prediction accuracy by 32% and 58%. New versions GSPM10 and GSPM13 were subsequently developed using data from January 1997–May 2010 and 1992–2013 respectively [38,39].

JPL's GIPSY-OASIS software studies show that GSPM-based methods yield more precise solutions for orbit overlap accuracy, laser validation residuals, and station coordinate repeatability, while ECOM1 provides higher accuracy for overlap clock errors and length-of-day variations. In 2008, 5–6 of 8 IGS analysis centers used ECOM1, with 1–2 not using a priori models [4]. For GPS Block IIF and IIR satellites, IGS third reprocessing recommended ECOM1, while for other GPS satellites, ECOM2 or GSPM were recommended. For BDS-3, ECOM1 with Box-Wing as a priori model achieves higher orbit accuracy than ECOM2 [40]. For empirical models, a seven-parameter ECOM2 model ignoring second-order terms yields good BDS-3 orbit accuracy, with overlap orbit accuracy and SLR validation accuracy of about 6 cm and 4 cm respectively [41].

Compared to analytical models, empirical SRP models estimate parameters simultaneously with other orbital parameters, no longer depending on satellite optical parameters, with simpler expressions and higher accuracy. Empirical SRP models are major factors affecting orbit accuracy [24]. However, they have disadvantages: (1) not based on force analysis of satellite SRP; (2) may couple with other non-conservative forces and correlate with parameters like phase center offsets at large Sun elevation angles; (3) lack universality, requiring different empirical models for different satellite types.

3.3.1 Adjustable Box-Wing Model

Since Box-Wing models require accurate satellite dimensions and optical parameters, they are difficult to apply without such information [13]. Rodriguez-Solano et al. (2012) developed an Adjustable Box-Wing (ABW) model that can directly estimate satellite optical parameters, belonging to semi-analytical models [42,43].

Based on equations (17) and (18), SRP on solar panels and satellite bus can be calculated. Using nominal attitude models and angle , partial derivatives of acceleration with respect to optical parameters can be obtained for parameter estimation. For solar panels, SRP is expressed in Sun-fixed coordinates where e_D aligns with e_N and cos = 1, leaving only one optical parameter 1 + + _d from equation (17). For illuminated bus surfaces (+X, +Z, −Z), SRP is expressed in body-fixed coordinates with two optical parameters each (_a + _d and _s), totaling six parameters. The ABW model estimates nine parameters total, plus two Y-axis bias parameters (small deviations between actual and designed solar panel alignment) and solar panel rotation lag angle parameters.

ABW model orbit accuracy is comparable to ECOM1, but systematic differences exist between orbits from the two models, indicating ECOM1 cannot fully model SRP. Currently, this model is not used in routine IGS analysis center strategies due to complexity, strong parameter correlations, and dependence on a priori satellite parameters and reasonable constraints. Therefore, ABW is mainly used to establish a priori SRP models for satellites with unknown optical parameters.

3.3.2 Construction of A Priori Models

Due to continuous changes in satellites and their environment, analytical models are difficult to correct precisely, while empirical models lack physical meaning. The common strategy uses an a priori model with empirical parameters to absorb unmodeled errors. Recent SRP model research has focused importantly on a priori model construction. Currently, the commonly used a priori model is Box-Wing, which achieves higher orbit determination accuracy than ECOM1 or ECOM2 alone for non-cubic satellites like Galileo, QZSS, BDS-3e, and BDS-3, but requires known geometric and optical parameters [1]. Additionally, scholars have proposed various other a priori model construction methods.

When using ECOM1 for Galileo-IOV precise orbit determination, periodic errors correlated with Sun elevation angle appear in satellite orbit series, caused by the rectangular bus structure where SRP is not effectively absorbed by the model [32]. Montenbruck et al. [32] at DLR developed an enhanced a priori SRP model (DLR model) based on the original Box-Wing model, dividing bus acceleration into three parts: cube, extension, and asymmetric structure, obtaining SRP perturbation accelerations on the bus in D and B (or X and Z) directions (Y direction is unaffected by SRP under nominal attitude; any force can be absorbed by ECOM1), abstracted as expressions with few parameters related to (_a + _d) and _s for each structural part, with a priori values estimated. ECOM1 with DLR model as a priori eliminates these periodic errors, demonstrating its applicability for non-cubic satellites.

Yan et al. [44] used a similar method to DLR, selecting six a priori SRP model parameters and obtaining final parameter estimates for BDS-3 MEO satellites through long time series. Implementation: first use ABW model for precise orbit determination to obtain precise SRP accelerations, then adjust to obtain six a priori SRP model parameters with precise SRP as observations, estimating one parameter set per day to obtain final fixed ABW model parameters, yielding SRP as a function of . Results show this a priori model significantly improves precise orbit determination accuracy and reduces systematic orbit-dependent variations in clock fitting residuals.

Wang et al. [45] combined advantages of ECOM1 and ABW models (considering yaw-steering mode, communication antenna, and occlusion effects) to construct an SRP model for BDS-2 GEO satellites. Using ABW model single-day solutions to calculate SRP; since solar panels are not perpendicular to sunlight in yaw-steering mode, the DYB frame is not fully suitable, so decomposition into DYB and DYB' directions is used (former for nominal yaw mode, latter for yaw-steering mode). Spectral analysis of SRP in different directions obtains periodic terms to establish trigonometric polynomial functions. To reduce parameters, comparative analysis of   and orbital angle u and sine/cosine effects on fitting accuracy was performed (selecting significant angles and terms: cosine terms only in D, B, and D' directions; sine terms only in Y and B' directions). Parameters were fitted based on ABW-calculated SRP, expressed as linear models or second-order polynomials in Sun elevation angle β. Results show this model significantly eliminates orbit errors related to solar azimuth and orbital angle, reducing systematic radial biases.

[Figure 4: see original paper] shows the development timeline of SRP models since navigation satellite launches. Based on the above analysis, numerous SRP models are currently used for navigation satellite orbit determination, with applicability mainly depending on satellite shape and design. lists information on commonly used SRP models for navigation satellites, where applicable satellites are based on literature analysis; further research is needed for optimal models for new satellites.

4 Conclusion

High-precision satellite orbits are prerequisites for high-precision navigation satellite applications. Only by carefully considering various perturbing forces can satellite orbits be precisely determined. SRP, as the main source of non-conservative forces on satellites, requires precise modeling. Attitude models are the foundation for SRP models, and their accuracy directly affects SRP model precision. This paper introduced attitude models of major navigation satellites and commonly used SRP models, analyzed specific model parameters and their applications, and compared advantages and disadvantages of different SRP models.

Currently, ECOM series or GSPM series models with corresponding a priori models are commonly used at analysis centers, basically meeting precise orbit determination requirements for various navigation satellites. However, further research is needed on navigation satellite attitude and SRP models: actual attitudes of some BDS-3 satellites are not completely consistent with currently published models, requiring further study of attitude variations during orbital maneuvers; due to lack of detailed structural parameters, SRP modeling accuracy is low during different operational phases, especially eclipse periods; analytical models require separate consideration for different satellite designs (e.g., BDS GEO satellites' large antennas significantly affect SRP); literature on SRP models for newly launched GPS III, GLONASS-K, and BDS-3 IGSO satellites is lacking and requires further research.

Acknowledgments: We thank the iGMAS analysis center at Shanghai Astronomical Observatory for support and assistance, and He Lina and Ruan Rengui for valuable suggestions.

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Note: Figure translations are in progress. See original paper for figures.