Irreversibility Evaluation Method for Cross-Seasonal Heat Storage Systems and Analysis of Cold-Hot Mixing Phenomenon in Thermal Storage Reservoirs

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Date: 2023-10-31T15:44:00+00:00

Abstract

Thermal energy storage technology can address the temporal, spatial, or intensity mismatch between supply and demand of zero-carbon thermal energy, and represents one of the key technologies for achieving carbon neutrality in building and urban energy systems. For district heating systems in northern urban areas of China, if various types of low-grade waste heat collected throughout the year can be stored via seasonal thermal energy storage technology, these heat sources can be fully utilized to meet the winter heating demand of northern urban areas, while also rationally leveraging the existing infrastructure advantages of district heating networks in these regions. However, seasonal water thermal storage (also referred to as thermal storage reservoirs) has yet to achieve technological breakthroughs or large-scale deployment, primarily because the fundamental principles governing internal flow and heat transfer processes in ultra-large-scale water storage bodies remain unclear, and models for temperature grade degradation resulting from cold-hot mixing have not been established. This has resulted in a current lack of sufficient theoretical tools for engineering researchers in China to design ultra-large-scale water thermal storage devices and to accurately predict and evaluate the performance of water thermal storage. China urgently needs to conduct fundamental research in fluid mechanics and heat transfer for large-scale seasonal thermal energy storage technology and establish a thermal performance evaluation system that includes seasonal thermal energy storage technology. To address these research needs, this report will discuss key fundamental theoretical issues in seasonal thermal energy storage technology through the following five chapters: Chapter 1 introduces the technical overview of seasonal thermal energy storage, enumerating the main technical pathways and characteristics, and focuses on elaborating the common problem of excessive temperature grade degradation in existing seasonal ther-

mal energy storage projects, identifying irreversible heat transfer as the key cause of temperature grade degradation; it also demonstrates the limitations of existing indicators, such as exergy efficiency, in evaluating thermal storage loss characteristics, and clarifies the research need to establish an irreversibility evaluation system for thermal storage devices. Chapter 2 elaborates the irreversibility evaluation system for thermal storage processes constructed in this study; first, based on thermal storage principles, a reclassification method for thermal storage technologies is proposed: fluid displacement thermal storage and heat exchanger thermal storage; the mutual conversion relationships and distinctions among entropy, entransy, and maximum work potential as evaluation parameters in describing irreversible internal heat transfer problems are comparatively analyzed, with particular emphasis on the advantages of entransy and entransy dissipation parameters in analyzing the irreversibility characteristics of thermal storage. Additionally, based on the physical characteristics that thermal storage processes are non-equilibrium states, the storage medium is a continuum, and there exist coupled momentum and heat transfer phenomena, an irreversibility analysis method using partial differential equations to describe entransy parameter transfer and dissipation phenomena is proposed. Chapter 3 presents an entransy dissipation analysis method for seasonal thermal storage media; first, entransy efficiency is defined based on the principle of entransy balance in the storage medium, and an entransy dissipation analysis method distinguishing ideal processes from actual processes is proposed according to the heat transfer principles of water thermal storage and borehole thermal energy storage; analytical solutions for medium entransy dissipation in both types of thermal storage processes under ideal and actual conditions are derived, and a general expression for entransy dissipation in turbulent thermal storage media is proposed by integrating turbulence theory and eddy viscosity models; the entransy dissipation levels of water thermal storage and borehole thermal energy storage are comparatively analyzed, identifying the essential reasons why water displacement thermal storage is more suitable for long-period thermal storage, and clarifying the engineering design direction of minimizing cold-hot mixing intensity and corresponding entransy dissipation levels. Chapter 4 introduces the flow and heat transfer phenomena in thermal storage reservoirs, summarizing the basic phenomena and governing equations from two aspects: the basic flow processes and heat transfer processes of thermal storage reservoirs with temperature stratification as the core characteristic. Chapter 5 delves into the mechanism of cold-hot mixing phenomena in thermal storage reservoir water bodies, first analyzing the basic criteria for transport phenomena corresponding to cold-hot mixing, and separately developing physical phenomena and model descriptions for three basic mechanisms of cold-hot mixing: macroscopic flow entrainment, microscale turbulent mixing caused by shear instability, and internal wave breaking mixing; by discussing cold-hot mixing phenomena in naturally stratified water bodies and cooling ponds that also exhibit temperature stratification characteristics, analogous analysis is performed on possible cold-hot mixing scenarios in thermal storage reservoirs. Finally, Chapter 6 summarizes the main research findings of this report and looks forward to future research by proposing four key issues for studying entransy dissipation in thermal storage reservoirs.

Full Text

Irreversibility Evaluation Framework for Seasonal Thermal Energy Storage Systems and Mixing Phenomena in Hot Water Storage Reservoirs

Postdoctoral Research Report, Tsinghua University

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Research Period: October 2021 – October 2023

Report Submission Date: October 2023

Tsinghua University, Beijing

Abstract

Heat storage technology can address the temporal, spatial, and intensity mismatches between supply and demand of carbon-free thermal energy, representing a key enabling technology for achieving carbon neutrality in building and urban energy systems. For district heating systems in northern Chinese cities, seasonal heat storage could enable full utilization of various low-grade waste heat sources collected year-round to meet winter heating demands while leveraging existing centralized heating network infrastructure. However, seasonal water thermal energy storage (also known as hot water reservoir storage) has yet to achieve technological breakthrough and large-scale deployment, primarily because the fundamental principles governing flow and heat transfer within ultra-large-scale storage water bodies remain unclear, and no temperature-grade loss model has been established for hot-cold mixing phenomena. This deficiency leaves Chinese engineering researchers without adequate theoretical tools for designing ultra-large-scale water heat storage systems or accurately predicting their performance.

China urgently needs fundamental research in fluid dynamics and heat transfer for large-scale seasonal heat storage technology, along with a comprehensive thermal performance evaluation framework. This research report addresses these needs through five chapters:

Chapter 1 introduces seasonal heat storage technology, surveys major technological pathways and their characteristics, and highlights the common problem

of excessive temperature-grade loss in existing projects. It identifies irreversible heat transfer as the key cause of temperature-grade degradation and demonstrates limitations of existing metrics like exergy efficiency, establishing the need for an irreversibility evaluation framework for heat storage devices.

Chapter 2 presents the irreversibility evaluation framework developed in this study. It first proposes a reclassification of heat storage technologies based on fundamental principles: fluid displacement-based versus heat exchanger-based storage. The chapter compares entropy, entransy, and maximum work potential as evaluation parameters, emphasizing the advantages of entransy and entransy dissipation for analyzing heat storage irreversibility. Considering that heat storage processes are non-equilibrium, storage media are continuous media, and momentum and heat transfer are coupled, the framework employs partial differential equations to describe entransy transfer and dissipation phenomena.

Chapter 3 details the entransy dissipation analysis methodology for seasonal heat storage media. Based on entransy balance principles, it defines entransy efficiency and develops methods to distinguish ideal from actual processes for both water and borehole storage. Analytical solutions for entransy dissipation are derived for both ideal and actual processes, with a general expression for turbulent entransy dissipation developed using turbulence theory and eddy viscosity models. The analysis compares entransy dissipation levels between water and borehole storage, revealing why fluid displacement storage is inherently more suitable for long-term storage and establishing design directions to minimize mixing intensity and corresponding entransy dissipation.

Chapter 4 introduces flow and heat transfer phenomena in storage reservoirs, outlining basic phenomena and governing equations for both flow processes and heat transfer in thermally stratified reservoirs.

Chapter 5 analyzes the mechanisms of hot-cold fluid mixing in water reservoirs. It examines fundamental transport phenomena associated with mixing, then details three primary mixing mechanisms: macroscopic flow-induced entrainment, micro-turbulent mixing from shear instability, and internal wave breaking. By discussing mixing phenomena in naturally stratified water bodies and cooling ponds, it draws analogies to potential mixing scenarios in storage reservoirs.

Chapter 6 summarizes key findings and proposes four critical research questions for future investigation of entransy dissipation in storage reservoirs.

Keywords: Seasonal thermal energy storage, entransy dissipation, hot water storage reservoir, mixing phenomena, heat transfer analysis

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Table of Contents

List of Tables

1.1 Seasonal Heat Storage and Carbon Neutrality in Heating

Achieving carbon neutrality in buildings requires low-carbon transformation of urban heating in northern China. By 2060, the heated building area in northern Chinese cities is projected to reach 20 billion m^2, with annual heat demand of approximately 5 billion GJ (Tsinghua University Building Energy Research Center 2022). Full electrification of northern heating would require adding about 800 GW of wind and solar capacity while comprehensively upgrading the power grid and demand-side heating equipment—an economically infeasible pathway. Conversely, northern China possesses abundant waste heat resources suitable for building heating, including nuclear power waste heat and low-grade waste heat from metallurgical, non-ferrous, chemical, building materials, and light industries (Wang Chunlin, Fang Hao et al. 2019). However, relying solely on winter waste heat recovery would leave a heating deficit of approximately 1.6

billion GJ. If various low-grade waste heat sources could be collected and stored year-round, they could fully meet winter heating demands in northern cities while utilizing existing centralized heating network infrastructure advantages.

As shown in Figure 1-1, low-grade waste heat utilization requires four categories of technologies: collection, conversion, storage, and long-distance transport (Wu Yanting 2022). China has successfully developed and begun large-scale application of technologies for the first three categories, but seasonal heat storage technology remains a critical bottleneck. The State Council's "Carbon Peaking Action Plan Before 2030" explicitly calls for "promoting low-grade waste heat heating development," while the "14th Five-Year New Energy Storage Development Implementation Plan" specifies that by 2025, long-duration thermal energy storage technology must achieve breakthroughs. Therefore, China urgently needs fundamental research on large-scale seasonal heat storage technology.

Figure 1-1 Low-grade waste heat heating system

1.2 Classification and Characteristics of Heat Storage Technologies

Four seasonal heat storage technologies have practical engineering applications, which can be classified into three categories based on storage principles:

(1) Pit Thermal Energy Storage (PTES) and Tank Thermal Energy Storage (TTES) Both technologies use water as both storage medium and heat transfer fluid with similar principles. During charging, hot water is injected from the top of the storage volume, displacing cold water originally stored; extraction reverses this process using cold water to displace hot water. The storage/extraction processes are primarily displacement rather than heat exchange processes.

(2) Borehole Thermal Energy Storage (BTES) This technology uses underground rock-soil as the storage medium and antifreeze or water as the heat transfer fluid. During charging, high-temperature fluid flowing through vertical or horizontal borehole heat exchangers transfers heat to the surrounding rock-soil; during extraction, low-temperature fluid recovers heat from the warmed rock-soil. The storage/extraction processes are heat exchange processes.

(3) Aquifer Thermal Energy Storage (ATES) An aquifer is water-saturated permeable rock where water can drain freely. ATES involves two aspects: (i) storage and displacement of hot water through wells (similar to PTES), and (ii) heat exchange between hot/cold water and underground rock formations (similar to BTES).

Figure 1-2 Classification of seasonal heat storage technologies

Sweden, Denmark, Germany, and the Netherlands pioneered seasonal heat storage research and applications in Europe, each developing distinct technology pathways (Figure 1-3). Denmark has built nearly ten large-scale PTES systems for municipal heating, with the largest in Vojens storing approximately 200,000 m^3 of hot water. Sweden favors borehole BTES systems, with the world's largest in Emmaboda storing 200,000 m^3. Germany has constructed numerous solar communities (e.g., Chemnitz, Eggenstein) integrating PTES and TTES systems (hundreds to 10,000 m^3 capacity) into district heating, plus high-temperature ATES projects like Neubrandenburg using 1,200 m deep wells. The Netherlands primarily develops low-temperature ATES (5–30°C) due to environmental regulations limiting maximum injection temperatures.

Figure 1-3 European seasonal heat storage technology pathways

Compared to Europe, northern Chinese cities have higher population density and more extensive centralized heating systems (85% coverage vs. 50–63% Nordic average). Consequently, China requires seasonal heat storage systems significantly larger than existing European installations. Tsinghua University's Professor Yang Xudong's team built and operates the world's largest BTES system in Chifeng, Inner Mongolia, with 500,000 m^3 capacity, while several PTES installations range from 1,000 to tens of thousands of m^3. Meeting winter heating demand for a 200,000-person northern city requires seasonal hot water storage volumes exceeding ten million m^3. Thus, China's planned seasonal heat storage systems face greater technical challenges and more urgent fundamental research needs.

A common problem across many seasonal storage systems is the need for auxiliary heating equipment (heat pumps, boilers) during heat extraction to compensate for insufficient extraction temperature. For BTES systems, Emmaboda could only directly supply 10–15% of stored heat to radiator heating systems until installing an electric heat pump booster in 2018, which increased usable heat threefold by raising 30–40°C extraction temperatures to 55°C (Olof Andersson 2021). Tsinghua University proposed coupling BTES with absorption heat pumps driven by high-temperature steam (Guo, Zhu et al. 2022). For PTES systems, monitoring data from Dronninglund (Sifnaios, Gauthier et al. 2023) and Gram (Winterscheid and Schmidt 2019) show electric heat pumps are needed throughout most of the winter extraction period. Some systems directly employ boilers on heating pipelines, such as Drake Landing's BTES (Mesquita, McClenahan et al. 2017) and Neubrandenburg's ATES (Kabus, Wolfgramm et al. 2021).

Two factors cause insufficient extraction temperature: heat loss to the environment and temperature-grade loss from irreversible heat transfer—an important but often overlooked cause. Both contribute to low extraction temperatures and significant grade loss in practice. Figure 1-4 illustrates five types of temperature-grade enhancement systems for seasonal storage, where heat pumps or boilers compensate for grade loss to meet heating demands. Greater irreversible storage losses require larger auxiliary power or fuel inputs.

Figure 1-4 Schematic of temperature-grade enhancement systems during heat extraction from seasonal storage. Abbreviations: STC—Solar thermal collectors, EHP—Electric heat pump, AHP—Absorption heat pump, ISH—Industrial surplus heat, CHP—Combined heat and power.

1.3 Existing Heat Storage Evaluation Metrics and Research Needs

Existing literature and engineering reports evaluate heat storage performance using thermal performance metrics and techno-economic indicators. Thermal metrics include thermal efficiency (first law, reflecting heat loss proportion) and exergy efficiency (second law, reflecting temperature-grade loss from irreversible heat transfer). Economic indicators include capital investment per storage capacity, per storage volume, and per storage power.

Current research focuses on thermal efficiency and economic metrics, with development directions summarized as: (1) reducing heat loss and improving thermal efficiency through larger volume, scale, and better insulation; (2) reducing costs for large-scale application. Figure 1-5 shows operating temperature ranges and investment levels for existing technologies.

Figure 1-5 Typical operating temperatures and initial investment costs per unit heat storage capacity for existing seasonal heat storage technologies

A critical development direction is reducing temperature-grade loss through proper design and operation strategies. Some studies use exergy efficiency to evaluate maximum useful work loss from stored heat. Dincer and Rosen (2021) illustrate its importance: two storage cycles with equal thermal efficiency (85°C inlet, 25°C outlet) but different flow rates yield 35°C vs. 75°C extraction temperatures, corresponding to 27% vs. 73% exergy efficiency (20°C reference). This clearly shows the higher irreversibility and grade loss in the first case. Exergy efficiency thus reasonably compares temperature-grade loss across technologies based on the second law.

However, exergy efficiency depends heavily on reference point selection. Figure 1-6 shows exergy efficiency variation with temperature-grade loss (dT) for three reference temperatures: 0°C (winter air), 10°C (annual mean ground temperature), and 20°C (storage inlet). For small temperature ranges, different references yield significantly different efficiencies. For example, with dT = 5 K and 15 K losses, the 0°C reference shows higher efficiency than the 20°C reference case. Moreover, exergy efficiency cannot directly reflect relative grade loss magnitudes when reference temperatures differ.

Figure 1-6 Exergy efficiency comparison for temperature-grade losses dT of 5 K and 15 K; reference temperatures: 0℃, 10℃, 20℃; heat storage temperature range: 10~100 K.

Literature presents exergy efficiencies for individual technologies but lacks cross-technology comparisons, complicated by varying reference points (annual soil temperature, ambient air, or inlet temperature) and different operating temperature ranges for BTES/ATES versus TTES/PTES. Therefore, a new irreversibility evaluation parameter is needed—one that accurately reflects temperature-grade loss characteristics and enables universal comparison across technologies with different temperature ranges and loss magnitudes.

The *entransy* parameter, proposed by Tsinghua University's Academician Guo Zengyuan (Guo, Liang et al. 2006), describes an object's heat transfer capability. Temperature-grade loss irreversibility can be characterized by entransy dissipation, though no evaluation framework yet exists for heat storage devices. As shown in Figure 1-7, heat transfers from heat source to storage medium during charging, then to heat sink during extraction. The associated temperature-grade loss can be described by entransy dissipation (upper diagram) or as the heat pump work required to restore the degraded heat to its original temperature grade—equivalent to useful work (exergy) loss.

Figure 1-7 Relationship between entransy dissipation and heat pump work input for temperature boosting

Alternatively, consider an ideal heat engine or heat pump (Figure 1-8). An ideal heat engine can produce work from a clear hot-cold temperature distribution; when all available work is extracted, the distribution becomes uniform. Conversely, an ideal heat pump can restore a uniform temperature distribution to a stratified one through work input.

Figure 1-8 Temperature distribution and work output/input for ideal heat engine (heat pump)

What kind of irreversibility evaluation framework should be established to compare different seasonal heat storage technologies? What are the fundamental differences between the most widely used water storage and borehole storage technologies in terms of principles and performance? Which key characteristics of heat transfer processes should guide parameter selection and evaluation method design? This report addresses these core questions through systematic research on irreversibility evaluation methods for heat storage devices, focusing on irreversible heat transfer phenomena in seasonal hot water storage reservoirs.

2.1 Reclassification of Heat Storage Principles Based on Irreversibility Characteristics

The second law of thermodynamics fundamentally describes process irreversibility. Clausius's formulation states: "Heat spontaneously flows from high to low temperature," but "heat cannot be transferred from low to high temperature without other effects." Heat storage is a dynamic process of heat transfer and accumulation. In sensible heat storage heat exchangers, heat transfers from high-temperature heat transfer fluid to low-temperature storage medium through convection and conduction—an inherently irreversible process. Hot water storage tanks represent another common sensible heat storage approach, relying on hot-cold water displacement rather than heat exchange. These include thermocline, dual-tank, and multi-tank designs where water serves as both heat transfer fluid and storage medium. Such devices function as hot water containers rather than heat exchangers, with irreversibility primarily from internal thermal diffusion due to molecular and turbulent motion.

Figure 2-1 illustrates the principle differences: fluid displacement storage "stores" high-temperature fluid medium during charging, "filling" the device; extraction "removes" the stored hot fluid. Heat exchanger-based storage transfers heat between heat transfer fluid and storage medium.

Figure 2-1 Classification of heat storage principles: fluid displacement-based (a-c) and heat exchanger-based (d-f)

Fluid displacement irreversibility originates from internal heat transfer and temperature distribution changes. Figure 2-2 shows three ideal temperature distributions with identical stored heat but different irreversibility levels, reflected in "maximum work capability from temperature differences." A clear hot-cold distribution (Figure 2-2a) yields maximum work W_1. Linear distribution (Figure 2-2b) can be viewed as W_1 after heat transfer from hot to cold regions, producing less work $W_2 < W_1$. Uniform temperature (Figure 2-2c) yields zero work capability ($W_3 = 0$).

Figure 2-2 Heat engine models for three ideal temperature distributions in thermally stratified storage

Thus, identical heat quantities contain different work capabilities. More heat transfer from hot to cold regions increases irreversibility and reduces work output. Macroscopic fluid displacement is reversible as "storage" and "removal" don't directly change thermodynamic states. However, thermocline displacement inevitably involves hot-cold contact, causing irreversible internal diffusion.

Heat exchanger storage irreversibility originates from heat transfer processes. Heat must "overcome" thermal resistance at heat exchanger boundaries and within the storage medium—an irreversible process whose reverse cannot occur spontaneously. Considering only internal medium diffusion, an ideal charging process under first-type boundary conditions is one-dimensional conduction (Figure 2-3). Intuitively, heat distributed through irreversible diffusion remains below the boundary temperature that drives it. In heat exchanger storage, every unit of stored heat undergoes this irreversible transfer, fundamentally differing from the essentially "reversible" ideal displacement storage.

Figure 2-3 Temperature distribution schematic for 1D conduction under first-type boundary conditions

The different irreversibility characteristics require different evaluation methods. For fluid displacement storage, focus on internal heat transfer at hot-cold interfaces, with parameters depending on temperature distribution changes and assessing irreversible transfer effects relative to reversible displacement. For heat exchanger storage, parameters should reflect equivalent thermal resistance and evaluate heat transfer difficulty from source to medium and within the medium. Design principles differ: displacement storage should minimize heat transfer rates between hot and cold fluids, while heat exchanger storage should enhance heat transfer performance.

2.2 Irreversibility Evaluation Parameters: Entransy, Entropy, and Useful Work

In traditional thermodynamics, entropy generation (ΔS) is the fundamental parameter for measuring irreversibility. For heat exchanger storage (non-adiabatic systems), entropy increase comprises entropy flow from boundary heat input and entropy generation from irreversible processes:

ΔS = + S_{gen} (2-1)

For fluid displacement storage with internal heat transfer (thermocline), the region can be treated as an isolated adiabatic system. Entropy generation is zero only for reversible transitions between equilibrium states; internal heat transfer between non-equilibrium states generates entropy:

ΔS = + S_{gen} = S_{gen} (2-2)

To describe irreversibility in conduction-based heat transfer, Academician Guo Zengyuan and colleagues proposed "entransy" (Guo, Liang et al. 2006). Entransy has "thermal potential" properties, representing a substance's total heat transfer capability. For solids with constant thermophysical properties, entransy is defined as:

G = (1/2) cVT dV (2-3)

where U = cV dV is internal energy; , V, c are density, volume, and specific heat. Volumetric entransy is:

g = (1/2) cT^2 (2-4)

Assuming density and specific heat are temperature-independent, the rate of change of volumetric entransy is:

dg/dt = cT dT/dt (2-5)

From the heat conduction equation:

c T/ t = - · q (2-6)

Thus:

dg/dt = -T · q = -[· (qT) - q · T] = - · (qT) + q · T (2-7)

The last term represents the product of heat flux and its driving force (temperature gradient), whose physical meaning is the local irreversibility generation rate. In entransy balance, q · T is defined as entransy dissipation rate, representing irreversible entransy loss due to material thermal resistance.

Treating the internal heat transfer region (thermocline) of fluid displacement storage as an isolated thermodynamic system, maximum output work can only be obtained when the system undergoes reversible transition between equilibrium states (zero irreversible entropy generation). Maximum work capability based on temperature distribution change is:

W = cT(x,y,z)dV - cT_{eq} dV (2-8)

where T(x,y,z) is the temperature field of the internal heat transfer region. The first integral term represents total internal energy; T_{eq} is equilibrium final temperature, with the second integral representing remaining internal energy after an ideal work-extraction process. Thus, maximum work capability also reflects storage irreversibility.

In summary, for storage systems with identical total heat but different temperature distributions, irreversibility can be evaluated using three parameters: entropy generation, entransy dissipation, and maximum work capability loss. What are their relationships? Which is most suitable?

Consider three temperature distributions for a storage volume V with total mass M and average temperature T (Figure 2-4): (a) clear hot/cold separation, (b) linear distribution, (c) uniform distribution, where $T_1 = T_2 = T_3$.

Figure 2-4 Schematic of three temperature distribution scenarios in storage media

Temperature deviation from average is:

(x,y,z) = T(x,y,z) - T (2-10)

A parameter measuring temperature heterogeneity (Oboukhov 1949) is:

G' = (1/2) c ^{2} dV (2-11)

This form is consistent with entransy G in Eq. (2-3), except G' uses squared deviation from average temperature (zero for uniform temperature) while entransy uses squared temperature relative to absolute zero. The temperature reference differs. We'll use "differential entransy" for G', with per-unit-mass differential entransy:

g' = (1/2)c ^{2} (2-12)

Maximum work capability can also be expressed as:

W = cT dV - cT_{eq} dV = Mc(T - T_{eq}) (2-13)

Entropy is a state function. With equal initial and final entropy, equilibrium final temperature is the geometric mean temperature. Simplifying maximum work capability:

W = MT[1 - (T_{eq}/T)] = MT[1 - exp(lnT dV/M)] (2-14)

The integrated logarithmic temperature function can be transformed:

lnT(x,y,z) = lnT + ln[1 + (x,y,z)/T] (2-15)

Taylor series expansion gives:

ln(1 + x) = x - x^2/2 + x^3/3 - … for x   (-1,1] (2-16)

Since /T is small, higher-order terms can be neglected:

 lnT dV   M[lnT - (1/2T^2) ^{2} dV] (2-17)

Thus, maximum work capability can be further simplified:

W   MT[1 - exp(-G'/(MT^2))] (2-18)

This shows maximum work capability can be expressed by differential entransy with conversion coefficient 1/T^2.

Conversely, uniform temperature distribution (Figure 2-4c) can be viewed as resulting from irreversible internal diffusion from an initially stratified state. This irreversible diffusion accompanies entropy generation:

ΔS = - c lnT dV + Mc lnT (2-19)

Neglecting higher-order terms:

ΔS   (c/T^2) ^{2} dV = G'/T^2 (2-20)

Thus, entropy generation from irreversible thermal diffusion can also be expressed by differential entransy with conversion coefficient 1/T^2.

In summary, three physical parameters describing temperature heterogeneity—differential entransy G' (J · K), maximum work capability W (J), and entropy generation ΔS (J/K)—can be interconverted (Eq. 2-21). For the uniform distribution case (c), $G'_3 = 0$, $W_3 = 0$, $\Delta S_3 = 0$; for clear separation (a), temperature heterogeneity exceeds linear distribution (b), so $G'_1 > G'_2$, $W_1 > W_2$, $\Delta S_1 > \Delta S_2$. The linear distribution can be considered a thermodynamic state after irreversible diffusion from the separated distribution.

W = TΔS = G'/T (2-21)

All three parameters reflect "irreversibility" and are interconvertible. Which best describes heat storage irreversibility? First, entropy and entransy definitions depend on temperature scale (increment units). Temperature concepts originated from empirical thermal expansion observations, leading to different "zero points" and increments based on reference substances. Fourier referenced Newton's linear laws to propose linear conduction heat flux, deriving relationships without entropy (not yet conceived), using only spontaneous high-to-low temperature heat transfer as fundamental law.

Subsequently, thermodynamics matured entropy concepts and thermal equilibrium definitions from macroscopic and statistical perspectives, giving thermodynamic temperature and entropy statistical meaning. For two closed systems A and B in thermal contact with conserved total internal energy, thermal equilibrium occurs when heat transfer ceases. At equilibrium, total entropy reaches maximum, with zero derivative of entropy generation with respect to internal energy:

(S_A + S_B)/ U_A = 0 → S_A/ U_A = S_B/ U_B (2-22)

Defining this equilibrium state as "equal temperature," the thermodynamic temperature should be:

τ = S/ U|_V,N (2-23)

Since S/ U = 1/T, the more natural temperature scale for entropy is the reciprocal of thermodynamic temperature. However, because traditional temperature was defined empirically before entropy, most entropy calculations still use thermodynamic temperature, though many studies use 1/T as the fundamental scale (e.g., thermodynamic beta β = 1/(k_B T) in statistical mechanics).

From another perspective, using 1/T as the basic temperature unit, the "average temperature" of a storage body becomes:

$\bar{\tau}$ = (1/M) τ dV (2-24)

with units [1/K]. Corresponding heat capacity units become [kg · K]. In this framework, entransy's cumulative change becomes:

ΔG = M c dτ = M c Δτ (2-25)

This shows entransy calculation in the 1/T scale takes the form of entropy generation in the T scale. Conversely, entropy generation in the 1/T scale becomes entransy dissipation. The temperature scale issue essentially concerns assigning numbers to isotherms according to certain rules. Entransy dissipation and entropy generation may share the same physical meaning for irreversibility but differ in measurement systems, requiring a conversion coefficient of T^2 between them. For isolated storage media internal heat transfer problems, the entropy-entransy conversion coefficient is T^2, consistent with their dimensional relationship.

2.3 Framework and Requirements for Irreversibility Evaluation System

Irreversibility parameters for heat storage systems must account for both medium transfer irreversibility and component (heat exchanger) irreversibility. Bejan (1996) proposed that thermal system entropy generation structure should include differential element, component, and overall equipment levels. As shown in Figure 2-5, storage devices exchange heat with energy system source/sink through system heat exchangers—the first major irreversible transfer link. Heat transfer fluid then exchanges heat with storage medium through storage heat exchangers (second irreversible link). Heat diffusion within the medium (third link) is also irreversible due to medium thermal resistance. Total irreversibility is the sum of three components:

I_{total} = I_{{{system}}{{HX}}} + I{{{storage}}{{HX}}} + I{medium} (2-31)

For system heat exchangers, temperature differences between hot and cold fluids vary along flow direction and charging time. At storage heat exchanger boundaries, temperature differences also vary spatially and temporally. Therefore, differential equations are needed to analyze irreversibility parameter distribution and totals. The general non-equilibrium irreversibility parameter calculation is the product of "flux (J)" and "driving force (X)" (Eq. 2-32). In heat transfer, "flux" is heat flow and "driving force" is temperature difference.

Figure 2-5 Irreversibility structure of heat storage devices

Figure 2-6 Schematic of differential equations for heat exchangers

For a counterflow heat exchanger (Figure 2-6), local heat flux is q' ' = U(T_1 - T_2). Using entransy dissipation as the irreversibility parameter, overall entransy dissipation rate is:

$\dot{G}$ = q' ' (T_1 - T_2) dA = U(T_1 - T_2)^2 dA (2-35)

Storage medium irreversibility parameter calculation (Eq. 2-36) involves local volumetric production rate g_i(x,y,z,t), such as volumetric entropy generation [$W/(K \cdot m^3)$] or entransy dissipation [$W \cdot m^{-3} \cdot K$].

I_{medium} = g_i(x,y,z,t) dV (2-36)

Entransy dissipation, maximum work capability, and entropy generation can all analyze storage system irreversibility. Evaluating irreversibility parameters for specific storage media and processes requires analyzing:

1. **Non-equilibrium state**: Charging/discharging are dynamic processes with continuously changing, irreversible temperature distributions. Non-equilibrium thermodynamics must analyze irreversibility parameter variation rates in temperature fields and calculate cumulative irreversibility. Internally, local equilibrium assumptions allow applying equilibrium thermodynamics principles to describe local behavior, with local irreversibility generation rate expressed as the inner product of "flux" and "force."

2. **Continuous medium**: Large temperature non-uniformity makes lumped parameter methods inaccurate for overall heat transfer coefficients and storage effects. Differential equations must describe continuous heat transfer, establishing conservation equations for storage medium microelements and solving temperature distribution as a linear PDE boundary value problem. Each infinitesimal control volume is treated as locally equilibrium, with irreversibility parameters analyzed via conservation laws and spatially integrated for overall irreversibility. Solid media follow the heat equation (no medium movement). Fluid storage requires continuity and momentum equations (Navier-Stokes), where heat transfer as a fluid "carrier" depends significantly on flow conditions. For water storage in tanks and reservoirs, the entire water body must be treated

as a continuum with coupled flow and heat transfer. Borehole storage in soil is primarily solid conduction without internal flow (ignoring phase change).

3. **Coupled transfer phenomena**: Fluid displacement storage involves: (i) fluid displacement (overall flow/advective momentum transfer); (ii) molecular and turbulent momentum diffusion (shear instability in stratified flow causes kinetic-to-internal energy dissipation—an important irreversibility); (iii) molecular and turbulent heat diffusion (heat as fluid "cargo" significantly affected by flow). Fluid motion (including turbulence) directly impacts temperature distribution, while temperature gradients create buoyancy forces affecting flow (natural convection). Irreversibility parameters must be built upon understanding these coupled phenomena and their mathematical models.

Additionally, the evaluation framework should connect easily to measured data, provide intuitive characterization of temperature-grade loss patterns, and guide storage device and overall heating system design and optimization.

2.4 Relationship and Distinction Between Entropy Generation and Entransy Dissipation

Both entropy generation and entransy dissipation can describe storage process irreversibility and are interconvertible via T^2 for internal distribution problems. Onsager and Prigogine developed mature non-equilibrium thermodynamics theory, and most existing storage irreversibility studies use entropy-based exergy efficiency. However, irreversibility of spontaneous heat diffusion from hot to cold need not be described by entropy generation.

Broadly, dissipation refers to energy conversion into unavailable forms. Turbulent viscous dissipation converts higher-order, work-capable kinetic energy into lower-order internal energy. The turbulent kinetic energy dissipation rate represents local kinetic-to-internal energy conversion rate. In Lagrangian mechanics, the Rayleigh dissipation function handles velocity-proportional friction effects. These quantities involve irreversibility analysis but operate outside entropy-based thermodynamics.

For heat storage, devices can be viewed as thermodynamic systems (using entropy generation) or as motion systems obeying Newton's linear laws (using dissipation). Which parameter more intuitively reflects temperature-grade loss? Hot water reservoirs constantly experience turbulence causing hot-cold mixing. Reynolds-Averaged Navier-Stokes (RANS) equations describe turbulent mean flow, derived from time/space averaging of Navier-Stokes equations. Herwig and Kock (2006) derived turbulent local entropy generation rate formulas from RANS, dividing it into four parts:

g__s,tot = g__s,CD + g__s,VD + g__s,CT + g__s,VT (2-37) where $g_{s,CD}$ is local entropy generation from mean temperature gradient conduction, $g_{s,CT}$ from temperature fluctuation conduction, $g_{s,VD}$ from mean flow viscous dissipation, and $g_{s,VT}$ from turbulent fluctuation viscous dissipation.

$g_{s,CD} = (k + k_T)(T)^2/T^2$ (2-38)

where k is thermal conductivity, k_T is turbulent thermal conductivity (2–3 orders of magnitude larger than molecular). Turbulent viscous dissipation rate converting kinetic to internal energy is:

$= \ s_{ij} s_{ij}$ (2-40)

where is kinematic viscosity, s_{ij} is fluctuating strain rate tensor. Entropy generation calculations require dividing by local mean temperature T to express irreversibility thermodynamically. The dimensionless Bejan number (Be) represents the proportion of entropy generation from heat transfer:

$Be = g_{s,\Delta T} / (g_{s,\Delta T} + g_{s,\Delta p}) = (g_{s,CD} + g_{s,CT}) / g_{s,tot}$ (2-42)

Large Be indicates heat transfer dominates entropy generation; small Be indicates viscous dissipation dominates.

Geophysical disciplines (atmospheric, oceanic, limnological) focus on mixing and turbulent transport in stratified fluids—mechanistically aligned with reservoir hot-cold mixing. Temperature is a stratified fluid "tracer" whose distribution is governed by flow, creating MOST similarity laws (Monin-Obukhov similarity theory). Corrsin (1951) used Fourier analysis to show temperature variance represents "potential" or "free energy" driving heat transfer. Though not actual energy, temperature variance exhibits dissipation characteristics from molecular thermal diffusion.

Temperature is easier to measure, so many studies analyze turbulent kinetic energy dissipation via temperature gradient squared. Scalar variance budget equations describe scalar variance evolution (Wyngaard and Coté 1971; Bradley, Antonia et al. 1981). The temperature variance budget equation is:

$^2 / t + U_j \ ^2 / x_j = - \ u_j \ ^2 / x_j - 2\alpha (\ / x_j)^2$ (2-43)

The last term is dissipation, proportional to temperature gradient squared, where α is molecular thermal diffusivity. For stratified fluids, temperature variance dissipation is:

$= 2\alpha (T'/ x_j)^2$ (2-44)

When temperature variance is expressed similarly to kinetic energy change rate, dissipation becomes:

$N = \alpha (T'/ x_j)^2$ (2-45)

Dissipation represents temperature inhomogeneity dissipation rate, analogous to viscous dissipation, showing similarity in transport laws (Monin and Yaglom

2013). Deacon (1959) and Taylor (1961) developed methods to estimate turbulent kinetic energy dissipation in naturally stratified fluids from temperature variance. Mestayer (1982) and Lin and Lin (1973) measured temperature spectral density variations. "Temperature spectral density" and "power spectral density" describe temperature signal energy distribution in frequency domain—Fourier transforms of temperature variance.

Thus, both entropy generation and entransy dissipation have theoretical foundations and applications for reservoir irreversibility. Both obey balance relationships with local change rates expressible as flux divergence plus source terms. Figure 2-7 shows the balance equation for a differential control volume:

(Î)/ t = - · (Îv) - · i + g_i (2-46)

where convective transport - · (Îv) represents irreversibility input/output from flow, diffusive transport - · i represents flux divergence, and g_i is generation/dissipation from transport phenomena. The change term splits into: (1) boundary input/output (convection + diffusion) and (2) generation/dissipation from transport.

Using the material derivative (Lagrangian derivative), which relates to local (Eulerian) derivative via:

D/Dt = / t + v · (2-47)

For a per-unit-mass irreversibility scalar Î, combining with continuity equation yields:

DÎ/Dt = -(1/) · i + g_i/ (2-48)

This shows that from a fluid particle perspective, irreversibility change results from diffusion and local generation. The term - · (Îv) represents reversible transport effects similar to fluid displacement.

Both entropy and entransy can serve as irreversibility parameters. Entropy flux and entransy flux are:

J_s = q/T (2-53)
J_g = q · T (2-54)

Both are vectors generated by heat flux q. Viscous dissipation also increases entropy. Internal energy balance is:

DU/Dt = - · q - p(· v) - (τ: v) (2-55)

where τ is viscous stress tensor. Internal energy and entropy change rates relate via:

DS/Dt = (1/T)[- · q - (τ: v)] (2-56)

Both heat transfer and viscous dissipation irreversibility require the 1/T conversion factor to express as "entropy change." Entropy material derivative becomes:

DŜ/Dt = -(1/) · J_s - (τ: v)/(T) (2-57)

Comparing with Eq. (2-52), entropy generation rate is:

$g_s = - \cdot J_s - (\tau: v)/T = q \cdot (1/T) - (\tau: v)/T$ (2-58)

For entransy as irreversibility parameter, based on $g_g = - \cdot J_g$, entransy material derivative is:

$D\hat{G}/Dt = -(1/\ ) \cdot J_g + g_g/\ = -(1/\ ) \cdot (qT) + q \cdot T/\ $ (2-59)

Thus, entransy dissipation rate is:

$g_g = - \cdot J_g + q \cdot T = q \cdot T$ (2-60)

Entransy dissipation rate is the dot product of heat flux and temperature gradient, presenting "flux-force" product form consistent with non-equilibrium thermodynamics definitions. Unlike entropy generation, entransy dissipation doesn't require the 1/T "correction factor," offering simpler form that better reflects irreversibility essence.

From Fourier's law q = -k T, the rates become:

$g_s = k(\ T)^2/T^2$ (2-62)
$g_g = -k(\ T)^2$ (2-63)

Figure 2-9 compares the frameworks: both have similar forms with temperature gradient squared multiplied by phenomenological coefficient (thermal conductivity). Entropy generation is positive (always increasing), while entransy dissipation is negative (always decreasing). Entropy generation includes the $1/T^2$ conversion term because its fundamental temperature scale is the reciprocal of thermodynamic temperature.

Figure 2-9 Comparison of entropy and entransy irreversibility evaluation frameworks

Which parameter should evaluate storage irreversibility? In heat transfer studies based on temperature T, Fourier's law, and Newton's cooling law, entransy dissipation more intuitively expresses irreversibility. Using cumulative entransy dissipation per unit stored heat (units: K) directly indicates average temperature-grade loss in degrees—physically intuitive. Entropy generation's [J/K] units are less direct for understanding temperature-grade loss.

Essentially, Eq. (2-62) calculates entropy generation rate via (1/T) gradient, meaning entropy generation uses 1/T as direct driving force. When expressed as T gradient, a "conversion coefficient" is needed. In non-equilibrium thermodynamics, Fourier's thermal conductivity k is a scalar phenomenological coefficient. To express entropy generation rate directly as flux-force dot product, thermal conductivity must be redefined as $L_{qq} = k/T^2$, giving:

$g_s = q \cdot (1/T) = L_{qq}(\ T)^2$ (2-65)

Thus, the essential difference remains temperature scale. Fourier's law and Fick's law are empirical, linear flux-force relationships independent of thermodynamics (phenomenological relations). Therefore, entransy dissipation better describes irreversibility in conduction heat transfer driven by linear temperature differences.

Second, reservoir mixing entropy generation involves two mechanisms: thermal diffusion (molecular + eddy) and viscous dissipation (Eq. 2-58). Large Bejan numbers indicate viscous dissipation dominates, but this isn't the main contradiction for overall storage irreversibility. While entropy generation measures generalized irreversibility increase, storage optimization focuses on heat transfer rates and reducing internal heat transfer—practical concerns about outlet temperature degradation relative to storage temperature. Entransy dissipation measures only thermal diffusion irreversibility, directly guiding engineering design.

Third, stratified fluid dynamics research has established experimental methods using temperature variance dissipation (Eq. 2-44) to determine turbulent kinetic energy dissipation and mixing states. This term has the same form as entransy dissipation, differing only in using thermal diffusivity versus conductivity as phenomenological coefficient. This not only demonstrates entransy dissipation's practical significance for stratified water bodies including storage reservoirs, but also enables direct measurement of temperature dissipation to calculate entransy dissipation levels in actual reservoir engineering—facilitating experimental research.

In conclusion, entransy-based parameters better align with transport phenomenon mathematical modeling, more intuitively reflect temperature-grade loss characteristics through equivalent dissipation temperature difference, and better support engineering measurements. Subsequent analysis will employ entransy dissipation parameters.

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3. Entransy Dissipation Analysis Methods for Seasonal Heat Storage Media

As shown in Figure 2-5, using entransy as the irreversibility parameter, total storage system entransy dissipation comprises three parts: (1) system heat exchanger, (2) storage heat exchanger, and (3) heat diffusion in storage medium:

G_{total} = G_{{{system}}{{HX}}} + G{{{storage}}{{HX}}} + G{medium} (3-1)

Chapter 2 derived entransy change rate for fluid storage media via local and material derivatives:

DĜ/Dt = - · (Ĝv) - · g + g_g (3-2)

The term - · (Ĝv) represents entransy transport from fluid motion and molecular diffusion (reversible like fluid displacement), while dissipation term g_g measures irreversibility from thermal diffusion. Storage devices should minimize entransy dissipation to preserve temperature grade and heat delivery capability.

Thus, key technology selection and optimization methods transform into establishing entransy dissipation mathematical models and comparing dissipation levels. This chapter develops entransy dissipation analysis for hot water storage technologies (reservoir, pit, tank), borehole storage, and phase change storage, building a comprehensive entransy dissipation evaluation framework.

3.1 Heat Balance and Entransy Balance in Storage Media

Seasonal heat storage primarily changes internal energy of solid or fluid media, ignoring work and mechanical energy effects. At low operating temperatures, radiation can also be neglected. Both heat and entransy parameters follow transport equation forms for differential control volumes:

{Parameter change rate} = {Boundary flux} + {Source/sink term}

Boundary flux results from convection or diffusion. For sensible heat storage media, heat source/sink terms are zero, but "entransy sink" (dissipation) exists. A key performance metric is thermal efficiency _q—ratio of actual heat stored to heat input:

ΔU_{st} = U_{in} - U_{loss} (3-4)
_q = ΔU_{st} / U_{in} = 1 - U_{loss} / U_{in} (3-5)

When _q = 100%, all heat input is stored in the medium.

Entransy also obeys conservation (Eq. 3-6), where ΔG_{st} is entransy stored, G_{in} is entransy entering with heat transfer fluid, G_{out} is entransy leaving, G_{loss} is entransy loss from surface heat loss, and G_{diss} is entransy dissipation from internal irreversible heat transfer. Though both reduce medium entransy, their mechanisms differ.

ΔG_{st} = G_{in} - G_{out} - G_{loss} - G_{diss} (3-6)

Two entransy efficiencies can be defined: dissipation entransy efficiency _g,diss focuses on internal transfer irreversibility (ignoring loss for comparison), while total entransy efficiency _g,tot focuses on actual entransy retention representing total heat transfer capability.

_g,diss = 1 - G_{diss} / (ΔG_{st} + G_{diss}) (3-7)
_g,tot = (ΔG_{st} - G_{loss}) / (G_{in} - G_{out}) = 1 - (G_{loss} + G_{diss}) / (G_{in} - G_{out}) (3-8)

For ideal conduction storage with zero heat loss (_q = 100%) and closed system (G_{loss} = 0), both efficiencies simplify to:

_g,diss = _g,tot = 1 - G_{diss} / ΔG_{st} (3-9)

Large dissipation terms indicate high irreversibility and low efficiency. The key to calculating thermal and entransy efficiencies is determining loss and dissipation relative to total changes. Fluid displacement and conduction storage require different energy balance equations due to different internal energy change mechanisms, primarily differing in convection and diffusion terms. Figure 3-1 illustrates analysis schematics for both types.

Figure 3-1 Schematic of heat balance and entransy balance analysis (a) conduction-based storage (b) fluid displacement-based storage

Both face third-type boundary conditions at environment interfaces, with heat flux expressed via overall heat transfer coefficient U and temperature difference:

q|*{surface} = -k T/ n*/{surface} = U[T_∞ - T(surface,t)] (3-10)

For conduction storage, internal energy change rate is:

c T/ t = - · q (3-12)

Using Gauss's divergence theorem, total internal energy change becomes surface integrals. Thermal efficiency is:

_q = 1 + (*{surface} q · n dA) / (* {HX} q · n dA) (3-13)

Entransy balance is:

G/ t dV = -   g · n dA +   g_g dV (3-14)

Total entransy efficiency:

_g,tot = [- *{HX} q · n · T dA* -   {surface} q · n · T dA +   g_g dV] / [_{HX} q · n · T dA] (3-15)

For fluid displacement storage (open system), internal energy change includes inflow/outflow:

c T/ t = - · (cTv) -   · q (3-17)

Total internal energy change:

c T/ t dV = -   cTv · n dA -   q · n dA (3-18)

Thermal efficiency:

_q = 1 + (*{in} cTv · n dA +*   {surface} q · n dA) / (_{out} cTv · n dA) (3-20)

Open system entransy balance:

G/ t dV = -   Gv · n dA -   g · n dA +   g_g dV (3-21)

Total entransy efficiency:

_g,tot = 1 - (G_{loss} + G_{diss}) / (G_{in} - G_{out}) (3-22)

The key is determining G_{diss}. For thermal efficiency, focus on surface heat flux. With good insulation or large volume, G_{loss} becomes negligible and G_{diss} determines _g,tot. Analyzing and minimizing entransy dissipation is crucial for performance optimization.

3.2 Turbulent Phenomena and Entransy Dissipation in Water Storage

Reservoir, pit, and tank storage all use hot-cold fluid displacement for charging/discharging. Though volumes differ vastly, all rely on natural thermal stratification to separate hot and cold regions, using inflow/outflow devices (pipes, distributors) to deliver/remove water. Thus, fundamental momentum and heat transport equations are consistent despite different design parameters. This section treats all as thermocline fluid displacement devices and develops unified entransy dissipation models.

The charging process is dynamic with time-varying temperature and flow fields, never reaching steady state. Figure 3-2 shows four flow stages: (1) momentum diffusion-dominated, (2) submerged outflow, (3) thermal diffusion and microscale turbulence-dominated, and (4) outlet entrainment.

Figure 3-2 Four flow stages during charging of fluid displacement storage device

Turbulent diffusion in stratified fluids causes mixing effects from turbulent fluctuations. Turbulent diffusion is the mixing and transport process of fluid "tracers" caused by turbulent fluctuations—much faster than molecular diffusion due to turbulence's irregularity and randomness. It can be described by diffusion coefficients related to turbulence intensity and scale. Turbulent fluctuations also cause irreversible diffusion and entransy dissipation. While Chapter 2 derived entransy dissipation expressions for laminar fluids and solids (from molecular diffusion), turbulent diffusion is widespread in actual operation and must be included. Therefore, turbulent models and source terms causing irreversible entransy dissipation must be analyzed.

Turbulent statistical models describe turbulence behavior statistically, decomposing flow variables into mean and fluctuating components. For incompressible turbulence, variables are:

$v = v + v'$ (3-23)
$T = T + T'$ (3-24)

Angle brackets denote time-averaged mean components (laminar component). Analyzing mean and fluctuation contributions to momentum, heat, and entransy transfer generally uses time-averaged transport equations. Assuming constant thermophysical properties, entransy change rate is:

$D\hat{G}/Dt = cT\, DT/Dt$ (3-25)

Turbulent fluctuation averages satisfy $v' = 0$, $T' = 0$. From continuity, $\cdot v = 0$. Temperature material derivative average becomes:

$DT/Dt = T / t + v \cdot T + \cdot v'T'$ (3-29)

Ignoring viscous dissipation effects on internal energy, turbulent mean temperature and internal energy change rate result only from molecular diffusion:

c DT/Dt = - · q_c = k ² T (3-31)

Turbulent velocity fluctuations cause thermal diffusion represented by turbulent heat flux:

q_t = c v'T' (3-32)

Turbulent energy balance becomes:

c DT/Dt = - ·(q_c + q_t) (3-33)

Solving turbulent entransy dissipation requires solving turbulent heat flux. The widely-used turbulence model is the eddy viscosity model based on Boussinesq hypothesis, which assumes turbulent heat flux aligns with mean temperature gradient (gradient-diffusion hypothesis):

v'T' = -α_T T (3-34)
q_t = -k_T T (3-35)

where α_T is turbulent thermal diffusivity and k_T = α_T c is turbulent thermal conductivity (similar magnitude to turbulent viscosity, Prandtl number Pr_T 0.7–1). Turbulent diffusivity is 1–3 orders larger than molecular due to enhanced transport.

Combining molecular and turbulent effects defines effective thermal diffusivity:

α_{eff} = α + α_T (3-37)

Turbulent mean entransy change rate is:

DĜ/Dt = c T DT/Dt = - T ·(q_c + q_t) (3-38)

= - ·(T q_c + T q_t) + (q_c + q_t)· T

Turbulent entransy dissipation (effective) is:

g_g,eff = (q_c + q_t)· T = (k + k_T)(T)^2 = k_{eff}(T)^2 (3-39)

Thus, determining turbulent heat flux and eddy diffusivity is key for displacement storage entransy dissipation analysis. Solving turbulent diffusivity requires solving eddy viscosity, which must consider both turbulent stress and heat flux fields to close the stress problem. Equation (3-39) provides the general form for entransy dissipation from irreversible thermal diffusion in turbulence, with different turbulence models and k_{eff} for various mixing processes (Figure 3-3).

Figure 3-3 Schematic of typical mixing processes

3.3 Ideal vs. Actual Processes in Entransy Dissipation Analysis

Section 3.2 derived general entransy dissipation expressions for turbulent water storage, concluding that specific turbulent models and thermal diffusivity are needed to calculate actual dissipation. If an "ideal" displacement reservoir had only laminar flow, molecular diffusivity could calculate dissipation. Similar "non-ideal" factors increase dissipation in other storage methods. Though ideal processes are unrealizable, an analysis framework distinguishing "essential" dissipation (from storage principle) from "deviation" dissipation (from design/operation) is valuable for comparing technologies.

This section performs ideal vs. actual entransy dissipation analysis for water and borehole storage to compare theoretical dissipation levels and identify key influencing factors. Table 3-1 lists dissipation/loss components for ideal and actual processes. Ideal processes assume _q = 100% (no heat loss, thus no entransy loss). Actual processes consider all factors reducing stored entransy, including loss and dissipation.

Table 3-1 Entransy loss and dissipation components in seasonal heat storage systems

Process	Ideal Conditions	Actual Conditions
Water Storage	1. _q = 100% 2. No entrainment at hot-cold interface, only conduction along flow direction (thermocline formation) 3. Only conduction causes dissipation 4. No radial conduction	1. _q < 100% 2. Entrainment at interface (thicker thermocline) 3. Radial conduction from sidewall heat loss
Borehole Storage	1. _q = 100% 2. No thermal resistance in borehole/backfill 3. Infinite heat transfer fluid flow rate, no heat transfer between hot/cold pipes 4. No vertical temperature gradient (T/ z = 0) 5. No groundwater flow (v_g = 0)	1. _q < 100% 2. Thermal resistance in borehole/backfill 3. Heat transfer between hot/cold fluids 4. Vertical temperature gradient exists (T/ z > 0) 5. Temperature distribution affected by finite borehole depth 6. Groundwater flow exists (v_g > 0)

Actual processes are complex, so ideal models first analyze main entransy dissipation characteristics, followed by numerical/experimental actual models. Ideal models use 1D transient conduction to obtain analytical temperature distribution and entransy dissipation solutions. Actual processes involve 3D transient heat transfer, generally requiring numerical solutions or measurements.

Cumulative entransy dissipation over any time period [t_0, t] is defined as:

G_{medium} = g_g(x,y,z,t) dV dt (3-40)

To evaluate dissipation per unit stored heat, define equivalent dissipation temperature difference δT_{diss} (units: K):

δT_{diss} = G_{medium} / Q_{stored} (3-41)

3.3.1 Water Storage Entransy Dissipation Analysis

Based on ideal vs. actual process division, ideal water storage assumes only vertical laminar flow of hot/cold water, with heat transfer at the interface driven only by molecular conduction. This ideal process is described by 1D advection-diffusion equation:

_w c_{pw} (T_w/ t + u T_w/ z) = k_w ^{2}T_w/ z^2 (3-42)

where T_w(z,t) is temperature distribution in a single hot water storage space (no partitions), u is average cross-section velocity, and _w, c_{pw}, k_w are water properties.

The analytical solution form is:

T_w = T_{cte} + erfc[(H_{pit} - uz)/(2√(α_w t))] + … (3-43)

Since squaring the z-derivative eliminates the time integral's analytical solution, we simplify by analyzing transient entransy dissipation physics. Ideal charging thermocline growth is equivalent to transient conduction between two semi-infinite spaces at different temperatures. With constant inlet temperature and flow rate, temperature distribution is:

T_w(z,t) = ΔT erfc[z/(2√(α_w t))] + T_{avg} (3-44)

where z is distance from thermocline midpoint, ΔT = T_{hot} - T_{cold}, T_{avg} = (T_{hot} + T_{cold})/2. Figure 3-4 shows the transient distribution. Thermocline average temperature remains constant because heat only transfers internally; thermocline thickness grows as heat continuously moves from hot to cold regions.

Figure 3-4 Temperature distribution during transient conduction between two semi-infinite spaces (Thot = 60°C, Tcold = 30°C)

Based on ideal temperature distribution, thermocline entransy dissipation is calculated via:

G_{medium} = k_w T ^{2}T/ z^2 dz dt (3-45)

For constant cross-section area A_{pit} and height H_{pit}, entransy dissipation per unit area G'_{medium} ($J \cdot K/m^2$) is:

G'_{medium} = ΔT^2 k_w √($\pi\alpha$_w t) (3-46)

The corresponding equivalent thermal resistance is:

R_w = ΔT / G'*{medium} = 1/(k_w _w c*{pw} t) (3-47)

This matches transient conduction resistance definitions, composed of √(_w c_{pw} k_w). Unlike steady conduction, heat capacity affects resistance in transient problems.

From semi-infinite heat flux definition, water storage equivalent dissipation temperature difference is:

δT_{diss},w = ΔT √(2α*w / (u H*{pit})) (3-49)

This shows that for displacement water storage, longer storage times yield smaller dissipation per entransy because diffusion timescale (√t) is slower than linear fluid displacement timescale (t). Displacement velocity significantly impacts dissipation.

Actual processes require turbulent thermal diffusivity α_{eff}. Influencing factors include inertial, viscous, and gravitational forces and thermophysical properties, expressed as functions of dimensionless numbers:

α_{eff}/α_w = f(Re, Ri, Pr, …) (3-50)

Hot-cold interface mixing may enhance heat transfer by tens to hundreds of times conduction alone. In turbulent heat transfer equations, eddy thermal diffusivity α*T or effective conductivity k*{eff} reflects comprehensive mixing effects. Cumulative entransy dissipation per unit stored heat considering turbulence is:

δT_{diss},w = ΔT √(2α*{eff} / (u H*{pit})) (3-51)

Define Fourier number for displacement storage: Fo = α*w t / H*{pit}^{2}. When Fo < 0.1, thermocline internal heat transfer can be treated as semi-infinite 1D conduction. Equivalent dissipation temperature difference per unit theoretical storage capacity is:

δT_{diss},w' = ΔT √(2 / Fo) (3-52)

This depends only on Fo, not on loss rate.

3.3.2 Borehole Storage Entransy Dissipation Analysis

Borehole storage uses soil as medium. In large borehole fields, each pipe exchanges heat with the medium, creating spatial superposition effects (Figure 3-5). Ideal processes treat single boreholes as infinite line sources (ILS) in cylindrical coordinates. Actual processes must consider borehole radius, spacing, depth, etc.

Figure 3-5 Borehole heat exchanger model: (a) mutual pipe influence; (b) single borehole (Lazzarotto 2015)

The 1D conduction equation in cylindrical coordinates is:

 _g c_{pg} T_g/ t = k_g (1/r) / r (r T_g/ r) (3-52)

where T_g(r,t) is soil temperature distribution around a single borehole, and _g, c_{pg}, k_g are soil properties.

With constant charging power q (W), analytical solution is:

T_g(r,t) = T_{g0} + (q/(4πk_g)) Ei(-r^2/(4α_g t)) (3-53)

where T_{g0} is initial uniform soil temperature, and Ei is exponential integral.

Using Laplace transform approximations, near-borehole wall temperature is:

T_b(t) T_{g0} + (q/(4πk_g)) [ln(4α_g t/r_b^2) - γ] (3-55)

where γ 0.577 is Euler-Mascheroni constant. Figure 3-6 shows transient temperature changes: both soil temperature and thermal penetration radius increase during heat diffusion.

Figure 3-6 Soil temperature variation around borehole with time and radius

Cumulative entransy dissipation for ideal charging in cylindrical coordinates is:

G_{medium} = k_g (T_g/ r)^2 2πrH_{bh} dr dt (3-56)

where H_{bh} is borehole depth. Per-unit-depth cumulative dissipation is:

G'_{medium} = (q'^{2}/(4πk_g)) [ln(4αg t/r{bh}^{2}) - γ] (3-57)

where q' is heat transfer rate per unit length (W/m). Equivalent dissipation temperature difference for borehole storage is:

δT_{diss},b = (q'/(4πk_g)) [ln(4αg t/r{bh}^{2}) - γ] (3-58)

Actual processes require g-functions to account for design parameters. The g-function describes temperature response around boreholes, depending on time, soil properties, and borehole layout (Eskilson 1987; Eskilson and Claesson 1988). Generally lacking analytical form, g-functions are represented integrally and solved numerically. They enable accurate temperature prediction for actual charging/discharging through temporal superposition.

For a given borehole design, g-function is g(t/t_s, B/H, D/H, …). When determined, instantaneous entransy dissipation is:

$\dot{G}$_b = q' ΔT = q'^{2} R_{bh}(t) = q'^{2} (1/(2πk_s)) g(t/t_s) (3-62)

Cumulative dissipation:

G_b = q'^{2}/(2πk_s) g(t/t_s) dt (3-63)

Thus, equivalent dissipation temperature difference is a g-function (Figure 3-8). For typical borehole layouts, g-function curve fitting yields:

 g(t/t_s) dt = 0.36 t_s [ln(t/t_s) + 2.24] (3-65)

Therefore:

δT_{diss},b,actual = (q'/(2πk_s)) · 0.36 [ln(t/t_s) + 2.24] (3-66)

3.3.3 Phase Change Storage Entransy Dissipation Analysis

Existing studies (Chen, Wang et al. 2012) propose that multi-stage phase change storage has lower entransy dissipation than single-stage, with optimal phase change temperature at arithmetic mean of hot/cold inlet temperatures for minimum generalized thermal resistance. Bellecci and Conti (1994) suggest geometric mean temperature minimizes entropy generation in Stefan problems. Current conclusions are based on numerical solutions; no analytical entransy dissipation solution exists.

This section builds an idealized Stefan problem model for cumulative entransy dissipation and analyzes main influencing factors.

Figure 3-9 Boundary condition schematic for 1D Stefan solidification problem (Bird, Stewart et al. 2002), with unknown function Z(t) at solid-liquid interface

Define dimensionless temperature = (T - T_m)/(T_0 - T_m), where T_m is phase change temperature, T_0 is initial temperature. Solid and liquid dimensionless temperatures are:

_s = C_1 + C_2 erf(z/√(4α_s t)) (3-68)
_l = C_3 + C_4 erfc(z/√(4α_l t))

The Stefan number Ste = c_p(T_0 - T_m)/ΔH represents sensible-to-latent heat ratio. For ideal processes considering only conduction:

Solid: _s/ t = α_s ^{2} _s/ z^2
Liquid: _l/ t = α_l ^{2} _l/ z^2

Key boundary conditions at solid-liquid interface Z(t) include temperature continuity and phase change energy balance. Solution yields interface position:

Z(t) = λ√(4α_s t) (3-71)

where λ depends only on phase change material properties and boundary conditions, increasing with Ste (Figure 3-10).

Figure 3-10 Variation of λ with Ste number under assumptions (T_0 = 60°C, T_1 = 30°C, T_m = 40°C)

Cumulative entransy dissipation for solid and liquid phases is:

G_{pcs} = k_s (T_s/ z)^2 dz dt + k_l (T_l/ z)^2 dz dt (3-80)

Per-unit-area dissipation is:

ΔG'*{pcs} = [ΔT_m erf(λ) + (ΔT - ΔT_m) erfc(λ)]* √(α{eff} t/π) (3-81)

The form is similar to sensible storage, but temperature squared terms differ: (ΔT_m erf(λ))^2 vs. (ΔT - ΔT_m)^2 erfc(λ). When latent heat → 0 (Ste → ∞),

$\lambda \to 0$, erf(λ) $\to 0$, erfc(λ) $\to 1$, and dissipation approaches sensible storage limit. When phase change is feasible, ΔT_m = ΔT/2 minimizes dissipation (Figure 3-11).

Figure 3-11 Variation with Ste number

3.4 Comparison of Entransy Dissipation Across Storage Technologies

The derived ideal and actual entransy dissipation models provide effective tools for comparing temperature-grade loss (irreversibility) across seasonal storage technologies. Higher entransy dissipation means greater loss of heat transfer capability, making it harder to recover all stored heat at source temperature grade. Thus, entransy dissipation level is an important reference for technology selection.

Based on ideal process analytical solutions and parameters in Table 3-2, Figure 3-12 shows equivalent dissipation temperature differences δT_{diss} for water and borehole storage. Solid curves represent different water storage heights H_{tank}, all decreasing with dimensionless time Fo = $\alpha w\ t/H$\{tank\}^{2} because thermocline growth rate √t while displacement volume t. Heat diffusion is "half-power" slower than constant-velocity displacement, making conduction dissipation increasingly insignificant relative to accumulated storage, hence decreasing δT_{diss}.

Figure 3-12 Comparison of equivalent dissipation temperature difference between displacement water storage (solid) and borehole storage (dashed)

In contrast, borehole storage δT_{diss} increases from ~25 K to ~40 K because borehole wall temperature rises, increasing temperature difference with average soil temperature. This driving temperature difference can be viewed as the heat transfer temperature difference. Unlike ideal displacement storage where conduction is a "side effect," conduction is the essential mechanism for borehole storage. For seasonal storage lasting up to half a year, water displacement storage theoretically offers lower dissipation.

Table 3-2 Parameters for entransy dissipation comparison between displacement water storage and borehole storage

Common Design/Operational Parameters	Specific Design Parameters	Case Study Variables
Water Displacement Storage	**Borehole Storage**	

Common Design/Operational Parameters	Specific Design Parameters	Case Study Variables
Charging temperature: T_{hot},in = 90°C	Borehole depth: L_{bh} = 50–150 m	Storage container height: H_{tank} = 10, 20, 50, 100 m
Discharging temperature: T_{cold},in = 20°C	Heat injection rate: q' = 233 W/m	Geology: limestone, granite, sandstone
Total theoretical capacity: Q_{total} = 4500 GJ	Borehole spacing: B/H = 0.12–3.9	Thermal conductivity: k_{soil} = 1.5–3.5 $W/(m \cdot K)$
Charging period: t_c = 180 days	Aspect ratio: D/H = 0.6	Volumetric heat capacity: c = 2.0–2.5 $MJ/(m^3 \cdot K)$
Temperature difference: ΔT = 70 K		Thermal diffusivity: α_{soil} = 0.6–1.5 × 10^{-6} m^2/s

Figure 3-12 also shows that for fixed volume, taller, smaller-cross-section water storage has lower dissipation because ideal dissipation only occurs at hot-cold interface cross-section—smaller area means higher displacement rate and lower conduction flux. For borehole storage, despite 1–2× differences in thermal properties across geologies, δT_{diss} curves are similar, showing medium properties aren't key factors. Overall, ideal water displacement storage has smaller δT_{diss}.

However, actual process comparison must consider hot-cold mixing-induced irreversible diffusion. Based on α{eff}*'s physical meaning in Eq. (3-50), define heat transfer amplification factor* N{mix} = α_{eff}/αw. *Larger* N{mix} increases water displacement storage entransy dissipation. To achieve lower dissipation than borehole storage, displacement velocity must exceed a threshold u_0:

u_0 = (π^2 αw N{mix}^{2} P_{bh}) / (2 αg t P{bh} - 3π αw N{mix} t) (3-83)

Figure 3-13 Critical displacement velocity (u_0) under different heat transfer amplification factors (N_{mix})

Figure 3-13 shows threshold variation with mixing intensity. For N_{mix} = 5, 120-day storage requires minimum 0.00024 m/d vertical velocity to beat borehole dissipation; for N_{mix} = 100, threshold drops to ~0.00005 m/d. Note: this comparison only considers medium dissipation and ideal single borehole.

Despite idealized assumptions simplifying complex transport phenomena, this entransy dissipation and equivalent dissipation temperature difference method provides an alternative to exergy efficiency, with mathematical foundation and computational feasibility for analyzing specific storage device losses.

Qualitatively, Table 3-3 compares displacement and heat exchanger storage characteristics. Quantitative comparison of charging/discharging rates and internal entransy dissipation is needed for comprehensive understanding and technology selection.

Table 3-3 Reclassification of heat storage principles and characteristic comparison

Characteristic	Fluid Displacement Storage	Heat Exchanger Storage
Principle	Hot fluid storage container	Heat exchanger between fluid and medium
Charging Rate	Determined by fluid displacement rate: Q = c_p V	Determined by heat transfer and internal resistance: $Q = UA\Delta T$
Irreversibility	Mixing at hot-cold interface; small dissipation when interface undisturbed	Conduction through thermal resistance; generally larger dissipation
Heat Transport	No heat transport between fluid and medium; small dissipation	Heat exchange between fluid and medium; high dissipation when mismatch is large

Methodologically, entransy analysis has been widely applied to steady-state heat exchanger optimization but rarely to transient processes or fluids, and never to seasonal storage. This study innovatively establishes a transient entransy dissipation analytical model-based evaluation framework for storage device irreversibility characteristics.

The analysis reveals water displacement storage's theoretical dissipation advantage for long-duration, seasonal operation due to decreasing δT_{diss} with time. However, this requires maintaining low-intensity hot-cold mixing. If water storage is the primary path forward, comprehensive research is needed on reservoir flow/heat transfer phenomena and quantitative entransy dissipation analysis of mixing mechanisms.

4. Flow and Heat Transfer Phenomena in Hot Water Storage Reservoirs

Hot water reservoirs store hot water through displacement flow of hot and cold water. This chapter discusses relevant flow and heat transfer phenomena and basic physical models.

4.1 Basic Characteristics of Water Storage

Hot-cold displacement is continuous macroscopic motion of fluids with temperature differences. Injected water differs in density from stored water—cold water is denser, hot water less dense. This creates gravity (buoyancy)-driven flow called density current, enabling vertical stratification and avoiding complete mixing—the principle of thermocline thermal energy storage. This flow forms a cold lower layer and hot upper layer. In stratified systems, vertical flow consumes energy (pumping), while horizontal flow doesn't, so flow extends far horizontally within narrow vertical layers.

Water enters/leaves reservoirs through inlets/outlets. Danish pit storage uses pipe distributors, while large reservoirs use river inflow, side weirs, or selective withdrawal structures. The primary goal is storing heat with minimal entransy dissipation and heat loss. Ideal operation maintains clear stratification without disruption from inflow/outflow. Design challenges include accurately predicting internal flow patterns and heat transfer processes.

Internal heat transfer is diffusion from high to low temperature regions, classified as molecular diffusion (conduction) or turbulent diffusion (orders of magnitude faster). Another mechanism is advection—overall fluid movement carrying heat, the principle of displacement storage.

4.2 Basic Flow Processes

Fluid Motion Equations

Based on continuum assumption, fluid properties are continuous functions of space and time. The primary variable is velocity vector v(x,y,z,t), with pressure p (scalar) and coupled scalars density and temperature T.

Flow description uses Eulerian approach. For unsteady storage processes, focus on fluid particle acceleration. Stress tensor τ describes internal stresses (normal and shear). Navier-Stokes equations describe fluid motion, considering pressure, viscous, and external (gravity) forces.

With Boussinesq approximation (density change only in gravity term) and incompressibility ($\cdot$ v = 0), constant density $_0$ and kinematic viscosity , the equation is:

$$v/\ t + (v\cdot\ )v = -(1/\ _0)\ (p -\ _0 g \cdot z) +\ ^2 v - g\alpha(T - T_0) \quad (4\text{-}1)$$

where α is thermal expansion coefficient, and the last term is buoyancy.

For ideal fluids (→ 0), viscous effects are negligible, simplifying to:

$$v/\ t + (v\cdot\ )v = -(1/\ _0)\ (p -\ _0 g\cdot z) - g\alpha(T - T_0)\ (4\text{-}2)$$

Though ideal fluids don't exist, viscous effects are often secondary in large-scale reservoirs with low velocities. When turbulence, boundary layers, and interface instability effects on overall flow are negligible, ideal fluid assumptions hold—Peclet and Richardson numbers dominate, viscosity can be ignored. When Reynolds and Richardson numbers dominate, viscosity must be considered.

Coriolis force from Earth's rotation also affects large reservoirs. Coriolis acceleration is perpendicular to velocity, changing direction but not magnitude. For small reservoirs, Coriolis effects can be ignored, making flow irrotational at macroscopic scales. For large lakes/reservoirs, Rossby radius L_R = √(g'H)/f determines importance (f is Coriolis parameter). At 40°N, with 2 m thermocline thickness and 70 K temperature difference, L_R 76.7 m, meaning very large reservoirs must consider Coriolis effects.

Rotating fluid equations become:

$$v/\ t + (v\cdot\ )v = -(1/\ )\ p + g - \omega\times(\omega\times r) - 2\omega\times v +\ ^2 v\ (4\text{-}4)$$

Centrifugal term can be merged with pressure. Coriolis term $-2\omega\times v$ is the key addition.

Turbulence Effects

These basic equations don't include turbulence. However, velocity differences inevitably create shear layers and various turbulence types: jet boundary turbulence from inlet/outlet turbulence, shear instability turbulence at stratified interfaces, and boundary layer turbulence near walls. Turbulence generates vortices and irregularities at scales much larger than molecular motion, dramatically enhancing heat, momentum, and mass transport efficiency compared to laminar flow, thus intensifying hot-cold mixing.

Turbulence also causes kinetic-to-internal energy dissipation. Statistical models decompose turbulence into mean and fluctuating components. For incompressible turbulence:

v = v + v' (4-5)

N-S equations with fluctuations become:

v_i / t + v_j v_i / x_j = - (1/) p / x_i + ^{2} v_i - v_i'v_j' / x_j (4-6)

The Reynolds stress term - v_i'v_j' / x_j (nine components) represents momentum transfer from turbulence. Solving requires Reynolds stress transport equations.

Turbulent kinetic energy (TKE) per unit mass is k = (1/2) v_i'v_i' . Its transport equation is:

k/ t + v_j k/ x_j = - v_j'k / x_j - v_i'v_j' v_i / x_j + v_i' ^{2}v_i' (4-7)

For storage reservoirs, temperature T is the key scalar. Scalar transport equation is:

$$T / t + v\_j \quad T / x\_j = - \ v\_j'T' / x\_j + \alpha\ ^2 T \quad (4\text{-}8)$$

Closure requires:

$$v\_i'T' \ = -\alpha\_T \quad T / x\_i \ (4\text{-}9)$$

where α_T is turbulent thermal diffusivity (not a property but phenomenological parameter). Scalar transport must be solved with momentum equations. Three main approaches exist: Direct Numerical Simulation (DNS) resolves all turbulence scales; Reynolds-Averaged Navier-Stokes (RANS) models mean fields; Large Eddy Simulation (LES) resolves large scales only. RANS models (zero-equation, k- , k-ω, RNG k-) are primary engineering tools.

Natural Stratified Flow

Thermally stratified flow is the core phenomenon. With good stratification, clear hot-cold interfaces form; with severe mixing, continuous density gradients make interfaces diffuse. Density differences from temperature mainly act vertically, creating horizontal layers. During charging, hot water injected at top spreads horizontally at its neutral buoyancy layer. Without strong vertical momentum, fluid remains confined to horizontal layers.

Density difference Δ / 0.003 for ΔT = 10 K, and only ~0.02 for extreme 70 K difference. Since density variation is small relative to reference density, its effect can be ignored in inertia terms but must be included in gravity terms. With Boussinesq approximation and linear density-temperature relationship:

$$= \ _0[1 - \alpha(T - T_0)] \ (4\text{-}10)$$

N-S equation becomes:

$$v/ t + (v \cdot \ )v = -(1/ \ _0) \ p + \ ^2v + g\alpha(T - T_0) \ (4\text{-}11)$$

Flow Distribution and Withdrawal Methods

Reservoirs require distributors to deliver/remove water. Flow patterns include jets, buoyant plumes, and buoyant jets—distinguished by dominant forces: inertia (jet), buoyancy (plume), or both (buoyant jet). Storage is unsteady, requiring analysis before and after outlet submergence.

Figure 4-2 summarizes common distribution methods: horizontal (disc distributors, horizontal outlets, slope wall flow) and vertical (pipe distributors, perforated plates). Disc and horizontal outlets typically form buoyant jets; slope wall flow with low momentum forms buoyant plumes. Large tanks use H-type or annular pipe distributors or perforated flow straighteners creating turbulent jets.

Figure 4-2 Common flow distribution methods: horizontal inflow (a-c); vertical inflow (d-e)

Buoyant jets combine jet and plume characteristics, influenced by both momentum and buoyancy. Inertia-to-buoyancy ratio is described by densimetric Froude number Fr_d. When Fr_d $\rightarrow \infty$, flow is momentum jet; when Fr_d = 0, pure plume. In storage applications, buoyant jets dominate early charging with large temperature differences; later stages become homogeneous jets when interface is far from outlet.

Momentum integral method is most common for engineering jet analysis. Along jet axis, establish entrainment flow rate equation per unit length (Eq. 4-12) with entrainment coefficient α_e, coupled with continuity, momentum, and density flux conservation equations to solve velocity, density distribution, and trajectory.

dQ/ds = 2πb α_e u_m (4-12)

Reservoir withdrawal methods include disc distributors, pipe extractors, and selective withdrawal structures from reservoir engineering. In spring irrigation, warm surface water is preferred over cold bottom water, requiring stratification-preserving or selective withdrawal. Four types exist: multi-level outlets, overflow outlets, pipe outlets, and curtain-controlled intake (Huang Yongjian 1986). Hydraulically, they divide into weir flow and orifice flow (Figure 4-3).

Figure 4-3 Schematic of weir and orifice withdrawal methods

Near orifice outlets, converging flow creates low-pressure zones that suction lower cold water upward (Figure 4-4), causing mixing. Predicting outflow mixing requires considering this suction effect.

Figure 4-4 Schematic of density current discharge from orifice (Qian Ning 1957)

Horizontal Partition Flow

Large reservoir surface area may cause excessive hot-cold interface area and mixing intensity. Horizontal partitioning divides the reservoir into N_{htr} sections with smaller cross-sections, reducing interface area to 1/N_{htr} (Figure 4-5). However, partitions introduce additional mixing at partition wall inlets/outlets and within partition channels.

Figure 4-5 Storage and extraction processes with longitudinal partition in reservoir storage

Table 4-1 compares single vs. horizontal partitioning across five mixing factors: outlet entrainment, inlet mixing, channel mixing, shear instability, and conduction. Horizontal partitioning is beneficial only if reduced shear instability and conduction mixing outweigh increased wall effects—requiring dynamic whole-reservoir numerical modeling.

Table 4-1 Comparison of single-partition vs. horizontal partitioning for storage reservoirs (red indicates advantage)

Mixing Factor	Single Partition	Horizontal Partition
Outlet entrainment	Once, at end of heating season	At each partition wall
Inlet mixing	Once, at start of heating season	At each partition wall
Channel mixing	None	Within each partition
Shear instability	Large diffusion range, high coefficient	Limited to single partition
Conduction	Large interface area, high dissipation	Small interface area, low dissipation

Effective thermal conductivity can be expressed as function of partition number:

k_{eff} = f(N_{htr}) (4-13)

Equivalent dissipation temperature difference:

δT_{diss}(N_{htr}) = ΔT √(2αw / (u H{tank} f(N_{htr}))) (4-14)

If f(N_{htr}) is linear, more partitions reduce dissipation; if cubic, more partitions increase dissipation (Figure 4-6). The key is understanding mixing phenomena, intensity, and entransy dissipation.

Figure 4-6 Effect of horizontal partition number on equivalent thermal conductivity

4.3 Basic Heat Transfer Processes

Heat transfer divides into: (1) reservoir-boundary exchange with environment (soil, air) causing heat loss, efficiency reduction, and entransy loss; (2) internal heat transfer from hot to cold regions, causing entransy dissipation and efficiency reduction.

Water Surface Heat Loss

Reservoir surfaces have coupled convective heat and evaporative mass transfer. Insulated floating covers are typically installed, making solar radiation negligible. Surface heat loss coefficient describes combined mechanisms (Eq. 4-14), where n is correction factor.

q_s = K_s (T_s - T_a) (4-14)

Evaporation is driven by forced (wind) and free (buoyancy) convection. On natural surfaces, forced convection dominates, especially with strong evaporation creating surface cooling, inverse temperature layers, and upper-layer vertical turbulence (Ryan, Harleman et al. 1974).

Danish pit storage studies show ~60% of total heat loss occurs at the top (Xie, Xiang et al. 2021), making insulated covers critical. Estimated overall loss coefficient is 0.21 W/($m^2 \cdot$K). Two main cover technologies exist: concrete covers and geomembrane floating covers—the latter being more economical and widely studied. Covers use polymer insulation with foam glass bases and edges. Key design points are sealing at walls and edges; otherwise evaporation losses are high. Condensation wetting insulation increases conductivity threefold (e.g., Studsvik storage: 0.24 W/(m $\cdot$ K) when wet (Heller 1997)), requiring vacuum valves to vent vapor and prevent condensation damage.

Figure 4-7 Floating cover design with vacuum valve (PlanEnergi, 1 et al. 2014)

With floating covers, shortwave radiation ϕ_{br} = $0.97\sigma T_0^4$ is negligible. Vacuum valves modify boundary conditions: (1) they maintain low humidity above water surface, increasing evaporation compared to saturated conditions without valves; (2) high water temperature (70–90°C) creates heated plate natural convection, forming cellular flows blocked by the cover; (3) covers reduce wind speed and convection coefficient, adding thermal resistance.

General surface heat loss formula (Ryan, Harleman et al. 1974) is:

ϕ_L = ϕ_e + ϕ_c = [λ(T_s - T_a) + π_0W_z] $\cdot$ [e_s - e_a + C(T_0 - T_a)] (4-15)

where λ is latent heat, W_z is wind speed, e_s and e_a are saturation and ambient vapor pressures, C is Bowen constant (~0.61 mbar/°C). Forced convection depends on wind speed; natural convection depends on Gr and Pr. Mixed convection must be considered with vacuum venting.

Peripheral Heat Loss

Factors include geomembrane waterproofing, insulation effectiveness, and groundwater. European large water storage uses two insulation structures: (1) Danish pit storage (20,000–200,000 m^3) uses only floating covers, no side insulation, with geomembrane to prevent water infiltration (Figure 4-9); (2) German pit/tank storage (thousands to 10,000 m^3) uses full peripheral insulation with geomembrane and drainage to protect insulation from hot water and groundwater (Figures 4-10, 4-11).

Figure 4-9 Construction structure of Danish Vojens pit storage

Figure 4-10 Insulation materials for German tank storage (Ochs, Nußbicker et al. 2008)

Figure 4-11 Peripheral insulation design for German storage system (Ochs, Nußbicker et al. 2008)

Composite wall thermal resistance analysis simplifies layers to series resistances plus contact resistance. Soil beyond 10 m is essentially unaffected, so 10 m thick outer boundary can be used in 1D steady analysis. Effective conductivity of porous insulation (foam glass granules) can be estimated via:

k_{eff} = [k_e + 2k_s - 2 (k_s - k_e)] / [k_e + 2k_s + (k_s - k_e)] $\cdot$ k_s (4-18)

Internal Heat Transfer

Internal heat transfer is the hot-cold mixing process. Figure 4-12 shows three main processes in natural lakes: (1) epilimnion vertical mixing and deepening; (2) hypolimnion vertical/horizontal mixing; (3) inflow/outflow entrainment. Analyzing these mechanisms and factors, combined with reservoir operation principles, identifies key mixing factors. Reservoirs have larger temperature gradients, stronger stratification, and higher per-unit-volume flow rates than natural lakes, with faster water replacement. Insulated covers reduce surface evaporation, radiation, and wind effects.

Figure 4-12 Schematic of internal heat transfer (mixing) mechanisms (Fischer, List et al. 1979)

(1) Surface Layer Mixing

In many lakes, wind is the primary kinetic energy source for vertical mixing; river inflow momentum is minor (Fernández Castro, Wüest et al. 2021). Wind stirring deepens the thermocline. Convection and wind-driven mixing are significant, creating uniform mixed layers.

Surface cooling rate is Q_{cool} (W/m^2). Without viscous effects, surface cooling reduces potential energy at rate αgQ_{cool}/(2 c_p). Entraining deeper cold water increases potential energy. Mixed layer kinetic energy change is dKE/dt = $_0$u_e^3/2, where u_e is entrainment velocity. Energy balance yields:

d(PE+KE)/dt = αgQ_{cool}/(2 c_p) - $_0$u_e^3/2 + αgΔTg $_0$u_e h/2 (4-19)

This allows calculating mixed layer depth h(t) and temperature after surface cooling:

T_{surf}   0.5 Q_{cool} t / (c_p h) (4-20)

Reservoir operation controls vertical flow, differing from natural lakes. In Danish pit storage (e.g., Dronninglund), 3–4 disc distributors at different heights enable selective withdrawal for different temperature distributions (Figure 4-14). Upper and middle distributors create surface and submerged outflows, with surface cooling affecting flow differently in each case.

Figure 4-14 Temperature distribution in Dronninglund pit storage (Sifnaios, Gauthier et al. 2023)

Figure 4-15 shows interface positions when stratification reaches upper distributor—at charging start and discharging end. Submerged outflow creates buoyant jets rising to surface, enhanced by surface cooling and natural convection, forming more mixed upper layer. Surface outflow has horizontal jet momentum dominating flow and directly affecting surface temperature. Continuous hot water withdrawal prevents surface cooling accumulation, making forced convection dominant.

Figure 4-15 Schematic of inflow/outflow method impacts

(2) Deep Water Layer Mixing

Stable thermocline acts as a “protective lid,” shielding deep water from surface disturbances, creating weakly stratified, uniform temperature layers (Figure 4-13). Deep layer vertical diffusion coefficients vary widely—from molecular (no turbulence) to 10× molecular under strong wind or high flow. Diffusivity generally decreases with increasing N (buoyancy frequency, representing stratification strength). Relationship forms include:

K__z = a N^{-n} (4-21)

where n = 0.2–2.0 depending on lake shape and flow. Length scales are constrained by turbulent kinetic energy diffusion and buoyancy frequency:

__z = m(N^c/H) = /N (4-22)

Deep layers experience intermittent, localized mixing when kinetic energy is introduced. Once energy depletes, mixing quickly subsides under buoyancy and viscous dissipation—making mixing local and intermittent. In small-medium lakes, boundary mixing is significant. Bottom roughness (0.01–0.2 m) creates turbulent boundary layers.

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5. Mechanisms of Hot-Cold Mixing in Storage Reservoirs

Based on Chapter 4’s flow and heat transfer descriptions, this chapter analyzes internal heat transfer (mixing) mechanisms and parametric models, quantitatively discussing actual mixing intensity and temperature distribution impacts.

Mixing mechanisms span three scales: molecular conduction (nanoscale), turbulent eddies (millimeter scale), and entrainment (meter scale). Comprehensive consideration of all mechanisms in numerical simulation is difficult. No research has “bottom-up” analyzed all possible mixing scenarios in ultra-large storage bodies or quantified irreversibility using entransy dissipation. Overall, hot-cold mixing research for seasonal storage is nascent.

This chapter first discusses dimensionless analysis principles from force balance (macroscale), then energy cascade and dissipation rates (microscale), and specific mixing forms.

5.1 Transport Phenomena in Storage Reservoirs

Force Balance and Dimensionless Numbers

Lake and reservoir flow/heat transfer are essentially mass, momentum, and energy transport phenomena described by dimensionless numbers representing ratios of effective diffusivities. Six dimensionless numbers characterize relative strengths of inertial, viscous, conductive, and buoyancy transport (Table 5-1).

Table 5-1 Dimensionless numbers for reservoir flow phenomena

Number	Physical Meaning
Re	Inertial vs. viscous forces; flow regime
Fr	Inertial vs. gravitational forces; wave phenomena
Pr	Momentum vs. thermal diffusion
Pe	Thermal advection vs. molecular diffusion
Ra	Buoyancy vs. viscous + conductive forces (natural convection)
Gr	Buoyancy vs. viscous forces (natural convection)
Ri	Buoyancy vs. inertial forces; stratification stability

Table 5-2 shows these numbers as ratios of transport mechanisms (e.g., Re = inertial/viscous). In reservoir fluid dynamics, Re is critical: low Re means viscous-dominated laminar flow; high Re means inertial-dominated turbulent flow.

Table 5-2 Transport phenomena comparison and dimensionless numbers

	Inertial	Viscous	Conduction	Buoyancy
Inertial	-	Re	Pe	Fr_d
Viscous	-	-	Pr · Re	Gr
Conduction	-	-	-	Ra/Pr

Density differences create buoyancy-driven flow. At large scales, viscous forces can be ignored, requiring densimetric Froude number Fr_d = u/√(g'L) and horizontal Froude number Fr_h = u/√(gL). When Fr_d is high (buoyancy low), inertia dominates (jet regime); when low, buoyancy dominates (plume regime).

Note: Ri and Fr have different definitions for macroscopic flow vs. internal waves. At small scales, Gr characterizes buoyancy vs. viscosity. When buoyancy overcomes viscosity, natural convection strengthens. Ra = Gr · Pr compares natural convection to conduction. Pe compares forced convection to conduction.

Diffusion coefficients for momentum ( ) and heat (α) have dimensions [L^2/T]. Advection velocity times length scale (vL) also has dimension [L^2/T], representing convective transport rate. Convection and diffusion are different mechanisms causing transport. Dimensionless numbers characterize ratios of these transport rates.

Energy Conservation Principles for Mixing

Analyzing hot-cold mixing aims to quantify instantaneous diapycnal mixing rates and kinematic energy dissipation rates, linking them to flow evolution. For a fluid element, energy conversion includes: (1) buoyancy flux, (2) irreversible mixing, (3) viscous dissipation, and (4) irreversible internal-to-potential energy conversion from heat transfer. Boundary effects add convective transport (Figure 5-1).

Figure 5-1 Energy conversion schematic for mixing in stratified Boussinesq flow (redrawn from Winters, Lombard et al. 1995)

(1) Buoyancy Flux

Internal waves in density-stratified fluids cause vertical motion and potential-kinetic energy conversion. Lorenz (1955) defined total potential energy as potential + internal energy, with potential energy split into available (APE) and background (BPE) components. APE is energy obtainable by adiabatically rearranging density to minimum potential energy state. At internal wave crests, kinetic energy maximally converts to APE; at troughs, the reverse. These processes create buoyancy flux. With purely vertical, stable stratification (no internal waves), potential energy is minimized and cannot convert to kinetic energy. In adiabatic flow, APE + KE is conserved. APE is determined solely by density distribution. BPE is minimum potential energy achievable by adiabatic rearrangement. Expressions are:

$E_p = E_a + E_b$ (5-1)
$E_p = g \quad z\, dV$ (5-2)
$E_b = g \quad z(x,t)\, dV$ (5-3)
$E_a = g \quad (z - z)\, dV$

where z* is reference height.

(2) Irreversible Mixing

Includes molecular and turbulent diffusion. Molecular mixing is irreversible, occurring even in static fluids. BPE change rate is proportional to molecular diffusivity (Lorenz 1955). For closed systems, only molecular mixing changes probability density function (PDF) and BPE. Stirring is reversible (no PDF change), but irreversible mixing changes reference points (PDF changes). Figure 5-2 shows thermocline thickening and gradient reduction (dashed vs. solid) after mixing, with PDF changes.

Figure 5-2 PDF changes in hot-cold stratified system (Winters, Lombard et al. 1995)

Internal energy-to-potential energy conversion rate (Boussinesq form) is:

$\Phi_i = -\alpha g \quad 'w'$ (5-4)

where is density diffusivity. Total potential energy change rate is sum of reversible buoyancy flux and irreversible mixing: $dE_p/dt = \Phi_z + \Phi_d$. Available potential energy change is $dE_a/dt = \Phi_z - (\Phi_d - \Phi_i)$. Background potential energy change is $dE_b/dt = \Phi_d$.

(3) Viscous Dissipation

Kinetic energy per unit mass is $KE = (1/2)u_i u_i$. Its change rate includes convective transport, molecular momentum flux work, reversible buoyancy conversion, and viscous dissipation:

dKE/dt = - [u_i(pδ{ij} + τ{ij})]/ x_j + g 'w' -   (5-5)

where   = 2 s_{ij} s_{ij} is dissipation rate, s_{ij} is strain rate tensor.

Energy balance provides an intuitive framework: separating reversible and irreversible processes reveals thermodynamic essence—buoyancy flux creates reversible potential energy changes, while mixing creates irreversible changes. Predicting mixing reduces to calculating each process's energy fraction: e.g., viscous dissipation fraction of TKE change, reversible vs. irreversible fractions of APE change.

5.2 Macroscopic Flow Entrainment

Artificial Inlet Mixing

Inlet mixing depends on discharge structure design, densimetric Froude number Fr_d, and ambient conditions. Table 5-3 classifies typical outflow regimes.

Table 5-3 Classification of reservoir outflow regimes

Regime	Schematic
Surface jet with free surface, small submergence (Stolzenbach & Harleman 1971)	
Surface jet with free surface, large submergence	
Submerged horizontal outflow into stratified ambient (Fan & Brooks 1969)	
Submerged vertical jet (Jirka & Harleman 1979)	
Submerged vertical jet into stratified ambient (Fernando 2012)	

Core objectives are velocity and temperature distribution in jet spread region, solved using momentum integral method with similarity assumptions. For buoyant jets, integral models predict trajectory, width, and dilution in uniform or stratified environments without ambient flow.

Mass conservation along jet axis:

dQ/ds = 2πb α_e u_m (5-8)

where Q is total volume flux, s is axial coordinate, b is jet half-width, α_e is entrainment coefficient, u_m is centerline velocity.

For slot outlets, velocity and temperature profiles are Gaussian:

$u(s,y) = u\_m(s) \exp(-y^2/(2b^2))$ (5-10)
$T(s,y) = T\_m(s) \exp(-y^2/(2(\lambda b)^2))$ (5-11)

where λ is profile shape factor (1.16). Jet axis dilution $S_0 = Q/Q_0$ describes concentration decay. Two key initial parameters are outlet angle $_0$ and initial momentum m_0 (determined by velocity).

Numerical integration is required. Fan and Brooks (1969) performed integrations for $_0 = 0°$ and 45° into stationary, unstratified ambient (Figure 5-4). Large $_0$ produces buoyant jet behavior; small $_0$ approaches vertical jet.

Figure 5-4 Buoyant jet trajectories for slot cross-section (horizontal and 45° discharge into stationary, unstratified ambient)

Dilution S_0 for vertical/horizontal slot jets is shown in Figure 5-5. For $= x/(\sqrt{m_0}\ b_0) > 50$ and $Fr\_d > 0.28$, simple plane plume relations give dilution. For given $\_m_0$, axis dilution S_0 can be read from curves.

Figure 5-5 Dilution for vertical and horizontal slot buoyant jets

Post-mixing average temperature rise is:

$\Delta T = \ \Delta T(s,y)\ u(s,y)\ dy\ /\ \ u(s,y)\ dy = \Delta T_0\ /\ S_0$ (5-22)

Entrainment coefficient α_e is ratio of entrainment velocity to jet cross-section average velocity—an empirical constant for homogeneous jets. However, for density-varying jets, α_e should be function of local Fr_d, decreasing with density difference.

Chinese Institute of Water Resources and Hydropower Research systematically studied mixing via theoretical analysis and experiments (Chen Huiquan 1963). For infinite-depth horizontal outflow, mixing coefficient E is function of Fr and Fr_d:

$E = f(Fr, Fr\_d)$ (5-24)

For buoyancy-dominated flows, Ri number (inverse of Fr_d^2) describes buoyancy-inertia interaction. Ellison and Turner (1959) showed entrainment coefficient E decreases rapidly with Ri, becoming negligible when $Ri > 0.8$ (Figure 5-8). Debler (1959) found flow separates into two horizontal layers when $Fr\_d < 0.28$.

Figure 5-8 Entrainment coefficient E vs. Richardson number Ri for surface jets (Ellison and Turner 1959)

Using K = entrainment velocity/jet velocity and M = Fr/Fr_d, empirical relationship K-M is shown in Figure 5-9.

Figure 5-9 Measured relationship between entrainment coefficients K and M

These empirical formulas guided cooling pond designs for Chinese power plants. For storage reservoirs, they can infer initial mixing near inlets/outlets during early charging/discharging. Energy balance before/after mixing (Eq. 5-30) predicts post-mixing water temperature.

$(T_0 - T_\infty)Q_0 = (T - T_\infty)(Q_0 + \Delta Q) + \Phi$ (5-30)

Density Current from River Inflow

In stratified reservoirs, inflow forms different density currents based on temperature: surface flow, interflow, or underflow. Interflow requires stable stratification. When inflow density matches ambient at neutral buoyancy layer, it separates from slope and intrudes horizontally (Figure 5-10). The process has three zones: (1) plunge line with initial mixing; (2) entraining underflow along bottom with boundary mixing; (3) subsurface intrusion when density equals ambient.

Figure 5-10 Surface flow and intrusion flow (Fischer, List et al. 1979)

Interflow phenomenon is shown in Figure 5-11. When inflow density decreases to ambient, it moves horizontally at neutral buoyancy layer, sometimes forming "cusped waves" (Figure 5-12). The plunge point densimetric Froude number $Fr_0 = u_0/\sqrt{(g'h_0)}$ determines interflow formation: $Fr_0 > 0.78$ is required; if depth is too small or velocity too low, interflow won't form.

Figure 5-11 Interflow phenomenon schematic (Ren Shi 2016)

Figure 5-12 Turbid interflow intrusion in De Gray Lake (Fischer, List et al. 1979)

Determining inflow-induced velocity, mixing, and interflow position is complex. Lake Temperature Model (Lake Thermocline Model) uses prescribed mixing layer thickness and entrainment ratio to estimate inflow (Fan Lenian, Liu Xinzhi 1984). Assuming uniform entrainment velocity u_0 in mixing layer:

$u_0 = P_{mix} Q_{in} / (B z)$ (5-31)

where P_{mix} is entrainment ratio, B is width at height z. Post-mixing inflow temperature is:

$T_{z,in} = [T_{in} + P_{mix} \quad T B dz] / (1 + P_{mix})$ (5-32)

GLM 3.0 lake model calculates entrainment coefficient $C_{S,inflow}$ via drag coefficient C_D and inflow densimetric Froude number (or inflow Reynolds number). Most intrusions have triangular cross-section. After momentum conservation derivations:

$C_{S,inflow} = C_D [1 + 0.21(C_D \sin\alpha_{inflow})] / (\sin\phi_{inflow})$ (5-34)

where α{*inflow*} *is half-angle of triangular flow,* ϕ{inflow} is slope angle of inlet channel (Figure 5-13).

Figure 5-13 GLM lake model (Hipsey, Bruce et al. 2019)

Selective Withdrawal Mixing

During charging/discharging, temperature-stratified fluid must be withdrawn at specific heights: cold water from bottom during charging, hot water from top during discharging. Ideal operation withdraws single-phase water without mixing. However, critical efflux velocity v_c causes interface mixing (Figure 5-14). For binary temperature difference, stratified fluid can be treated as two miscible, incompressible fluids with similar viscosity but different density. Critical velocity is:

v_c = 0.178 (^{2}g/Δ /)^0.33 for Re > 450 (5-38)
v_c = 0.127 (^{2}g/Δ /)^0.33 for Re < 450

Figure 5-14 Critical interfacial flow velocity (Huang Yongjian 1986)

For disc distributors, axisymmetric withdrawal applies (Figure 5-15c). For horizontal partitions, side withdrawal must avoid critical velocity.

Figure 5-15 Schematic of water intake problem (Streeter and Kestin 1961)

During actual discharging, the interface takes long periods to approach upper outlets, rarely maintaining sharp binary interface. More likely is continuous stratification. If withdrawal layer thickness is large, multiple density layers may be involved, increasing mixing. Smaller withdrawal thickness enables more precise selective withdrawal, reducing mixing. Withdrawal layer thickness δ_w is key parameter.

For continuous stratification (Figure 5-16), δ_w is (Imberger and Fischer 1970):

δ_w = G (q^2/βgQ)^(1/3) (5-40)

where G is Hino-Ohashi coefficient (0.324 for top/bottom, 0.134 for middle withdrawal), q is flow rate per unit width, β is thermal expansion coefficient, Q is total flow. When thermocline has large gradient, it blocks withdrawal range (Li Guangning 2015).

Figure 5-16 Selective withdrawal from continuously stratified water body (Huang Yongjian 1986)

Outlet temperature prediction requires mass and energy balances based on velocity and temperature distributions within withdrawal layer:

T = T(h) u(h) dh / u(h) dh (5-41)

Natural Convection

Temperature differences cause density-driven natural convection. In storage water bodies, two thermally unstable conditions occur:

1. **Horizontal temperature non-uniformity from side heat loss**: Far from inlets/outlets, forced convection is weak and natural convection dominates. Boundary cooling creates horizontal temperature gradients—unstable states inducing natural convection that destroys thermocline, especially in small tanks.

2. **Inverse stratification from surface heat loss**: Large top heat loss creates "cold-over-hot," "heavy-over-light" unstable condition, triggering vertical circulation. Rayleigh-Bénard convection model describes this, creating irregular cellular flows above thermocline (Figure 5-20).

Figure 5-20 Rayleigh-Bénard convection schematic

For small tanks, enclosure assumptions apply with laser Doppler velocimetry measuring near-wall velocity fields (Hess and Miller 1982). For large reservoirs, semi-infinite space assumptions apply. Inlet/outlet buoyant jets are affected by both natural and forced convection.

Sidewall Cooling Model

A basic model is steady 2D boundary layer on heated vertical plate with Boussinesq approximation. Governing equations are continuity, x-momentum, y-momentum, and energy (Otto and Cierpka 2021):

$u/ x + v/ y = 0$ (5-42)
$u\ u/ x + v\ u/ y = {}^2u/ y^2 + g\beta(T - T_{ref})$
$u\ T/ x + v\ T/ y = \alpha\ {}^2T/ y^2$

Natural convection is controlled by Pr and Ra. Boundary layer Pr 5–7. Transition to turbulence occurs at Ra 10^9 (Bejan 2013). Scaling shows wall jet velocity depends on sidewall-water temperature difference and stratification factor $s(Y,\tau)$:

$s(Y,\tau) = -(1/\Delta T)\ T/ Y$ (5-44)

Larger temperature difference or smaller stratification factor creates stronger natural convection.

Experimental results (Otto and Cierpka 2021) show two wall jets: hot layer flowing downward, cold layer upward (Figure 5-19). Jet thickness and velocity increase with distance from top/bottom plates. Internal insulation suppresses natural convection; porous inner surfaces increase flow resistance, also suppressing vertical convection.

Figure 5-19 Flow and heat transfer schematic (left: natural convection direction; right: stratified temperature and fluid-sidewall conditions) (Otto and Cierpka 2021)

Surface Cooling Convection

Surface cooling creates cold boundary layer that becomes unstable when local Rayleigh number exceeds critical value Ra_c 658–1708 (Drazin 2002). Cold plumes form and penetrate water column as Rayleigh-Bénard convection (Figure 5-21). In shallow regions, faster cooling creates large-scale circulation.

Figure 5-21 Unsteady natural convection from surface cooling (cavity length L=0.3 m, width 0.06 m, height H=0.015 m) (Bednarz, Lei et al. 2008)

Critical time for instability onset is t_B ~ $(Gr \cdot Pr)^{-2\ 3}\ H^2/\alpha$. Ra represents ratio of diffusion timescale to buoyancy-driven convection timescale. In Rayleigh-Bénard convection, Ra measures instability—when Ra > Ra_c, convection initiates. High viscosity or diffusivity (small temperature gradients) may prevent convection by diffusing heat faster than buoyancy can organize it.

Studies on Danish pit storage (Chang, Wu et al. 2017; Dahash, Ochs et al. 2020) model boundary cooling using enhanced effective conductivity:

k_w,eff = Nu $\cdot$ k_w (5-45)
Nu = C_1 Ra^n (5-46)
Ra = $g\beta\Delta T\ z^3/(\ \alpha)$ (5-47)

where ΔT is temperature difference across convection zone, z is characteristic length. For Dronninglund (75,000 m^3), n = 0.5. Current modeling uses parameterized approaches to represent natural convection's enhancement of vertical heat diffusion.

5.3 Shear Instability and Microscale Turbulent Mixing

In inviscid ideal fluid with velocity discontinuity, perturbations create wave-like interfaces. Bernoulli's equation shows pressure differences that amplify waves into vortices. In viscous fluids, shear layers with velocity gradients become unstable to perturbations, especially when velocity profiles have inflection points. Vortices develop into turbulence.

Shear Instability Theory

Shear instability occurs when kinetic energy releasable from density-stratified fluid exceeds work needed to overturn stratification. A simplified stratified shear flow model (Chandrasekar 1961) analyzes critical conditions (Figure 5-22).

Figure 5-22 Stratified shear flow analysis model

Work to lift heavy fluid by is:

W = g [(z) - (z+)] dz (5-48)

Total work for mixing is gΔ dz. Kinetic energy difference before/after mixing is (1/4) $(\Delta U)^2$ dz. If kinetic energy exceeds mixing work, stratification becomes unstable. Local Richardson number Ri_g is:

Ri_g   (g/) (/ z) / (U/ z)^2 (5-50)

When Ri_g < 1/4, stratification cannot suppress microscale shear turbulence. Brunt-Väisälä frequency N = √[-(g/)   / z] measures vertical displacement stability. Shear rate S = U/ z. The condition becomes $N^2/S^2 < 1/4$. For reservoir operation, minimize horizontal velocity gradient and maximize vertical temperature gradient near thermocline to prevent shear instability.

Turbulent Froude number Fr_T = u'/L_T N and turbulent Reynolds number Re_T = u'L_T/ characterize turbulent flow, where u' is characteristic velocity, L_T is length scale. Energy cascade transfers energy from large to small eddies until Kolmogorov scale where viscosity dissipates it. Turbulent kinetic energy dissipation rate is:

= 2 s_{ij} s_{ij} (5-55)

Turbulence activity parameter I = /(N^2) = Re_T Fr_T^2 represents ratio of turbulent mixing to buoyancy+viscosity stabilization. High I means weak buoyancy influence, fast turbulence development. Buoyancy Reynolds number Re_B = /(N^2) has same expression.

Ozmidov scale L_O = (/N^3)$^{1\ 2}$ is largest eddy scale where buoyancy affects turbulence. Kolmogorov scale L_ = (^{3}/)$^{1\ 4}$ is smallest scale where viscosity dissipates eddies. In strongly stratified turbulence, L_O L_ . Turbulence requires large Re_B to develop, allowing viscous force neglect.

Figure 5-23 shows three mixing regimes based on (K_ + K)/K vs. I: molecular, transitional, and energetic (Table 5-4). Turbulent density diffusivity K_ is:

K_ = Γ /N^2 (5-67)

where Γ is mixing efficiency (0.2 upper limit). In stratified lakes, Γ 0.1–0.15, meaning only ~10–15% of TKE converts to potential energy; the rest dissipates to internal energy.

Table 5-4 Mixing conditions in stratified fluids (Ivey, Winters et al. 2008)

Regime	Re_T/Ri	I = /(N^2)	K_ /K
Molecular	< 7	< 0.5	0.2
Transitional	7–150	0.5–100	0.2–0.015 (Re_T/Ri)
Energetic	> 150	> 100	0.4 (Re_T/Ri)

Table 5-5 Turbulent diffusivity fitting formulas

Formula	Range	Reference
K_ = 0.2 (/ N)	I < 0.5	Barry (2002)
K_ = 5 (/ N)$^{1\ 2}$	0.5 < I < 100	Osborn (1980)
K_ = 2 (/ N)	I > 100	Shih, Koseff et al. (2005)

Density variance dissipation rate = 2 | '|^2 is equivalent to buoyancy flux B. Thus, = Γ , showing linear relationship between temperature variance dissipation and TKE dissipation. In oceans, ~90% of TKE doesn't contribute to buoyancy flux, dissipating via viscosity.

Kelvin-Helmholtz vs. Holmboe Instability

In stratified shear flow, density stratification and velocity gradient interact to produce two instability types with different vortex structures (Figure 5-26).

Figure 5-26 Mixing process from shear instability (Fernández Castro, Wüest et al. 2021)

Kelvin-Helmholtz (KH) Instability: Creates horizontally parallel, co-rotating vortices (billows) with vorticity perpendicular to mean flow (Figure 5-27a). Dominant when shear layer thickness δ_u < 2δ_ (density gradient thickness). KH billows overturn, causing strong mixing.

Holmboe Instability: Creates alternating horizontal vortices with opposite rotation above/below interface, forming "cusped waves" (Figure 5-27b). Dominant when δ_u > 2δ_ (strong stratification). Mixing is weaker than KH.

Figure 5-27 Vortex structures: (a) KH instability; (b) Holmboe instability (Strang and Fernando 2001)

DNS shows detailed structures (Figures 5-28, 5-29). KH instability causes isotherm overturning; Holmboe creates cusp-shaped isotherms with less mixing (Figures 5-30, 5-31).

Figure 5-28 Density stratification under Holmboe instability (Carpenter, Tedford et al. 2010)

Figure 5-29 KH instability development (Thorpe 2007)

Figure 5-30 Vortex structure comparison: (a) KH instability; (b) Holmboe instability

Figure 5-31 Transverse vorticity development: top (a-c) KH; bottom (d-f) Holmboe (Caulfield 2021)

Shear layer thickness relative to density gradient thickness determines instability type (Figure 5-32). When δ_u/δ_ < 2, KH dominates; when > 2, Holmboe dominates. In strong stratification (Ri_g 1/4), Holmboe appears as special instability form (Peltier and Caulfield 2003).

Figure 5-32 Relationship between shear instability characteristics and layer thicknesses (Fraunié, BERRABAA et al.)

KH instability generally causes stronger mixing due to parallel vortices. Holmboe's smaller vortices and cusp waves produce weaker mixing. Thorpe scale L_T = RMS vertical displacement of fluid parcels measures mixing intensity:

L_T = √(d_i^2) (5-74)

Ellison scale L_E = ($'^2$)$^{1\ 2}$ / ($^-$/ z) also measures mixing. Figure 5-33 shows KH turbulent density diffusivity slightly exceeds Holmboe's at same Re_b, but both are comparable.

Figure 5-33 Mixing intensity comparison between KH and Holmboe instabilities (Caulfield 2021)

5.4 Internal Wave Breaking Mixing

Internal waves occur only in density-stratified fluids. Amplitudes are tens times larger than surface waves. Lakes/reservoirs develop stable stratification from temperature gradients; inflow and wind create disturbances. Basin-scale internal waves (internal seiches) evolve into sub-basin scale waves. Stable degeneration doesn't significantly affect vertical transport. Unstable degeneration creates shear instability and enhances mixing. Nonlinear internal solitary waves propagating shoreward break on slopes.

Internal wave breaking is the process where internal gravity waves reach large amplitudes, become nonlinearly unstable, and break, accompanied by unsteady turbulent dissipation and mixing. Wave breaking relates to instability: waves can trigger fluid instability when encountering topography or other waves, creating mixing and turbulence. Common wave-related instabilities include KH (shear) and Holmboe (density gradient) instabilities.

Boundary-Induced Mixing

Boundary mixing occurs when internal waves break on slopes, destroying vertical density/temperature gradients. Shallow water layers propagate as internal bores into deeper layers. Slope topography easily triggers wave breaking. Figure 5-35 shows planar laser-induced fluorescence of internal wave breaking on boundary. Turbulence intensity evolves from laminar to turbulent and back over wave period t/T = 0.10 (Ivey, Winters et al. 2008).

Figure 5-35 Background two-layer density field for boundary internal wave breaking (Ivey, Winters et al. 2008)

Figure 5-36 shows the process: downslope flow concentrates near leading edge (a); trailing edge steepens (b); downslope and upslope flows interact creating shear and initial breaking (c); post-breaking, dense fluid surges upslope creating interfacial shear instability (c-d); after maximum run-up (f), dense fluid flows down (g-i), returning isopycnals to near-original positions.

Figure 5-36 Density distribution changes during boundary internal wave breaking (Arthur, Koseff et al. 2017)

Laboratory observations (Chen, Hsu et al. 2007) show internal wave breaking on 30° slopes changes density gradient near interface (Figure 5-37). Wunsch & Ferrari (2004) suggest boundary wave breaking explains why global ocean mixing rates are 10× higher near topography than in open ocean.

Figure 5-37 Density distribution before/after breaking (Zhu Hai, Wang Lingling et al. 2014)

Mixing efficiency during slope wave breaking is 0.25–0.37 (Arthur, Koseff et al. 2017). Key parameter is normalized pycnocline thickness kδ (k = horizontal wavenumber, δ = pycnocline thickness). As kδ increases, KH billow scale grows relative to upslope surge. Maximum mixing efficiency occurs at kδ 1 (Fr_T 0.4) (Figure 5-38).

Figure 5-38 Overall mixing efficiency vs. kδ (Arthur, Koseff et al. 2017)

Internal wave breaking mixing can also be described by turbulent diffusivity K_ and Cox number C_x = | '$|^2$ / (̄/ z$)^2$. Numerical studies (Li Bingrui 2006) show C_x 10 during breaking.

5.5 Mixing Characteristics in Natural Temperature-Stratified Water Bodies

Stratification and Mixing Causes

Many lake stratification studies focus on Loch Ness, Scotland—long and narrow (Figure 5-39), similar to potential seasonal storage reservoirs. Volume 7.4 km^3, mean depth 140 m.

Figure 5-39 Loch Ness shape schematic (Thorpe and Deacon 1977)

Vertical temperature distribution (Figure 5-40) shows: uniform 0–30 m epilimnion; steep gradient 30–35 m thermocline; weak gradient below 35 m hypolimnion (Simpson and Woods 1970).

Figure 5-40 Vertical temperature distribution in Loch Ness (Simpson and Woods 1970)

Figure 5-41 summarizes lake mixing causes. Wind-induced turbulent eddies, surface cooling, and wind-driven flow occur year-round. Other processes occur only during stratification, especially internal wave-topography interactions causing instability and mixing.

Figure 5-41 Natural temperature stratification and mixing in lakes (Macintyre and Jellison 2001)

Surface Layer

Wind is primary kinetic energy source; river inflow contributes minimally. Wind stirring deepens the thermocline. Convection and wind mixing are significant.

Thermocline

Main feature is thin laminar layers separated by weakly turbulent layers (Woods 1968). Structure and thickness vary with flow conditions (Chandra and Matuska 2019).

Deep Water Layer

Surface wind alone cannot fully mix deep lakes. In early summer when stratification is weak, wind kinetic energy creates weak deep vertical structure via momentum transfer. Once strong stratification forms, wind cannot alter deep structure. Only very shallow waters (< few meters) remain unstratified all summer.

Intermittent Localized Mixing

Deep layers also experience intermittent, localized mixing from shear-generated turbulent eddies (e.g., KH billows). Evidence of KH waves exists in Loch Ness (Thorpe and Hall 1974; Thorpe, Hall et al. 1977). KH waves are transient, repeatedly forming and dissipating in changing velocity/temperature fields.

Figure 5-42 Isotherm schematic for instability development stages (record length 25.8 m) (Thorpe and Hall 1974)

Figure 5-43 shows density stratification under different stratification strengths. At low N^2 (2.5$\times10^{-3}$ s^{-2}), rapid KH billow mixing creates transient intermediate layer with uniform temperature, distinct from high-gradient thermocline. KH billow breakdown enhances mixing, creating thicker, weaker-gradient thermocline where new KH billows can form (Strang and Fernando 2001). At higher N^2 (5.2$\times10^{-3}$ s^{-2}), stratification suppresses KH, and Holmboe instability becomes more important.

Figure 5-43 Density stratification for different N^2 values (solid: 2.5; dashed: 5.2) (Strang and Fernando 2001)

When hypolimnion velocity is high, bottom boundary layer turbulence must be considered, especially for thin deep layers (Gorham and Boyce 1989).

Mixing Intensity Parameterization

Direct measurement of mixing intensity parameters like eddy diffusivity is impossible, so oceanographic studies rely on dissipation rate measurements (Ivey, Winters et al. 2008). However, ~90% of TKE doesn't contribute to buoyancy flux, dissipating via viscosity.

Key parameter describing mixing intensity is vertical diffusivity K_z (m^2/s). Its relationship with stratification parameters is fundamental. Characteristic parameters fall into three groups: (1) fluid properties (, α); (2) stratification state (N, S); (3) turbulence properties (TKE,). Figure 5-24 shows possible Γ = K_ / vs. I = /(N^2) relationships. No consensus exists on Γ variation in strong stratification (Caulfield 2021). Flux may approach upper limit in strong stratification (blue curve), or decrease continuously as stratification strengthens, representing mixing suppression. Alternatively, mixing may cease at critical Ri_g ($\Gamma = 0$).

Figure 5-24 Mixing efficiency as function of Ri_g and Re_b (Caulfield 2021)

Measured Diffusivity Ranges

For Castle Lake (California, 0.2 km^2, 35 m deep), hypolimnion diffusivity is ~10^{-2} cm^2/s, 14× molecular diffusivity, indicating weak turbulence even below thermocline. For large lakes, diffusivity ranges from 8.64 m^2/day (large lakes, Lewis 1983) to 0.086 m^2/day (small lakes, Hondzo and Stefan 1993)—700× to 7× molecular diffusivity, respectively (Figure 5-44). Stratification and lake geometry are decisive factors.

Figure 5-44 Relationship between maximum hypolimnion turbulent diffusivity and lake surface area (Hondzo and Stefan 1993)

Table 5-6 Typical mixing, stability, and diffusivity ranges for stratified water bodies (Fernández Castro, Wüest et al. 2021)

Region	(W/kg)	N^2 (s^{-2})	K_ /K_{molecular}
Ocean thermocline	10^{-10}–10^{-8}	~10^{-4}	10^{-6}–10^{-9}
Lake surface layer	10^{-8}–10^{-6}	~10^{-3}	10^{-6}–10^{-4}
Lake interior (no BBL)	10^{-10}–10^{-8}	10^{-8}–10^{-3}	10^{-6}–10^{-4}
Metalimnion (basin scale)	10^{-10}–10^{-8}	~10^{-3}	10^{-6}–10^{-4}
Near-shore metalimnion	10^{-12}–10^{-10}	10^{-8}–10^{-6}	10^{-6}–10^{-4}
Deep hypolimnion	10^{-12}–10^{-10}	10^{-8}–10^{-6}	10^{-6}–10^{-4}

Temperature variance dissipation rate _T ranges 10^{-11}–10^{-6} K^2/s; diffusivity K_T ranges 10^{-7}–10^{-4} m^2/s; temperature gradient T/ z ranges 10^{-3}–10^{-1} K/m—much smaller than storage reservoir thermocline gradients (~10 K/m).

Key Findings for Storage Reservoirs

Mixing in deep water layers is primarily caused by shear instability eddies. When gradient Richardson number Ri_g (based on buoyancy frequency) is low, KH instability dominates. As Ri_g increases, KH weakens; when Ri_g > 0.9, Holmboe instability becomes important. At Ri_g > 1.5, KH is suppressed and Holmboe dominates. In natural water bodies, multiple instabilities and processes may coexist.

Mixing depth is influenced by geometry (surface area, maximum depth) and aspect ratio. Large and medium lakes experience strong dissipation and mixing in surface and bottom layers (Alfred Wüest 2003). Wave-induced complexities are more pronounced in lakes and reservoirs than open oceans (Alfred Wüest 2003). Thermocline depth correlates with surface area: for lakes < 25 km^2, thermocline depth h A^0.43 (Gorham and Boyce 1989).

However, artificial flow distribution design may dominate mixing in storage reservoirs. Natural stratification studies haven't addressed such large density gradients, and Γ variation in strong stratification remains unresolved (Caulfield 2021). Therefore, mixing insights from natural water bodies cannot be directly applied to storage reservoirs without further research.

5.6 Similarities and Differences in Mixing Between Heat Storage Reservoirs, Lakes, and Storage Tanks

The water volume of seasonal heat storage reservoirs is comparable to medium-sized lakes, but the vertical temperature difference (70 K) far exceeds what occurs in natural water bodies (15 K). Conversely, while the temperature difference in storage reservoirs is similar to that in storage tanks, their water volume is 3–6 orders of magnitude larger. Consequently, mixing phenomena exhibit both similarities and significant differences.

Stratified flows can be categorized into two types: one where fluids of different densities have a distinct internal interface, and another where they intermix to form a continuous density field.

Characteristics of Stratification and Mixing in Storage Tanks

Overall, water tank heat storage features small volume and short storage cycles (non-seasonal), resulting in a relatively large flow rate-to-volume ratio. Common inlet/outlet designs for water tank thermal storage include:

(1) **Orifice Jet Design**: This design reduces non-uniform axial flow distribution, and turbulence-induced mixing intensity decays with distance from the orifice plate. As shown in [FIGURE:5-46], distance L refers to the distance from the orifice plate to where the jet fully merges and the average velocity becomes completely uniform. In the range of L/2 to 3L, a fully mixed region forms and creates a thermocline, known as the jet mixing region. Overall, while the orifice plate induces jet mixing, the horizontal velocity gradient is completely eliminated after sufficient mixing, making shear instability less likely to occur.

[FIGURE:5-46] Schematic of orifice jet in storage tank (Villermaux and Hopfinger 1994)

The axial temperature distribution is expressed by equation (5-80), and the turbulence intensity (amplification factor of thermal diffusivity) varying with distance from the outlet can be expressed as equation (5-81) (Pilotelli, Grassi et al. 2022).

$$T(x) = T_0 + \Delta T \cdot f\left(\frac{x}{L}\right) \quad \text{(5-80)} \tag{1}$$

$$\frac{\alpha_{eff}}{\alpha_w} = g\left(\frac{x}{L}\right) \quad \text{(5-81)} \tag{2}$$

(2) **Disk Diffuser Design**: This design aims to reduce mixing between fluids of different temperatures or densities. The primary function of the disk diffuser is to guide cold and hot fluids to stratify at the inlet, minimizing their direct contact and mixing. The disk diffuser design creates inlet mixing effects based on an engulfment mechanism (van Berkel, Rindt et al. 2002).

[FIGURE:5-47] Disk diffuser design for Danish pit heat storage (Gram)

Some storage reservoir simulation studies have used commercial CFD software to model disk diffuser performance, but without in-depth analysis of specific mixing mechanisms. For example, Chang, Wu et al. (2017) used a 2D model to simulate 40-minute flow conditions but did not explain how water cooled by heat loss at boundaries mixes with stratified water. Jianhua Fan (2017) used a 3D model to simulate a 75,000 m^3 storage pit with a 1-day calculation time; the study identified flow characteristics near the diffuser but did not focus on thermocline development throughout the entire storage cycle. Simulation results showed that cold water beneath the diffuser would be entrained upward to mix with outflowing hot water, as shown in [FIGURE:5-48].

[FIGURE:5-48] Flow field simulation around disk diffuser in Danish pit storage (Jianhua Fan 2017)

(3) **Octagonal and Annular Diffusers**: As shown in [FIGURE:5-49], some large chilled water storage tanks employ octagonal diffusers to achieve uniform distribution of inflow/outflow across the tank cross-section. Such diffusers also cause jet mixing in nearby regions.

[FIGURE:5-49] Common diffuser designs for chilled water storage tanks (ASHRAE 2012)

Zurigat, Liche et al. (1991) experimentally analyzed mixing effects caused by different diffuser configurations in small storage tanks and presented axial water temperature and thermal diffusivity amplification factor (α_{eff}/α_w) conditions. [FIGURE:5-50] shows the variation of α_{eff}/α_w with dimensionless height H* when using an impingement jet for distribution; α_{eff}/α_w is calculated from empirical formulas listed in equation (5-82).

[FIGURE:5-50] Diffusion coefficient calculation results from Zurigat model (Zurigat, Liche et al. 1991)

$$\frac{\alpha_{eff}}{\alpha_w} = C_1 \cdot H^{*n_1} + C_2 \cdot H^{*n_2} \quad \text{(5-82)} \tag{3}$$

Characteristics of Artificial Temperature-Stratified Water Bodies

Due to fewer than 10 large-scale seasonal water heat storage projects exceeding 10,000 m^3 worldwide (Xiang, Xie et al. 2022), available data resources are extremely limited. Currently, only the following internal temperature stratification measurement data for seasonal heat storage water bodies are available in literature:

(1) The Institute of Electrical Engineering, Chinese Academy of Sciences, constructed a 3,000 m^3 water storage body in Huangdicheng, Zhangjiakou (Bai, Wang et al. 2020, Li Xiaoxia 2021). Using temperature sensors spaced 0.8 m vertically, they studied temperature distribution in a 5 m water column. Results showed a linear vertical temperature distribution

without forming a distinct thermocline separating cold and hot water sections, as shown in [FIGURE:5-51(a)].

(2) The Dronninglund storage reservoir in Denmark has a volume of approximately 60,000 m^3. Internal temperature measurements showed that a relatively clear thermocline only formed during the early heat extraction period (October–December) (Schmidt and Sørensen 2018), with a thickness of 2–3 meters, as shown in [FIGURE:5-51(b)]. Theoretical calculations revealed this thermocline thickness far exceeds what would result from thermal conduction alone. These two pioneering experimental studies demonstrated the feasibility of water heat storage and revealed actual internal temperature distributions, but did not conduct in-depth mechanism discussions on the cold-hot mixing phenomena reflected in the measurements.

[FIGURE:5-51] Temperature distribution in actual water storage projects

Cooling ponds are artificial processes with larger vertical temperature differences than lakes. They can be divided into deep and shallow cooling ponds. Cooling ponds also exhibit severe surface mixing while bottom turbulence mixing is negligible (Stolzenbach and Harleman 1971). There exists a dimensioning issue for water distribution.

[FIGURE:5-52] Schematic of flow phenomena at cooling pond inflow (Harleman and Stolzenbach 1972)

In summary, the flow and heat transfer processes inside heat storage reservoirs are complex and coupled. Based on a clear understanding of fundamental flow phenomena principles, it is necessary to construct a reasonably simplified CFD model that can accurately predict dynamic temperature distribution changes within the water body. However, comprehensive numerical models for storage reservoirs that consider all mixing mechanisms are still lacking. Therefore, we must start from theories on internal flow conditions and thermocline formation mechanisms in naturally stratified water bodies comparable in scale to storage systems, infer main factors affecting temperature stratification and flow conditions in ultra-large storage reservoirs, and reconstruct physical models for heat storage and extraction. Based on physical phenomena of cold-hot mixing, reasonable simplifications should be adopted to establish numerical models that comprehensively reflect internal mixing conditions.

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6. Discussion and Conclusion

Current research lacks a reasonable evaluation system for temperature quality loss in seasonal heat storage technology, and studies on flow and heat transfer in ultra-large-scale temperature-stratified storage reservoirs remain insufficient. These issues constrain the engineering application development of storage reservoirs in China. This report starts from existing thermal performance and system design of seasonal heat storage devices, identifies the common problem of excessive temperature quality loss during heat storage processes in current projects, and emphasizes the unreasonable aspects of using exergy parameters to evaluate temperature quality loss in existing literature. Accordingly, this report constructs an irreversibility evaluation system for heat storage processes based on huoji (火积) and huoji dissipation parameters, pointing out that using huoji dissipation to analyze heat transfer irreversibility in storage processes offers advantages including clear physical meaning, concise form, and ease of guiding practical engineering.

Furthermore, based on physical characteristics of heat storage processes as non-equilibrium states with continuous media and coupled momentum and heat transfer phenomena, this report proposes a differential equation analysis method for describing huoji parameter transfer and dissipation phenomena, and defines a general expression for huoji efficiency in continuous medium heat transfer problems based on huoji balance principles in storage media.

Using this evaluation system, this report proposes a huoji dissipation analysis method that distinguishes ideal from actual processes, and presents a new classification of heat storage technologies based on heat storage principles and huoji dissipation characteristics: fluid displacement heat storage and heat exchanger heat storage. The report focuses on comparing huoji dissipation levels between water storage and borehole storage, concluding that water storage reservoirs are more suitable for long-cycle heat storage because their displacement principle is inherently reversible, while cold-hot mixing is merely a "side effect" of the displacement process. Therefore, water storage reservoirs are superior to borehole storage in terms of huoji dissipation characteristics.

This further clarifies the importance of analyzing flow-heat transfer phenomena and mixing processes in storage reservoirs. The report describes three fundamental cold-hot mixing mechanisms—macroscopic flow entrainment, shear instability, and internal wave breaking mixing—along with their physical phenomena and basic mathematical models, and enumerates cold-hot mixing conditions in temperature-stratified water bodies including storage tanks, lakes, and cooling ponds, achieving preliminary understanding of mixing phenomena and physical models that may occur in storage reservoirs.

Based on this research, the author proposes four key questions regarding huoji dissipation in storage reservoir operation processes to guide future research directions:

(1) **How to eliminate theoretical misunderstandings about the physical meaning of huoji and huoji dissipation in the academic community?**

Whenever attempting to use huoji dissipation to describe "temperature quality loss" in heat storage and other heat transfer processes, it faces skepticism: "Entropy generation or exergy destruction should be used as they conform to classical thermodynamic definitions; huoji is an immature physical concept that should be used cautiously." Through derivations and discussions in Chapter 2, this report proves that under conditions of "constant total internal energy but changing internal temperature distribution," huoji dissipation, entropy generation, and maximum work capability loss can be interconverted through average temperature as a coefficient—precisely corresponding to conditions where temperature quality loss occurs due to mixing in stratified water bodies. Section 2.3 also proves that when entropy generation uses the reciprocal of thermodynamic temperature as a scale, it presents a huoji dissipation form in the traditional thermodynamic temperature system. Therefore, is it possible that the physical meanings of entropy generation and huoji dissipation are fundamentally unified in a generalized sense when describing heat transfer problems? This report provides mathematical connections between the two, but more rigorous demonstration and extensive promotion are needed to achieve full academic recognition and acceptance of huoji dissipation's physical meaning.

(2) **What is the essence of huoji dissipation caused by turbulent mixing phenomena?**

The various cold-hot mixing physical phenomena presented in this report—whether macroscopic, bulk fluid motion causing entire portions of cold and hot water to converge and intertwine in a region, or micro-level mixing of fluid parcels with different temperatures due to eddies—are currently collectively termed "cold-hot mixing" processes because they all result in mixing and formation of water at intermediate temperatures. From an overall heat storage perspective, these processes indeed cause huoji dissipation because temperature distribution changes irreversibly. However, from analyses in Sections 2.2 and 5.1, we have distinguished between reversible temperature distribution changes and irreversible mixing processes. So among phenomena generally called "cold-hot mixing," which are irreversible processes directly causing huoji dissipation, and which indirectly promote increased huoji dissipation?

First, heat conduction in storage media results from molecular thermal motion. Molecular thermal motion causes collisions and interactions between molecules. Molecules in high-temperature regions have higher energy and velocity, transferring energy to surrounding molecules through collisions, causing energy to propagate gradually through the material. Energy flow in heat conduction is driven by temperature gradients. Larger temperature gradients result in faster energy transfer rates because molecular thermal motion in high-temperature regions transfers energy more vigorously to low-temperature regions. Molecular thermal motion causes continuous random vibration, rotation, and translation, driving materials toward more disordered states—an irreversible trend. Huoji theory for solid storage media and stationary or laminar-flow fluid media also indicates that heat conduction (molecular thermal diffusion) is the essential cause of huoji dissipation.

When turbulence exists in fluid storage media, although the actual molecular thermal diffusivity does not change, turbulence's effect on heat transfer can be viewed as an effective enhanced diffusion process. Turbulence breaks up and transports fluid parcels with different temperatures (or concentrations) to adjacent regions, effectively increasing the diffusion surface area in the fluid. This effect results from irregular, random turbulent fluid motion rather than increased thermal diffusivity, manifesting as enhanced heat transfer. Typical turbulent phenomena appear in buoyant jets and shear-unstable stratified interfaces, spanning the entire storage body. While this effect resembles increased thermal diffusivity, it actually achieves mixing through turbulence. In turbulent eddy viscosity models, we consider turbulence as a form of Brownian motion to some extent, thus using turbulent viscosity or turbulent thermal diffusivity to equivalently describe turbulence's amplification effect on momentum or heat diffusion rates. However, this describes the effect of expanded molecular thermal motion interface area and reduced diffusion distance after turbulence—the truly irreversible process remains molecular thermal motion.

When scaling up further, we focus on stirring or convective motions that transport fluid portions from one region to another, such as thermally driven flow and inertial flow from diffusers. Assuming fluid medium thermal diffusivity (thermal conductivity) is zero, the displacement process itself would not cause any internal state changes and could be considered reversible. However, stirring processes stretch and distort entire regions, increasing temperature gradients and contact areas where molecular thermal motion occurs. Therefore, in fluids with thermal diffusivity, stirring processes promote molecular thermal motion, achieving "cold-hot mixing" and enhancing overall huoji dissipation effects. The nature of macroscopic stirring/convection processes in promoting huoji dissipation is the same as micro-scale turbulence, but because the latter's characteristic length is several orders of magnitude smaller, it is generally represented by equivalent diffusion coefficients in most models. Thus, we need to understand that "cold-hot mixing" phenomena cause macroscopic huoji dissipation in storage water bodies, but essentially, they result from amplified reversible thermal motion speeds of storage medium molecules by flow phenomena at different characteristic lengths.

(3) **How to predict mixing intensity under actual engineering conditions for storage reservoirs?**

This report comprehensively summarizes mixing phenomena in naturally stratified lakes, oceans, and small thermocline storage tanks. However, we must clearly recognize essential differences in boundary conditions between these stratified water bodies and ultra-large storage reservoirs: (1) Temperature differences between cold and hot water in natural stratified water bodies are only about 10 K, while storage reservoirs can have differences up to 70 K; (2) Lakes and oceans have objective conditions such as surface wind momentum input and complex edge morphology that easily cause disturbance and mixing, whereas storage reservoirs are constructed with insulation covers and smooth water structures to reduce disturbance and mixing possibilities. Currently, academic consensus on turbulence mixing formation mechanisms and intensity under large temperature difference and strong stratification conditions remains controversial, making many existing research conclusions inapplicable for predicting actual mixing in ultra-large storage reservoirs.

Therefore, we must construct fundamental turbulence mixing models from fluid mechanics mechanisms: identify dimensionless numbers involving fluid stratification characteristics and turbulence mixing intensity, clarify interactions between buoyancy (gravity), inertial forces, and viscous forces in mixing formation mechanisms such as stratification instability and internal wave breaking, and elucidate turbulence mixing causation mechanisms under engineering conditions. We must also establish turbulence mixing intensity expressions: define flow-heat transfer boundary conditions in ultra-large seasonal water storage projects, including cold-hot temperature differences, horizontal velocity distributions, shear layer thickness and shear rates, and investigate mixing intensity under actual storage conditions through numerical simulation or similar flow experiments. Finally, establish overall flow mixing intensity: summarize inlet/outlet design schemes applicable to high-flow conditions and corresponding diffuser characteristics, clarify similarity laws for overall water movement in binary temperature-density currents, and summarize parameters and empirical formulas describing drainage jet entrainment effects and intake withdrawal effects in temperature-density currents, along with dimensionless number expressions for diffuser design parameters and operating parameters such as cold-hot temperature differences and flow velocities (flow rates). Overall, more fundamental research on mixing phenomena is needed.

(4) **How to construct dynamic huoji dissipation calculation methods coupled with temperature field numerical solutions for engineering design problems, and flow design principles with minimized total mixing huoji dissipation as the objective function?**

Under the premise of clarifying huoji dissipation models for various cold-hot mixing processes, huoji dissipation analysis can be performed on the entire storage water body to investigate mixing activity characteristics and corresponding huoji dissipation magnitudes at different storage stages and flow path regions, and analyze the main contradictions causing huoji dissipation from an overall perspective. This requires innovatively establishing a domain-wide huoji dissipation calculation method coupled with temperature field simulation, comprehensively reflecting the influence of various mixing phenomena and corresponding boundary conditions on flow and temperature fields in mathematical models based on convection-diffusion equations, while also dynamically extracting time-dependent flow and temperature field information to calculate huoji dissipation caused by three mixing modes at each instant. Existing literature has established optimization analysis theory for huoji dissipation minimization in transient heat transfer problems, but the involved heat transfer mechanisms only include one-dimensional heat conduction, which is not applicable to water body temperature stratification flow conditions. Therefore, it is necessary to establish a domain-wide huoji dissipation calculation method coupled with temperature field simulation for optimizing water storage design, and construct an optimization design process for storage reservoirs targeting huoji dissipation minimization as the objective function.

Acknowledgments

I sincerely thank my collaborative advisor, Professor Jiang Yi, for providing me the opportunity to conduct postdoctoral work and study at Tsinghua University. Professor Jiang's great dedication to China's building energy conservation cause and meticulous scientific research spirit deeply inspire me. You have pointed out the direction for my future research: seasonal heat storage technology for low-carbon heating; you have also guided and motivated me to explore the essence of heat storage processes, helping me break through my thinking limitations and refine my scientific understanding of building heating and cooling technologies. Over the past two years, I feel extremely honored and grateful to have received your personal teaching, care, and support!

I sincerely thank Associate Professor Xie Xiaoyun for her careful guidance and concern! It is your support and the help from junior colleagues in the group that enabled me to step-by-step conduct research on seasonal heat storage and evaporative cooling technologies. In your research group, I have learned much new knowledge and technology, and have benefited immensely over the past two years.

I also thank Dr. Hu Shan for her guidance and encouragement! Without Dr. Hu's help, I could not have completed the work "Comparison of Building Carbon Emissions Between China and Sweden and Its Implications for China's Building Carbon Neutrality Path," which makes me very proud.

Thanks to my family and friends for their unconditional support! This postdoctoral experience at Tsinghua University will benefit me for life and has clarified my direction and purpose moving forward. May I remain true to my original aspiration and forge ahead.

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Note: Figure translations are in progress. See original paper for figures.