Postprint: Simulation Analysis of High-Resolution Reconstruction Errors for Solar Polarization Images

Authors: Lü Zhuo, Wang Xiqun, Wang Yuanfangzhou, Jin Zhenyu, Jin Zhenyu

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Abstract

Ground-based solar telescope magnetographs are subject to atmospheric turbulence effects during polarimetric measurements, resulting in inaccurate measurement results. The method of synchronously detecting wavefront aberrations and performing deconvolution reconstruction on solar narrow-band polarization images can overcome issues such as the low photon count level in polarization measurement channels caused by narrow-band filters, thereby applying high-resolution image reconstruction algorithms to solar polarization image reconstruction. During the reconstruction process, inaccurate wavefront estimation can cause crosstalk from I in the reconstructed polarization images, leading to deviations from the true polarization signal. To investigate the influence of wavefront reconstruction accuracy on polarization image reconstruction quality during synchronous reconstruction, a simulation model was established to simulate the crosstalk from I to polarization signals under various seeing conditions and different wavefront reconstruction accuracies. The results demonstrate that the reconstruction quality of polarization images is positively correlated with wavefront reconstruction accuracy; under certain conditions, methods such as increasing the number of frames used for image reconstruction and reducing image resolution can also mitigate the crosstalk from I to polarization signals.

Full Text

Simulation Analysis of High-Resolution Reconstruction Errors in Solar Polarization Images

Authors: Lv Zhuo[123], Wang Xiqun[123], Wang Yuanfangzhou[1245], Jin Zhenyu[1]
Affiliations:
1. Yunnan Observatories, Chinese Academy of Sciences, Kunming 650216, China

2. School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China
3. Yunnan Key Laboratory of Solar Physics and Space Science, Kunming 650216, China
4. School of Physics and Electronic Information, Yunnan Normal University, Kunming 650500, China
5. Yunnan Key Laboratory of Opto-electronic Information Technology, Kunming 650500, China

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Abstract

Ground-based solar telescope magnetographs are affected by atmospheric turbulence during polarimetric measurements, leading to inaccurate results. By synchronously detecting wavefront aberrations and applying deconvolution reconstruction to solar narrow-band polarization images, problems such as low photon levels in polarization measurement channels caused by narrow-band filters can be overcome, enabling the application of high-resolution image reconstruction algorithms to solar polarization images. During the reconstruction process, inaccurate wavefront estimation causes crosstalk from intensity (I) to the reconstructed polarization images, creating deviations from the true polarization signals. To investigate the impact of wavefront sensing accuracy on polarization image reconstruction quality during synchronous reconstruction, we established a simulation model to analyze the crosstalk from intensity to polarization signals under different seeing conditions and wavefront reconstruction accuracies. The results demonstrate that polarization image reconstruction quality is positively correlated with wavefront sensing accuracy. Under certain conditions, increasing the number of frames used for image reconstruction and reducing image resolution can also mitigate crosstalk from intensity to polarization signals.

Keywords: solar polarimetry; high-resolution image reconstruction; wavefront sensing; error analysis

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1. Introduction

Ground-based solar telescope magnetometers operate primarily based on the Zeeman effect. A magnetograph is an imaging magnetic field measurement instrument that uses tunable narrow-band filters and polarimetric analyzers to obtain four polarization signals (I, Q, U, and V) for deriving solar vector magnetic field information [1]. Relying on large-aperture solar telescopes to perform polarimetric measurements of magnetically sensitive Fraunhofer spectral lines enables two-dimensional observations of solar magnetic fields with high spatial and spectral resolution. During actual observations, since detectors are only sensitive to intensity information, polarization signals must be modulated onto intensity signals for acquisition, with the polarization signals then demodulated from the captured images. However, due to Earth's atmospheric turbulence, solar images exhibit blurring, jitter, and distortion, causing the two frames used for demodulating polarization signals to suffer from different degrees of aberration and misalignment, preventing accurate polarization signal extraction.

To overcome atmospheric turbulence effects on ground-based observations, two main approaches have been developed: adaptive optics technology [2] and post-facto image reconstruction algorithms [3]. Adaptive optics systems, consisting of wavefront sensors, controllers, and correctors, can detect atmospheric wavefront aberrations in real time and correct low-order aberrations. Post-facto reconstruction algorithms can complement adaptive optics by compensating for uncorrected high-order aberrations or work independently by constructing cost functions based on statistical information or constraints to recompute target information. The New Vacuum Solar Telescope (NVST) is equipped with a high-resolution image reconstruction system based on speckle interferometry and speckle masking for its imaging channel, providing the solar community with high-resolution photospheric and chromospheric images for scientific research [4].

Fraunhofer spectral lines used for solar magnetic field measurements are absorption lines, requiring narrow-band filters with bandwidths reaching 0.01 nm for scanning observations [5]. The extremely low transmission results in low photon levels in the narrow-band channel, producing poor signal-to-noise ratios during short exposures. Long exposures reduce temporal resolution for polarimetric measurements and smooth out high-frequency information in images due to atmospheric turbulence. To address the difficulty of directly reconstructing narrow-band channel images, a broadband channel with a central wavelength close to that of the narrow-band polarimetric measurement channel was first introduced. The optical transfer function obtained from the broadband channel via speckle imaging was used to deconvolve the narrow-band degraded images. Compared with long-exposure images, the spatial resolution of reconstructed images improved by a factor of two [6]. Subsequent solar telescopes have adopted this approach in magnetograph design, adding a broadband wavefront sensing channel adjacent to the narrow-band polarimetric measurement channel for wavefront aberration estimation and precise alignment of different polarization state images [7-9].

High-resolution reconstruction of narrow-band polarization images can recover lost mid- and high-frequency information and improve spatial resolution. However, reconstruction is essentially an estimation based on a specific imaging model, and the reconstruction process modifies data far more complexly than flat-fielding or dark-field correction. When the reconstruction quality of the two modulated images used for demodulating polarization signals is inconsistent, difficult-to-remove crosstalk from intensity signals is introduced. This paper investigates the crosstalk from intensity signals to polarization signals under different seeing conditions and wavefront aberration estimation accuracies through simulation modeling, and examines how factors such as frame number and resolution affect polarization image reconstruction quality.

2. Narrow-Band Image High-Resolution Reconstruction

The synchronous acquisition system requires strict simultaneous capture of narrow-band polarization measurement channel images and broadband wavefront detection channel images. Since both channels experience the same optical aberrations before the beam splitter, high signal-to-noise broadband wavefront detection channel images can be used to estimate atmospheric wavefront information for reconstructing narrow-band observation targets. Figure 1 shows the optical structure of the synchronous acquisition system.

[Figure 1: see original paper]

In a linear space-invariant system, the image captured by the detector is the convolution of the observation target with the atmospheric-telescope system point spread function. Let o be the observation target, s the actually observed image, and h the corresponding point spread function, with representing the convolution operation. In the spatial domain, the channel images satisfy:

$$s = o \otimes h$$

In the frequency domain, they satisfy:

$$S = O \cdot H$$

where O and H are the spectra of s, o, and h, respectively.

The broadband wavefront detection channel estimates the wavefront aberration $\hat{\phi}_b$ for that channel. After converting to the aberration $\hat{\phi}_n$ corresponding to the narrow-band channel's central wavelength, the optical transfer function $\hat{H}$ for the polarimetric measurement channel is calculated using $\hat{\phi}_n$ combined with the telescope aperture function. The polarimetric measurement channel target is then reconstructed using the deconvolution method:

$$\hat{O} = \frac{S\hat{H}^*}{|\hat{H}|^2 + \sigma}$$

where σ is a regularization factor introduced to avoid issues caused by zero-frequency points, and $\hat{H}^*$ is the complex conjugate of $\hat{H}$.

To improve reconstruction accuracy and more precisely estimate target information, multi-frame images are typically used. The multi-frame reconstruction formula is:

$$\hat{O} = \frac{\sum S\hat{H}^*}{\sum |\hat{H}|^2 + \sigma}$$

3. Polarization Signal Reconstruction and Demodulation

During polarimetric measurements, since detectors are only sensitive to intensity information, polarimetric analyzers must be used to modulate polarization signals onto intensity signals for observation, with the polarization information then demodulated from the observed images. To obtain circular polarization v signals, two sets of modulated signals must be measured within a short time interval:

$$i_+ = i + v$$
$$i_- = i - v$$

where i is the background intensity signal and v is the circular polarization signal. The v signal is obtained by:

$$v = \frac{i_+ - i_-}{2}$$

To eliminate atmospheric turbulence effects and obtain high-resolution solar polarization images, we need to reconstruct the modulated signals. The reconstructed v signal can be expressed as:

$$\hat{V} = \frac{\hat{I}_+ - \hat{I}_-}{2}$$

where $\hat{I}_+$ and $\hat{I}_-$ are the reconstructed spectra of the modulated images. Substituting the reconstruction formula yields:

$$\hat{V} = \frac{I+V}{2} \cdot \frac{H_i \hat{H}_i^*}{|H_i \hat{H}_i^*|^2 + \sigma} - \frac{I-V}{2} \cdot \frac{H_j \hat{H}_j^*}{|H_j \hat{H}_j^*|^2 + \sigma}$$

where I and V are the spectra of the true intensity and circular polarization signals, respectively. H_i is the optical transfer function corresponding to the wavefront aberration experienced by the p modulation state images in different frames, and H_j is the optical transfer function corresponding to the p modulation state images in different frames. $\hat{H}$ is the estimated value of the true optical transfer function H, and $\hat{P}$ is the estimated result of the modulated target P. $\hat{V}$ is the reconstructed polarization signal.

This can be further simplified to:

$$\hat{V} = I\sum \frac{H_i\hat{H}_i^*}{|H_i\hat{H}_i^*|^2+\sigma} - I\sum \frac{H_j\hat{H}_j^*}{|H_j\hat{H}_j^*|^2+\sigma} + V\sum \frac{H_i\hat{H}_i^*}{|H_i\hat{H}_i^*|^2+\sigma} + V\sum \frac{H_j\hat{H}_j^*}{|H_j\hat{H}_j^*|^2+\sigma}$$

The first part is a variable related to I that introduces crosstalk from I to the V signal, while the second part is a variable related to V. This crosstalk affects the accuracy of the V signal. After estimating the optical transfer function $\hat{H}$ of the polarimetric measurement channel images, the reconstructed polarization signal $\hat{V}$ consists of two parts. When the optical transfer function estimation is inaccurate, crosstalk from the intensity signal occurs.

4. Simulation Setup

To study the impact of wavefront aberration estimation errors on polarization image reconstruction quality, we simulated the process described in the previous section. We used magnetohydrodynamic (MHD) simulations to generate solar quiet region intensity and circular polarization signals as observation targets i and v. The images were convolved with the transmission curve of a Lyot filter with 0.01 nm bandwidth and normalized by the maximum intensity value. The images, sized 1008 × 1008 pixels with a field of view of 8 × 8 (approaching an isoplanatic patch size), contained polarization signals of varying strengths with V/Ic [-0.214, 0.111].

A set of Zernike polynomial coefficients was generated to represent the true wavefront aberrations for the simulation. The corresponding seeing conditions were characterized by the empirical formula:

$$\text{seeing} = 0.134\left(\frac{D}{r_0}\right)$$

where σ^2 is the variance of the wavefront aberration, D is the telescope aperture, and r_0 is the estimated seeing parameter.

Since the first three Zernike polynomial terms represent piston and tilt errors in the x and y directions, which cannot be sensed by wavefront sensors but can be eliminated through cross-correlation methods, the wavefront aberrations used in this study exclude piston and tilt. We fitted the true wavefront aberrations using fewer Zernike polynomial orders than the true aberrations, making the wavefront estimation error ϕ primarily consist of inaccurately estimated low-order aberrations and residual high-order aberrations.

The true optical transfer function H and its estimate $\hat{H}$ were calculated from the wavefront aberrations ϕ and $\hat{\phi}$ combined with the telescope aperture function. The crosstalk from i to v was computed using the first part of the formula, while the reconstructed polarization signal $\hat{v}$ was obtained from the complete expression. A reference image was introduced for comparison, using the true wavefront aberration as the wavefront estimation result during reconstruction.

[Figure 2: see original paper]
[Figure 3: see original paper]

5. Simulation Results and Discussion

5.1 Impact of Frame Number on Polarization Image Reconstruction Quality For a seeing condition of 10 cm and wavefront reconstruction residual RMS of 0.107λ, Figure 4 shows the single-frame short-exposure reconstructed v image. Compared with the reference image with accurate wavefront estimation, numerous pseudo-structures appear in the reconstructed image, primarily arising from crosstalk from i. These structures result from inaccurate wavefront aberration estimation, causing certain frequency components to be amplified and preventing accurate estimation of the target spectrum, with severe crosstalk effects.

[Figure 4: see original paper]

In this reconstruction method, we can obtain the wavefront aberration corresponding to each captured polarimetric modulation image. Within short time intervals, solar structures can be considered unchanged, allowing reconstruction using multiple frames via the multi-frame formula. Increasing the number of frames used for polarization image reconstruction can resolve the problem of certain frequency component amplification caused by single-frame wavefront estimation inaccuracies and improve reconstruction quality.

Figure 5 shows reconstruction results for frame numbers of 1, 3, 6, 10, 15, 21, 28, 36, and 45. As the frame number increases, the strong polarization signals increasingly approach the reference image, while weak polarization signals remain unrecoverable. For the area 2 reconstruction, most polarization signals are overwhelmed by crosstalk, except in regions with stronger signals. If used for magnetic field inversion, this would lead to erroneous magnetic field measurements.

[Figure 5: see original paper]

Figure 6 compares the reconstructed v signal and i crosstalk in two regions. For area 1, the pv value and standard deviation of i crosstalk are both smaller than those of the v signal, allowing the v signal to be distinguished. For area 2, the pv value and standard deviation of i crosstalk are close to those of the v signal, meaning the true v signal is always affected by i crosstalk and cannot be accurately measured within certain wavefront reconstruction accuracy and limited frame numbers.

[Figure 6: see original paper]

Figure 7 quantitatively compares the v signal and i crosstalk in the two regions at different frame numbers. The results show that increasing the frame number reduces crosstalk levels, but the reduction rate gradually decreases. For weak polarization signals, they remain difficult to distinguish from crosstalk even with increased frames.

[Figure 7: see original paper]

5.2 Polarization Image Reconstruction Quality Under Different Seeing Conditions Seeing is a crucial parameter measuring the impact of atmospheric turbulence on ground-based observations. Fuxian Lake Solar Observatory is one of the world's best solar observing sites, with mean seeing of 11.9 cm in summer and 7.8 cm in winter. This section simulates reconstruction quality under different seeing conditions and wavefront sensing accuracies.

Table 1 lists the RMS of wavefront aberrations ϕ and the RMS error of their estimates $\hat{\phi}$ for different seeing conditions used in the experiments. Figure 8 shows the reconstructed polarization signals under different seeing conditions using 45 short-exposure modulated images. Under better seeing conditions, the power spectrum of the reconstructed v signal more closely approaches the ideal case in both regions with different polarization signal strengths, indicating more accurate estimation of mid- and high-frequency v signal information.

[Figure 8: see original paper]

To distinguish the specific impacts of seeing conditions and wavefront reconstruction accuracy on v signal reconstruction, an additional dataset was generated with seeing conditions of 80 mm, 100 mm, and 120 mm, all with the same wavefront reconstruction accuracy of 0.128λ. The power spectra of the reconstructed v signals for these wavefront aberrations are shown in Figure 10. Under identical wavefront reconstruction accuracy, the crosstalk levels in reconstructed images are comparable, but better seeing conditions enable more accurate estimation of mid- and high-frequency v signal information because the observed degraded images are less affected by atmospheric turbulence and retain richer information.

[Figure 9: see original paper]
[Figure 10: see original paper]

5.3 Impact of Reduced Sampling Resolution on Polarization Image Reconstruction Errors To meet the demands of precise astronomical measurements, detectors typically operate in "binning" mode, where multiple pixels are combined into one to collect more photons. This approach sacrifices spatial resolution for higher measurement precision [12]. We simulated the crosstalk from intensity signals in binned mode.

Figure 11 shows two sets of images: one corresponding to the diffraction-limited resolution of a 1 m aperture telescope, and the other corresponding to that of a 50 cm aperture telescope. The polarization signals reconstructed from 45 short-exposure images and their corresponding crosstalk are displayed. Reduced resolution decreases the variation range of reconstructed signals and significantly lowers i signal crosstalk. Under any seeing condition, reducing resolution can decrease both the pv value and standard deviation of crosstalk.

Table 2 presents the pv values and standard deviations of crosstalk under different seeing conditions and resolutions. In practice, we can select appropriate resolution and polarimetric measurement precision based on target intensity to obtain more accurate reconstructed polarization signals and develop corresponding observation plans.

[Figure 11: see original paper]

6. Conclusions

This paper presents a simulation analysis of errors in high-resolution reconstruction of solar polarization images. Due to limited wavefront sensor orders, wavefront estimation results from different sensing methods deviate from the true atmospheric wavefront. The primary impact of wavefront estimation errors on high-resolution reconstruction is the inability to accurately estimate mid- and high-frequency polarization signal information and the introduction of intensity signal crosstalk.

Simulation results indicate that for strong solar active region magnetic fields or quiet region magnetic fields reaching tens of gauss (with polarization signal intensity exceeding 5×10^{-3}), reconstructed polarization images suffer relatively low crosstalk from background intensity. However, for weaker quiet region magnetic fields, particularly in linear polarization measurements where polarization signals are weaker, reconstructed polarization images suffer severe crosstalk from intensity signals.

Increasing the number of frames used for image reconstruction can reduce intensity signal crosstalk, though the reduction rate decreases with additional frames. In practical polarimetric measurements, using too many frames for polarization image reconstruction sacrifices temporal resolution. While reducing pixel resolution through binning decreases intensity signal crosstalk, it also reduces spatial resolution. Therefore, specific observation plans must balance temporal resolution, spatial resolution, and polarimetric measurement precision based on the observation target. Additionally, an observation site with excellent seeing conditions is essential for high-resolution solar polarimetric measurements.

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Note: Figure translations are in progress. See original paper for figures.