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Full Text

Preamble

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Anisotropic emission from magnetized quark-gluon plasma

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Abstract: In these proceedings, we study the polarization effects of a strongly magnetized quark-gluon plasma at finite temperature. We find that a background magnetic field can have a strong effect on the photon and dilepton emission rates, influencing not only the total rate but also the angular dependence. In particular, Landau-level quantization leads to nontrivial momentum dependence of the photon/dilepton ellipticity coefficient on transverse momentum. We propose that the anisotropy of photon and dilepton emission may serve as an indirect measurement of the magnetic field.

Keywords: heavy ion collision; electromagnetic probes; strong magnetic field; finite temperature theory

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1 Introduction

Over the last half-century, there has been growing interest in the properties of Quantum Chromodynamics (QCD) matter. Heavy-ion collision experiments at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven and the Large Hadron Collider (LHC) at CERN provide strong experimental evidence for studying such matter on Earth, particularly the quark-gluon plasma (QGP) produced in these collisions. The broad goal of the ongoing research program is to investigate the fundamental properties of this deconfined state of matter.

Electromagnetic probes (i.e., photons and leptons) play a unique role in heavy-ion collision experiments. Unlike strongly interacting hadrons, they have long mean free paths that greatly exceed the size of the fireballs created by the collisions, enabling them to carry invaluable information about the plasma directly to the detectors. Recently, several measurements of electromagnetic emissions have been reported by both RHIC and LHC [?]. In general, thermal electromagnetic emission rates have been regarded as thermometers of the QGP. In this study, we propose that these rates can also serve as magnetometers for the hot QGP in the presence of a strong background magnetic field.

In noncentral heavy-ion collisions, the QGP is produced with an extremely strong magnetic field. In the early stages of the collision, the strength of the magnetic field has been estimated [?]. However, how strong the magnetic field remains in the latter stages of the collision, when the hot plasma forms and expands, remains an open question. If the field remains relatively strong, it can trigger anomalous phenomena, modify the flow of the plasma, induce new types of collective modes, and affect particle emission. Additionally, large background effects make experimental observation of the magnetic field extremely challenging.

In the case of a vanishing magnetic field, the leading-order photon production is given by gluon-mediated $2 \to 2$ processes that are linear in the strong coupling constant α_s. In a strongly magnetized plasma, however, the photon rate is dominated by three single-photon processes: (i) quark splitting ($q \to q + \gamma$), (ii) antiquark splitting ($\bar{q} \to \bar{q} + \gamma$), and (iii) quark-antiquark annihilation ($q + \bar{q} \to \gamma$), which are no longer forbidden by energy-momentum conservation. Consequently, the photon rate is nonzero already at leading zeroth order in α_s [?].

For dilepton production in a vanishing magnetic field, the dominant contributions in the intermediate range of dilepton invariant masses come from thermal radiation from the QGP, the Drell–Yan process, and semileptonic decays of heavy quarks. However, the signature effects of a magnetic field are rate enhancement and strong anisotropy, whose measurements could provide valuable constraints on the field strength in the plasma produced by heavy-ion collisions [?].

The main goal of this paper is to present recent theoretical understanding of photon and dilepton emission from a strongly magnetized hot QGP. Our results reveal nontrivial anisotropic emission caused by the background magnetic field for a range of model parameters. As we will argue, such anisotropy carries information about the magnitude of the magnetic field in the early stages of heavy-ion collisions.

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2 Anisotropic Flow

In noncentral heavy-ion collisions, the anisotropic flow coefficients (v_n) are defined through the following Fourier decomposition of the azimuthal particle distributions [?]:

$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{2\pi p_T dp_T dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_{RP})] \right)$$

where E is the particle energy, p is the momentum, p_T is the transverse momentum, ϕ is the azimuthal angle, y is the rapidity, and Ψ_{RP} is the reaction plane angle. By definition,

$$v_n(p_T, y) = \langle \cos[n(\phi - \Psi_{RP})] \rangle$$

where the angular brackets denote the average over all particles (or all events, or both) in a given bin of transverse momentum (p_T) and rapidity (y). Note that the second coefficient in the Fourier decomposition, v_2, characterizes the elliptic flow.

The direction of the magnetic field in a noncentral collision is approximately perpendicular to the reaction plane. In this paper, we assume the z-axis points along the magnetic field direction, the x-y plane is the reaction plane, and the x-axis points along the beam direction. The azimuthal angle ϕ measures the angle between the particle momentum $\mathbf{p}$ and the reaction plane. The particle four-momentum is given by $p^\mu = (p^0, \mathbf{p})$, with transverse components $p_y = p_T \cos(\phi)$

and $p_z = p_T \sin(\phi)$, where $p_T = \sqrt{p_y^2 + p_z^2}$ is the magnitude of the transverse momentum.

The anisotropy coefficients v_n can be evaluated from the differential distribution of particles as follows:

$$v_n = \frac{\int d\phi \cos(n\phi) \frac{dN}{p_T dp_T d\phi dy}}{\int d\phi \frac{dN}{p_T dp_T d\phi dy}}$$

where the normalization factor is given by the particle production rate integrated over the angular coordinate ϕ.

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3 Thermal Photons in a Magnetized QGP

Using quantum field theory, the thermal photon production rate can be expressed in terms of the imaginary part of the retarded polarization tensor as follows [?]:

$$\frac{dR_\gamma}{d^{3p}} = \frac{1}{(2\pi)^3} \frac{1}{2E_p} \mathrm{Im}[\Pi^\mu{}_\mu(p)] \frac{1}{e^{E_p/T} - 1}$$

Because of the quantizing background magnetic field, the quark and antiquark states in the one-loop photon polarization are characterized by Landau-level quantum numbers, the integer indices n and n'. For simplicity, we assume that the masses of both light quarks are the same, namely $m_u = m_d = m = 5$ MeV. The flavor-dependent quark charges are q_f, where $q_u = 2/3e$, $q_d = -1/3e$, and e is the absolute value of the electron charge. We define the modified fine structure constant $\alpha_f = q_f^2/(4\pi)$ for different quark flavors. $N_c = 3$ is the number of colors, T is the temperature of the system, and n_F is the Fermi-Dirac distribution function.

The imaginary part of the (Lorentz-contracted) polarization tensor at leading order with a background magnetic field can be written as follows [?]:

$$\mathrm{Im}[\Pi^{\mu,R}_{\mu}] = \sum_{f=u,d} \sum_{n,n'=0}^{\infty} g(n,n') \left\{ \theta(p^2 - p_0^2 + p_3^2 - p_{-,f}^2) - \theta(p_0^2 - p_3^2 - p_{+,f}^2) \right\} \times \frac{[p_{-,f}^2 + p_{+,f}^2 + p_z^2 - p_0^2]}{\sqrt{(p_0^2 - p_z^2 - p_{-,f}^2)(p_0^2 - p_z^2 - p_{+,}^2}}$$

where $\theta(x)$ is the Heaviside step function, the transverse momentum thresholds are given by $p_{\pm,f} = |\sqrt{m^2 + 2n|q_f B|} \pm \sqrt{m^2 + 2n'|q_f B|}|$, and we have used the shorthand notations:

$$g(n,n') = 2 - \delta_{n,n'}, \quad g_0(n) = g(n,n)$$

The explicit form of the corresponding function reads:

$$\mathcal{I}^f_{n,n'}(\xi) = 8\pi \left[\mathcal{I}^{0,f}_{n,n'}(\xi) + \mathcal{I}^{0,f}_{n-1,n'-1}(\xi)\right] \times \left[\mathcal{I}^{0,f}_{n,n'}(\xi) + \mathcal{I}^{0,f}_{n,n'-1}(\xi) + \mathcal{I}^{0,f}_{n-1,n'}(\xi)\right]$$

where the functions $\mathcal{I}^{0,f}_{n,n'}(\xi)$ are defined in terms of generalized Laguerre polynomials [**?**] as:

$$\mathcal{I}^{0,f}_{n,n'}(\xi) = (-1)^{n+n'} e^{-\xi} L^{n}_{n'-n}(\xi) L^{n'}_{n-n'}(\xi)$$

with $\xi = (k_\perp \ell_f)^2/2$ and the flavor-specific magnetic length $\ell_f = 1/\sqrt{|q_f B|}$.

For photons and dileptons, we assume that the mean free path is larger than the system size so that the particles leave the plasma region without reabsorption. The anisotropy coefficients v_n can be evaluated from the differential distribution.

The total rate integrated over the angular coordinate is plotted in Fig. 1 [Figure 1: see original paper] for two different magnetic fields and two different temperatures: (1) $|eB| = m_\pi^2$, $T = 200$ MeV (red dashed line); (2) $|eB| = 5m_\pi^2$, $T = 350$ MeV (purple dotted line); (3) $|eB| = m_\pi^2$, $T = 350$ MeV (black solid line); (4) $|eB| = 5m_\pi^2$, $T = 350$ MeV (blue dot-dashed line). In the case of the stronger field, $|eB| = 5m_\pi^2$, well-resolved peaks appear in the photon production rate at $p_T \simeq 0.06$ GeV when $T = 0.2$ GeV and at $p_T \simeq 0.04$ GeV when $T = 0.35$ GeV. For each case, the transverse momenta were taken in the range between 0.01 GeV and 1 GeV with a discretization step of 0.01 GeV, and the azimuthal angle was considered in the range between $10^{-4}\pi$ and $\pi - 10^{-4}\pi$ with a discretization step of $10^{-3}\pi$.

As argued in Ref. [**?**], similar peaks should also appear in the case of weaker fields. The existence of such maxima is a necessary consequence of Landau-level quantization of quark states in a strongly magnetized plasma. However, due to numerical calculation difficulties at very small p_T, the peak is not shown here. The maxima are larger when the temperature is higher and the magnetic field is weaker. In addition, when the transverse momentum increases further, the photon production rate starts to decrease rapidly. Higher temperature and larger magnetic field induce a higher rate at larger p_T.

To better understand the role of the magnetic field on direct photon emission from hot quark-gluon plasma, we show the ellipticity v_2 dependence on transverse momentum in Fig. 2 [Figure 2: see original paper]. For all the physics parameters we plot, we find a common feature: at large p_T, the ellipticity reaches and saturates at a relatively large positive value, i.e., $v_{2,\max} \simeq 0.2$. In the small p_T regime ($p_T \lesssim \sqrt{|eB|}$), the ellipticity becomes negative. The sign of v_2 changes at around $p_T \simeq \sqrt{|eB|}$. These results mean that the emission rate at small values of p_T has an overall tendency to peak at $\phi = \pi$, i.e., in the direction perpendicular to the reaction plane. When p_T is larger than about $\sqrt{|eB|}$, the emission tends to be highest at $\phi = 0$, i.e., in the direction along the reaction plane. Such anisotropic emission is caused by a strong magnetic field and has nothing to do with the hydrodynamic behavior of the quark-gluon plasma (see Ref. [**?**] for details).

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4 Dileptons in a Magnetized QGP

The dilepton rate is given by [**?**]:

$$\frac{dR_{l\bar{l}}}{d^{4K}} = \frac{2\pi e^2 e^{-\beta k_0}}{(2\pi)^3 E_1 (2\pi)^3 E_2} L_{\mu\nu}(Q_1, Q_2)\rho^{\mu\nu}(k_0, \mathbf{k})$$

where the three-momenta and energies of the two leptons are denoted by q_i and E_i with $i = 1, 2$, respectively. Note that, to distinguish from photons in the previous section, we use $k^\mu = (k_0, \mathbf{k})$ as the four-momentum of the dilepton in this section. To leading linear order in the electromagnetic coupling constant, the electromagnetic spectral function $\rho_{\mu\nu}(k_0, \mathbf{k})$ is expressed in terms of the imaginary part of the photon polarization tensor as:

$$\rho_{\mu\nu}(k_0, \mathbf{k}) = -\frac{1}{e^{\beta k_0} - 1}\mathrm{Im}[\Pi_{\mu\nu}(k_0, \mathbf{k})]$$

In Eq. (12), the leptonic tensor for the final plane-wave states has the following explicit form:

$$L_{\mu\nu}(Q_1, Q_2) = \sum_{\text{spins}} \mathrm{tr}[\bar{u}(Q_2)\gamma_\mu v(Q_1)\bar{v}(Q_1)\gamma_\nu u(Q_2)]$$

By using the explicit form of the leptonic tensor and setting the lepton masses m_l to zero, the corresponding differential rate reads [**?**]:

$$\frac{dR_{l\bar{l}}}{d^{4K}} = \frac{\alpha}{6\pi^4}\frac{n_B(p_0)}{M^2}\mathrm{Im}[\Pi^\mu{}_\mu(k_0, \mathbf{k})]$$

where $\alpha \equiv e^2/(4\pi) = 1/137$ is the fine structure constant and $n_B(k_0) = (e^{k_0/T} - 1)^{-1}$ is the Bose-Einstein distribution function. $M^2 = K^2 = k_0^2 - \mathbf{k}^2 \equiv k_0^2 - k_x^2 - k_\perp^2$ is the square of the invariant mass of the lepton pair. By definition, $k_\perp = \sqrt{k_y^2 + k_z^2}$ is the magnitude of the momentum component perpendicular to the magnetic field. Note that $d^{4K} = MdMk_T dk_T dyd\phi$, where $p_T = \sqrt{k_y^2 + k_z^2}$ is the transverse momentum (with respect to the beam direction) and $y = \frac{1}{2} \ln \frac{k_0+k_x}{k_0-k_x}$ is the rapidity.

After applying the imaginary part of the Lorentz-contracted photon polarization tensor from Eq. (7), the final result reads:

$$\frac{dR_{l\bar{l}}}{d^4K} = \frac{\alpha n_B(\Omega)}{3\pi^3 M^2} \sum_{f=u,d} \sum_{n=0}^{\infty} g_0(n)\theta(\sqrt{M^2+k_\perp^2} - k_{+,f}) \frac{\mathcal{I}_{n,n}^f(\xi)}{(M^2+k_\perp^2)[M^2+k_\perp^2-k_{+,f}^2]} + \sum_{n,n'=0}^{\infty} g(n,n')\theta(k_{-,f} - \sqrt{M^2+$$

where massless quarks and leptons are assumed.

It is useful to compare these results with the corresponding rate in the zero-field limit. In the Born approximation, the rate is given by [**?**]:

$$\frac{dR_{l\bar{l},\mathrm{Born}}}{d^4K} = \frac{\alpha^2}{6\pi^4} n_B(k_0) \ln \frac{\cosh \frac{k_0+|\mathbf{k}|}{2T}}{\cosh \frac{k_0-|\mathbf{k}|}{2T}}$$

The dependence of the rate on the invariant mass is presented in Fig. 3 [Figure 3: see original paper]. The corresponding data was calculated for the whole range of invariant masses between $M_{\min} = 0.02$ GeV and $M_{\max} = 1$ GeV, using a discretization step of $\Delta M = 0.01$ GeV. Individual panels show results for two representative temperatures, $T = 0.2$ GeV and $T = 0.35$ GeV, and two magnetic field values, $|eB| = m_\pi^2$ and $|eB| = 5m_\pi^2$. Each panel contains rates for the same set of fixed transverse momenta: $k_T = 0$ (black), $k_T = 0.1$ GeV (blue), $k_T = 0.2$ GeV (orange), $k_T = 0.5$ GeV (green), and $k_T = 1$ GeV (red). For comparison, we also show the zero-field Born rate (dashed lines).

By comparing the overall profiles of dilepton rates in a strongly magnetized plasma with the benchmark zero-field Born rate, we find that the rates at $B \neq 0$ remain approximately the same as those at $B = 0$ when the invariant mass is sufficiently large, i.e., $M \gtrsim \sqrt{|eB|}$. On the other hand, the magnetic field has a dramatic effect on dilepton production in the region of small invariant masses, i.e., $M \lesssim \sqrt{|eB|}$, where the rates are strongly enhanced. As seen in Fig. 3, the rate can increase by several orders of magnitude when M decreases by only half. The dilepton rate is on average a decreasing function of the invariant mass M. Generically, the rate in the magnetized QGP approaches the zero-field Born result (17) when the invariant mass is sufficiently large (i.e., $M \gg \sqrt{|eB|}$). By comparing the plots for different values of transverse momentum, we see that the dilepton rate tends to decrease with increasing k_T. As one can verify, the suppression of the rate at large M or k_T (or both) comes primarily from the overall Bose distribution function $n_B(\Omega)$ in Eq. (16).

The dependence of the ellipticity parameter v_2 on the dilepton invariant mass M is shown in Fig. 4 [Figure 4: see original paper]. In the case of small invariant masses, $M \lesssim \sqrt{|eB|}$, there is a clear tendency for v_2 to become positive.

This behavior is particularly well pronounced at large k_T, analogous to photon emission. Such behavior occurs because the angular dependence of the rate is systematically larger for azimuthal directions near $\phi \approx 0$ (in the reaction plane) and systematically smaller for $\phi \approx \pi/2$ (out of the reaction plane). On the other hand, the value of the ellipticity parameter v_2 for the dilepton rate at small invariant masses, $M \lesssim \sqrt{|eB|}$, and large transverse momenta is positive.

The overall ellipticity is harder to discern from the data at small values of transverse momentum, i.e., $k_T = 0.1$ GeV (blue lines) and $k_T = 0.2$ GeV (orange lines). For sufficiently small invariant masses, $M \lesssim \sqrt{|eB|}$, the ellipticity parameter appears to be generally nonvanishing. However, as seen from Fig. 4, its sign may change from being positive at intermediate values of M (i.e., $M \simeq \sqrt{|eB|}$) to being negative at sufficiently small values of M (i.e., $M \ll \sqrt{|eB|}$). This is particularly clear in the case of the stronger magnetic field $|eB| = 5m_\pi^2$. It should also be noted that the average ellipticity parameter v_2 is consistent with zero for sufficiently large invariant masses, $M \gtrsim \sqrt{|eB|}$.

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5 Summary

Electromagnetic probes in relativistic heavy-ion collisions provide an ideal environment to study the properties of QCD matter under extreme conditions. By using the Landau-level representation for the imaginary part of the photon polarization tensor, we obtained an explicit expression for the photon and dilepton emission rates from a hot QGP in a quantizing background magnetic field.

For photon emission from a strongly magnetized hot quark-gluon plasma, there is a nonzero ellipticity coefficient v_2 that depends on transverse momentum. v_2 is negative at small momenta ($p_T \lesssim \sqrt{|eB|}$) and positive at large momenta ($k_T \gtrsim \sqrt{|eB|}$). While the ellipticity coefficient v_2 is an overall growing function of p_T, it reaches a relatively high positive value $v_{2,\mathrm{max}} \simeq 0.2$ at large p_T. One of the most exciting results is the negative value of v_2 at small p_T, which differs from predictions of hydrodynamic or transport calculations that predict positive v_2 for any p_T [**?**]. If such a negative value could be measured, it would provide definitive evidence for the existence of a magnetic field in the QGP.

For dilepton production, the anisotropy is most pronounced in the regime of small dilepton invariant mass ($M \lesssim \sqrt{|eB|}$). When transverse momenta are large, the ellipticity parameter v_2 tends to be positive and large. For small transverse momenta, the ellipticity parameter v_2 may still remain nonzero in general, but its sign gradually changes from positive at intermediate values of M (i.e., $M \simeq \sqrt{|eB|}$) to negative at small M (i.e., $M \ll \sqrt{|eB|}$). Under optimal conditions, we find that the magnitude of v_2 could be as large as 0.2. If such large values are measured experimentally, they will most likely indicate the presence of a nonzero magnetic field in the QGP.

Furthermore, the background field strongly enhances the rate at small values of the dilepton invariant mass ($M \lesssim \sqrt{|eB|}$). At large values of the invariant mass ($M \gtrsim \sqrt{|eB|}$), the role of the magnetic field decreases, and the results gradually approach the isotropic zero-field Born rate. Such significant enhancement of the integrated dilepton rate at small invariant masses is a unique signature of a nonzero background magnetic field. Thus, measuring the corresponding rate at $M \lesssim 0.2$ GeV, for example, could provide sufficient information to confirm or rule out fields of order $|eB| \simeq m_\pi^2$ in relativistic heavy-ion collisions.

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Note: Figure translations are in progress. See original paper for figures.