Preliminary Investigation of Geometric Distortion Variations in the 2.4 m Telescope at Yunnan Observatories: Postprint

Authors: Chen Xiao, Guo Bifeng, Zheng Zhongjie, Peng Qingyu, Guo Bifeng

Date: 2023-10-20T00:00:00+00:00

Abstract

Geometric Distortion (GD) is a non-negligible factor in high-precision measurements with Charge-Coupled Devices (CCD), which typically requires meticulous observational efforts to derive, inevitably occupying valuable observation time.

By utilizing CCD images of dense star fields observed with the 2.4m telescope at the Lijiang Observatory, preliminary discovery was made of variation patterns in image geometric distortion with Zenith Distance (ZD). Specifically, this paper references the positional information provided by the Gaia DR3 catalog and performs data reduction, corrects for Differential Color Refraction (DCR) effects, solves for the geometric distortion model of the CCD at different observation times, and uses fourth-order polynomial fitting. The results indicate that there exists a certain correlation between the geometric distortion of CCD images and the zenith distance at the time of observation. In the pixel x-direction, the fourth-order terms x^4, x^{3y}, x$^{\{\{\{2y\}\}}${2,}} and xy^3, as well as the second-order terms x^2 and xy of the geometric distortion model exhibit a relatively obvious linear relationship with zenith distance; in the y-direction, the coefficient terms that have a linear relationship with zenith distance are the fourth-order terms x^4 and xy^3, the third-order term xy^2, and the second-order term xy.

Full Text

Preliminary Exploration of Geometric Distortion Variation in the 2.4 m Telescope at Yunnan Observatories

Guo Bifeng[1], **Zheng Zhongjie**[23], **Peng Qingyu**[1]
[1]College of Information Science and Technology, Jinan University, Guangzhou 510632, China
[2]Sino-French Joint Laboratory for Astrometry, Dynamics and Space Science, Jinan University, Guangzhou 510632, China

[3]School of Mathematics and Computer Science, Guangdong Ocean University, Zhanjiang 524088, China

Abstract: Geometric distortion is a non-negligible factor for high-precision CCD astrometric measurements, typically requiring careful observational work to derive, which inevitably consumes valuable observing time. This paper explores the variation pattern of geometric distortion with zenith distance using CCD images observed with the 2.4 m telescope at the Lijiang station of Yunnan Observatories. Specifically, we reference the Gaia DR3 catalog for data reduction and correct for differential color refraction (DCR) effects. Geometric distortion models at different observation times are derived and fitted with fourth-order polynomials. The results show that the geometric distortion models of CCD frames are correlated with the zenith distance (ZD) of observation. In the pixel x-direction, there are obvious linear relationships between zenith distance and the quartic terms (x^4, x^3y, x^2y^2, xy^3) and quadratic terms (x^2, xy) of the geometric distortion model. In the y-direction, the coefficient terms that show linear relationships with zenith distance are the quartic terms (x^4, xy^3), cubic term (xy^2), and quadratic term (xy).

Keywords: geometric distortion; astrometry; image processing; astrometric calibration

1. Introduction

Geometric distortion in telescope optical imaging systems causes target star images to deviate from their original pixel coordinate positions in observed images. This is a ubiquitous phenomenon that produces non-negligible effects on high-precision astrometric measurements. The phenomenon exists not only in space telescopes—for example, the Hubble Space Telescope (HST) exhibits severe geometric distortion in its Wide Field and Planetary Camera 2 (WFPC2), with maximum distortion reaching approximately 5 pixels—but also in ground-based telescopes. To accurately solve for HST' s geometric distortion effects and improve the precision of the Wide Field Camera (WF) to 0.01 pixel, Anderson & King (2003) proposed a method to correct geometric distortion by improving linear term precision. Anderson et al. (2006) applied HST' s geometric distortion solution method to the 2.2 m telescope at the European Southern Observatory, achieving a position measurement accuracy of 7 mas and improving ground-based telescope observation accuracy and precision to 0.02 pixel. Peng et al. (2012) proposed a geometric distortion solution method based on reference catalog position measurement information, which was successfully applied to Phoebe' s position measurement. After correcting geometric distortion, the positions of near-Earth asteroid Apophis and Triton were measured. With the release of the latest Gaia DR2 catalog, this method was applied to multiple studies. Lin (2021) systematically compared the two methods from aspects of accuracy and precision. With the release of Gaia DR3, the application of these methods will become more widespread, enabling more precise measurement of celestial positions.

Both methods have strict requirements for observational data when solving geometric distortion: the data must adopt observation modes with different orientations or pointings, or uniform dithering observations. This work typically requires sufficient observation time and certain observational experience. Telescope observation opportunities are precious, and observation time is limited. To make full use of limited and valuable observation time, this paper investigates the variation pattern of geometric distortion in images from the 2.4 m telescope at Yunnan Observatories' Lijiang station from the perspective of zenith distance.

2. Observations and Data

2.1 Telescope and CCD Parameters

The observations used the 2.4 m telescope at Yunnan Observatories' Lijiang station. The detailed parameters of the telescope and CCD are shown in Table 1.

Table 1 Specifications of the telescope and the CCD chip

2.2 Observation Overview

We selected observations of the open cluster NGC 1664 and near-Earth asteroid Apophis on February 4-6, 2013, to explore the variation pattern of the telescope' s geometric distortion models. The original purpose of the NGC 1664 dense star field dithering observations was field calibration, while the Apophis observations were for high-precision position measurement. Each frame contains approximately 273 stars. The observational data are summarized in Table 2.

Table 2 Observations overview

3. Data Reduction and Analysis

3.1 Astrometric Reduction Steps

The following steps were performed for astrometric reduction:

1. Flat-field and bias corrections were applied to the images. The edge regions of the images were cropped, resulting in a pixel size of 1900 × 1900 for the cropped images.
2. The pixel coordinates of each star image were measured using a two-dimensional Gaussian centering algorithm. These pixel coordinates were then matched with position information provided by the Gaia DR3 catalog. During catalog matching, to avoid position errors caused by binary systems, stars with Renormalised Unit Weight Error (RUWE) > 1.4 were excluded.

3. The matched stars were transformed to topocentric apparent positions at the observation epoch, considering astrometric effects such as atmospheric refraction. Standard coordinates were obtained for each star image through central projection formulas.
4. A weighted four-constant plate model was used to solve for the plate constants.
5. Observed positions were calculated for each star image based on the solved plate constant model.
6. Residuals (Observed Minus Computed, O-C) were calculated for each star.

3.2 Differential Color Refraction Correction

Since the refractive index of light in air is wavelength-dependent, light of different wavelengths experiences different refraction in the zenith direction. This causes light from different stars to be refracted into different spectra when passing through Earth’ s atmosphere, a phenomenon called differential color refraction. This can cause systematic errors in target position measurements along the zenith direction.

All observations used a B filter, which is significantly affected by differential color refraction. We first eliminated the effect of differential color refraction on each star image using the correction method proposed by Zheng et al. (2022). Based on the astrometric and photometric data in the Gaia DR3 catalog, each star image was transformed to apparent positions in the horizontal coordinate system (including atmospheric refraction). The altitude and azimuth of each measured star were obtained. A weighted fourth-order polynomial plate model was used to calculate residuals for each star. A first-order polynomial was used to fit the relationship between color index (BP-RP) and residuals in the altitude direction:

$$\text{Residual} = \text{a} + \text{b} \times \text{color}$$

where a and b are fitting parameters solved through weighted least squares. The differential color refraction correction coefficients a and b were obtained, and the pixel coordinates of each star image were corrected. Figure 1 shows the residuals in the zenith direction versus color index for observation set 3 before and after differential color refraction correction. After correction, the systematic errors were significantly improved.

[Figure 1: see original paper] Figure 1 Residuals change with the color index BP-RP for observation set 3. The blue solid line represents the fitted result of Equation (1). (a) Before differential color refraction correction; (b) After differential color refraction correction.

3.3 Geometric Distortion Solution Method

Regarding the method for solving geometric distortion, we use a self-calibration approach to directly construct the pixel coordinates of images into a self-consistent, distortion-free reference frame.

The solution steps are as follows:

1. For each set of observational data, after correcting for differential color refraction effects using the method described in Section 3.2, a weighted four-constant plate model is used for reduction (see Section 3.1). Residuals for each star image in the image are calculated.
2. For stars appearing in N different images, let i and j represent two different images. The mathematical relationship between the geometric distortion at the pixel position of the star in the ith and jth images is:

dx_i = e_i cos ϕ_i / _i - e_j cos ϕ_j / _j
dy_i = e_i sin ϕ_i / _i - e_j sin ϕ_j / _j

where e_i and e_j represent the geometric distortion at the pixel position of the star image in the ith and jth images, ϕ and are the orientation and scale, respectively.

3. The geometric distortion values at the pixel positions of all stars are calculated. The image is divided into 19 × 19 square regions. The geometric distortion values of all stars whose pixel coordinates fall in the same region are averaged, and this average is taken as the geometric distortion at the central pixel coordinate position of the region. Bilinear interpolation is then used to correct the pixel coordinates of all stars in each image.
4. The above solution process is repeated iteratively to obtain geometric distortion increments until the increments reach a predetermined precision.
5. The final geometric distortion model is obtained by accumulating the geometric distortion from each iteration.

4. Results

4.1 Geometric Distortion Models

We solved for geometric distortion models for 10 sets of data (observation sets 1, 3, and 8). Figure 2 shows typical geometric distortion vector diagrams for each night, with distortion values exaggerated by a factor of 200.

[Figure 2: see original paper] Figure 2 The vector graphs of geometric distortion models. At the top of each panel, the observational date, maximum distortion, and median distortion are listed in units of pixels.

We used fourth-order polynomials to represent the geometric distortion models:

$dx = \Sigma a_{ij} x^i y^j$
$dy = \Sigma b_{ij} x^i y^j$

where x and y are normalized positions in the range [-1, 1], making it easier to see the effect of each term on distortion values.

Tables 3 and 4 show the fourth-order, third-order, and second-order polynomial coefficients of the geometric distortion models in the x- and y-directions for observations at different zenith distances.

Table 3 Fourth-order polynomial coefficients of geometric distortion models in the x-direction

Table 4 Fourth-order polynomial coefficients of geometric distortion models in the y-direction

4.2 Correlation with Zenith Distance

We initially found that the cubic term coefficients were relatively stable, while some coefficients of the quartic and quadratic terms showed certain patterns with zenith distance variation. Figures 3 and 4 show the variation of geometric distortion model coefficients with zenith distance in the x- and y-directions, respectively, with blue solid lines representing linear fitting results.

[Figure 3: see original paper] Figure 3 In the x-direction, the coefficients of geometric distortion models change with zenith distance. Blue lines are the results obtained from linear fitting.

[Figure 4: see original paper] Figure 4 In the y-direction, the coefficients of geometric distortion models change with zenith distance. Blue lines are the results obtained from linear fitting.

We analyzed the correlation strength between each coefficient and zenith distance using Pearson correlation analysis. The Pearson correlation coefficient r is calculated as:

$r = \Sigma(X_i - \bar{X})(Y_i - \bar{Y}) / \sqrt{}[\Sigma(X_i - \bar{X})^2 \Sigma(Y_i - \bar{Y})^2]$

where X_i and Y_i are the observed values of the two variables, $\bar{X}$ and $\bar{Y}$ are their means, and n is the sample size. The standard deviations σ_X and σ_Y are the sample standard deviations. We consider correlation significant when the absolute value of the correlation coefficient is greater than 0.5, indicating a medium or stronger correlation.

In the x-direction, the significantly correlated coefficient terms are the quartic terms x^4, x^3y, x^2y^2, and xy^3, and the quadratic terms x^2 and xy. In the y-direction, the significantly correlated terms are the quartic terms x^4 and xy^3, the cubic term xy^2, and the quadratic term xy.

Tables 5 and 6 list the Pearson correlation coefficients and standard deviations of the linear fitting results, respectively.

Table 5 The Pearson’ s correlation coefficient of the linear fitting results

Table 6 The standard deviation of the linear fitting results

5. Discussion

Geometric distortion phenomena are related to telescope gravity deformation and atmospheric refraction errors. From a systematic error perspective:

Ground-based telescopes are affected by gravity deformation. The telescope support structure and optical components bend or deform under gravity, causing errors in the optical system. This deformation from the telescope structure typically varies with the elevation angle of the support structure. Optical component deformation also contributes to instability in ground-based telescope distortion solutions to some extent.

Ground-based telescopes are also affected by atmospheric refraction during observations. This effect becomes increasingly significant with larger zenith distances, causing the observed target altitude to be higher than the actual altitude. Although we corrected for atmospheric refraction models based on the average temperature and pressure during observations, these corrections may not be accurate, as factors such as actual temperature, pressure, and target spectral type all affect atmospheric refraction error correction.

The constant term of the geometric distortion model represents the offset of the telescope coordinate axes, while the linear terms represent the scale and orientation of the telescope. These deviations cause higher-order terms in the geometric distortion model. Typically, telescope bending varies with observation time and position. The orientation term of the model aligns the measurement coordinate system with the standard coordinate system. Actual scale changes are usually caused by thermal variations in the telescope and are related to calibration of the plate measuring instrument and inaccurate modeling of higher-order terms.

Quadratic terms correct for plate tilt, while cubic terms mainly correct for optical field-angle distortion (OFAD). Optical field-angle distortion is typically expressed as a cubic form related to coordinates, with its differential being a quadratic polynomial. Errors generated during the solution of optical field-angle distortion coefficients introduce quadratic and cubic terms during conversion to tangent plane coordinates. Since optical field-angle distortion is related to the angle of light relative to the optical axis, we hypothesize that for celestial objects at different zenith distances, the refraction angle of their light passing through the telescope optical system differs, causing some quadratic and cubic term coefficients to also correlate with zenith distance.

Quartic terms can correct for systematic offsets caused by incomplete atmospheric refraction correction. Objects at small zenith distances are less affected by atmospheric refraction, while those at large zenith distances are more affected. In the x-direction, the influence of coefficients for quartic terms x^4, x^2y^2, and xy^3 on distortion gradually increases. In the y-direction, the coefficients for quartic terms x^4 and xy^3 show similar behavior.

The higher-order terms of the geometric distortion model exhibit central symmetry in their coefficients between the x- and y-directions. In the x-direction, the coefficients of quartic terms x^3y and xy^3 and cubic terms x^3 and xy^2 have greater influence on distortion. In the y-direction, the coefficients of quartic terms x^2y^2 and y^4 and cubic terms x^2y and y^3 have greater influence. Our preliminary findings show that larger zenith distances have greater impact on celestial position measurements.

6. Summary and Outlook

This paper investigated the variation pattern of geometric distortion models for the 2.4 m telescope at Lijiang station using dithered observations of dense star fields on February 4-6, 2013. We derived geometric distortion models for each set of data and fitted them with fourth-order polynomials to examine the relationship between image geometric distortion and pixel coordinates.

We preliminarily found that some coefficients of the quartic and quadratic terms of the geometric distortion model correlate with zenith distance. In the x-direction, quartic terms x^4, x^2y^2, and xy^3 and quadratic terms x^2 and xy show obvious linear relationships with zenith distance. In the y-direction, quartic terms x^4 and xy^3, cubic term xy^2, and quadratic term xy show obvious linear relationships with zenith distance.

We hope this pattern can save time for future calibration field observations with the 2.4 m telescope at Lijiang station by deriving geometric distortion for other observation times through fewer calibration field observations, thereby effectively saving valuable telescope time. This work only preliminarily explores the variation pattern of geometric distortion from the perspective of zenith distance. In follow-up work, we will continue in-depth research on the 2.4 m telescope' s geometric distortion variation from aspects such as temperature and pressure.

Acknowledgments

We thank all staff members of the Lijiang 2.4 m telescope. The telescope is funded by the Chinese Academy of Sciences and the Yunnan Provincial Government.

References

Anderson, J., & King, I. R. 2003, *Publications of the Astronomical Society of the Pacific*, 115, 113

Anderson, J., Bedin, L. R., Piotto, G., et al. 2006, *Astronomy & Astrophysics*, 456, 1029

Peng, Q. Y., Vienne, A., Zhang, Q. F., et al. 2012, *The Astronomical Journal*, 144, 141

Fu, X. Q., Peng, Q. Y., & Feng, J. 2021, *Astronomical Research & Technology*, 18, 382

Wang, N., Peng, Q. Y., Zhang, X. L., et al. 2017, *Monthly Notices of the Royal Astronomical Society*, 468, 3805

Shang, Y. J., Peng, Q. Y., et al. 2019, *The Astronomical Journal*, 158, 141

Zheng, Z. J., Peng, H. W., et al. 2020, *Monthly Notices of the Royal Astronomical Society*, 497, 6216

Lin, F. R. 2021, *Monthly Notices of the Royal Astronomical Society*, 505, 6216

Bernstein, G. M., Armstrong, R., & Plazas, A. A. 2017, *Publications of the Astronomical Society of the Pacific*, 129, 114

Lin, F. R., & Peng, Q. Y. 2022, *Monthly Notices of the Royal Astronomical Society*, 511, 314

Zheng, Z. J. 2022, *Monthly Notices of the Royal Astronomical Society*, 517, 1415

Fu, L., Dong, J., Ling, Q. B., et al. 2023, *Chinese Journal of Radio Science*, 38, 258

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.