Ballistic Early Warning Information Generation Method Supported by Elevation Database (Postprint)

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Date: 2023-10-07T00:00:00+00:00

Abstract

In the extrapolation of early warning information based on target orbital motion characteristics, support from an elevation information database is introduced to transform the Earth assumption from a reference ellipsoid to a more realistic shape, and methods for improving the prediction accuracy of launch and impact point locations and times under this scenario are investigated. Numerical experimental results using a digital elevation model demonstrate that prediction accuracy can be enhanced with only a minor addition of elevation iterative calculations to the original method, and the improvement is more pronounced in regions where the elevations of launch and impact points are higher.

Full Text

Preamble

Acta Astronomica Sinica, Vol. 64, No. 5, September 2023
doi: 10.15940/j.cnki.0001-5245.2023.05.008

Method for Generating Ballistic Early Warning Information with Elevation Database Support

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Abstract

In the calculation of early warning information based on the orbital motion characteristics of targets, this study introduces elevation database support to transform the Earth's model from a reference ellipsoid to a more realistic shape, and explores methods for improving the prediction accuracy of launch and landing positions and times. Numerical experiments using a Digital Elevation Model (DEM) demonstrate that forecast accuracy can be enhanced with only a small number of additional elevation iteration calculations based on the original method, with more pronounced improvements in regions where the launch and landing points have higher elevations.

Keywords: ballistic trajectory, methods: numerical, height
CLC number: P135
Document code: A

1 Introduction

Ballistic missile early warning has broad application prospects in aerospace technology and strategic defense, and is increasingly becoming a hot research topic both domestically and internationally. The entire flight process of a ballistic missile is divided into three phases: boost phase, midcourse phase, and reentry phase. The midcourse phase, also known as the free-flight phase, occurs after the missile's engine shuts down and it travels outside the atmosphere. This phase accounts for over 80% of the total flight time and is influenced only by natural forces, making its orbital characteristics relatively easy to model and understand. Consequently, the midcourse phase is the primary stage during which various wireless measurement devices perform acquisition, tracking, identification, and calculation of early warning information (such as launch and landing times and positions).

Previous research has treated target identification during the midcourse phase and the calculation of launch/landing point information as two separate problems. Studies [1–3] have focused on the physical characteristics of ballistic missile warheads reflected in radar echoes. For instance, the micro-motion of warheads (such as precession, nutation, and variations in Radar Cross Section (RCS)) exhibits periodic features that can be analyzed and identified through the micro-Doppler effects present in radar echoes. Additionally, High Resolution Range Profile (HRRP) imaging, which offers fast imaging speeds and high resolution, provides distribution information of strong scattering centers along the radar line of sight and contains structural features of the target. By comparing these extracted features with templates in a target feature library, target identification can be accomplished.

Research on launch and landing point calculation, on the other hand, has typically assumed the target is a ballistic missile and exploited the fact that the trajectory intersects with the Earth. Most calculation methods, such as those described in [4], employ Kalman filtering or improved Kalman filtering to extrapolate the target's equations of motion. These extrapolation methods are essentially trial-and-error approaches: after each extrapolation, one must check whether the target has entered the Earth's interior, and if so, use a bisection method to repeatedly extrapolate until the solution is found. The computational efficiency of these methods is closely related to the choice of filter step size, yet the optimal step size varies with specific trajectories, making it difficult to select a universally optimal value. Moreover, such methods do not consider perturbation forces acting on the target during flight, nor do they incorporate elevation information support.

Some studies have proposed simplifying the missile trajectory to a two-body elliptical orbit and solving for the intersection with the Earth through geometric relationships [5], but this approach similarly neglects perturbation factors and elevation effects. Reference [6] takes a different approach, proposing a method for rapid target identification and early warning information generation based on the characteristic that missile trajectories intersect with the Earth while satellite trajectories do not. This method offers fast computation, simple and robust processing, high efficiency, and considers the Earth's main perturbation terms. However, when generating early warning information, the Earth is crudely simplified as a reference ellipsoid, with elevation information still omitted. While this reduces computational load and improves speed, it degrades the accuracy of early warning information if the missile's launch or landing points are located in mountainous or plateau regions where the surface deviates significantly from the reference ellipsoid. The greater the elevation, the more pronounced this accuracy degradation becomes.

Therefore, it is necessary to improve the early warning information calculation method presented in [6] to eliminate errors caused by elevation factors while maintaining high computational efficiency and robustness, thereby enhancing the accuracy of early warning information. This paper addresses this problem through theoretical analysis and discussion.

The elevation information database used to support launch and landing point calculation can be a global Digital Elevation Model (DEM), which provides a discrete mathematical representation of the Earth's surface topography. The DEM principle divides a region into b rows and c columns of quadrilaterals (where b and c are positive integers), calculates the average elevation of each quadrilateral, and stores these elevations as a three-dimensional matrix. This can be described functionally as:

$$V_j = (L_j, \psi_j, H_j), \quad j = 1, 2, \cdots$$

where L_j and ψ_j are geodetic longitude and latitude coordinates, and H_j is the elevation at coordinate (L_j, ψ_j). Currently, high-resolution global DEMs are primarily generated from satellite remote sensing, satellite altimetry, and shipborne bathymetry data [7]. Generally, the side length of the small quadrilaterals in a DEM represents its resolution, with common resolutions including 90 m, 30 m, 12.5 m, and 5 m. Resolution is an important indicator describing terrain accuracy: smaller side lengths mean more quadrilaterals for the same area, indicating higher resolution and more precise terrain description.

2 Original Method for Ballistic Missile Early Warning

For convenience in subsequent exposition, we briefly introduce the method proposed in [6]. When a missile is at its launch or landing point, its trajectory position intersects with the Earth's surface, and the missile's geocentric distance $r(f)$ equals its sub-satellite point geocentric distance $R(f)$:

$$r(f) = R(f) \quad (1)$$

where f is the true anomaly of the target at time t. Solving equation (1) is the core content of target identification and early warning information calculation based on orbital motion characteristics. A solution for f proves that the target intersects with the Earth, and the launch/landing point early warning information can be calculated from the orbital elements.

Directly solving equation (1) is difficult, so an iterative two-step approach is used:

Step 1: Solve for the initial value under the two-body motion model. The iteration initial value is the target's true anomaly f_0 obtained from the initial orbital elements. The convergence criterion is $|r_k - R_k| < \varepsilon$, where $|r_k - R_k|$ is the absolute difference between the target's geocentric distance and sub-satellite point geocentric distance after the k-th iteration, $\varepsilon > 0$ is the convergence threshold, and k denotes the k-th iteration. Under the two-body model, r can be expressed as:

$$r = \frac{a(1-e^2)}{1+e\cos f} \quad (2)$$

where a and e are the semi-major axis and eccentricity in the Keplerian elements, respectively. R can be expressed as:

$$R = R_p\sqrt{\frac{1-(2\delta-\delta^2)}{1-(2\delta-\delta^2)\sin^2 i \sin^2(\omega+f)}} \quad (3)$$

where i and ω are the orbital inclination and argument of perigee in the Keplerian elements, R_p is the Earth's polar radius, and δ is the Earth's flattening.

Step 2: Introduce the effects of first-order secular and short-period terms to correct the initial value obtained in Step 1, yielding the precise intersection time. The convergence threshold for this step is $|r'_k - R'_k| < \varepsilon$, where k' denotes the k'-th iteration. After convergence of this iteration, the precise intersection time T_0 is obtained, from which the target's position vector in the Earth-fixed coordinate system and the geographic longitude L and latitude ψ of the precise intersection point between the trajectory and Earth's surface can be further calculated.

3 Improved Algorithm for Precise Intersection with DEM Support

After the original method obtains the intersection information between the trajectory and the reference ellipsoid, this paper proposes a new method that performs an additional round of iteration based on these results to achieve a more precise intersection under DEM support.

In this new iteration, the convergence condition is set as $|H_D - H_C| < \varepsilon_2$, where H_C is the elevation calculated for the target at time t, H_D is the elevation obtained by querying the DEM at the target's latitude and longitude at time t, and $\varepsilon_2 > 0$ is a given threshold different from that used in the original method. Since this round of iteration uses elevation difference below a specific threshold as the convergence criterion, it can be called the "elevation iteration" component. The specific iteration procedure is as follows:

1. Query the DEM using the geographic longitude L and latitude ψ of the intersection point calculated by the original method to obtain elevation H_D. Then, through rotational and translational transformation from the Earth-fixed coordinate system to the topocentric horizontal coordinate system, obtain the vertical velocity component v_{hz} of the target at time t in the horizontal coordinate system with the target itself as the origin. The time correction Δt is calculated as:

$$\Delta t = H_D / v_{hz}$$

2. Set $T = t + \Delta t$, introduce the effects of first-order secular and short-period terms, and re-extrapolate to obtain the instantaneous orbital elements σ of the target at time t. From these elements, calculate the target's geocentric distance r and its position vector $\mathbf{r}$ and velocity vector $\dot{\mathbf{r}}$ in the orbital coordinate system.

3. Through coordinate rotation transformation from the orbital coordinate system to the Earth-fixed coordinate system, obtain the target's position vector $\mathbf{r}_b$ in the Earth-fixed coordinate system at time t. From $\mathbf{r}_b$, calculate the geographic longitude L, latitude ψ, and elevation H_C of the precise intersection point between the target trajectory and Earth's surface according to [8]:

$$\tan\psi = \frac{z(1-\delta) + (2\delta - \delta^2) R_e \sin^3\alpha}{(1-\delta)[\sqrt{x^2+y^2} - (2\delta - \delta^2) R_e \cos^3\alpha]}$$

$$\sin L = \frac{y}{N\cos\psi}, \quad \cos L = \frac{x}{N\cos\psi}$$

$$H_C = \frac{\sqrt{x^2+y^2}}{\cos\psi} - N$$

where R_e is the Earth's equatorial radius, x, y, and z are the three components of the target's position vector in the Earth-fixed coordinate system, and:

$$N = \frac{R_e}{\sqrt{1-(2\delta-\delta^2)\sin^2\psi}}$$

$$\tan\alpha = \frac{z(1-\delta)}{\sqrt{x^2+y^2}}$$

4. Query the DEM using L and ψ to obtain elevation H_D, and through coordinate transformation, obtain the vertical velocity component v_{hz} of the target at time t in the horizontal coordinate system with itself as the origin. Calculate the correction Δt at time t as:

$$\Delta t = (H_D - H_C)/v_{hz}$$

5. Set $T = t + \Delta t$ and return to Step 2 of this iteration method to continue calculating t and Δt. When returning to this step, if the iteration condition $|H_D - H_C| < \varepsilon_2$ is satisfied, terminate the iteration process; otherwise, continue iterating from Step 2.

After iteration convergence, the calculation results provide the more precise time t when the target is at the intersection of the trajectory and Earth's surface, along with the launch/landing point longitude L and latitude ψ, and the elevation value H_D obtained by querying the DEM at these coordinates.

As mentioned at the beginning of this paper, the elevation points stored in the DEM database are discrete. This means that when the elevation of the intersection region between the target trajectory and Earth's surface is higher, the slope is steeper, and the DEM resolution is higher, the difference between elevation data H_D obtained from consecutive queries may be large, making convergence difficult. To handle such non-convergence situations, one can first increase the convergence threshold ε_2. Second, an optimal selection strategy can be employed: set a maximum iteration number $k_{\max}$, and when the iteration count reaches $k_{\max}$, stop the iteration. Select the L and ψ calculated from the iteration with the smallest $|H_D - H_C|$ value among all iterations, along with the corresponding elevation H_D queried from the DEM, as the optimal approximate values and treat them as the final results.

4 Numerical Experiments and Analysis

Due to the lack of measured ballistic data, we conducted simulation tests and analysis to evaluate the computational performance of the proposed method. The DEM data used in the tests were obtained from the SRTMDEM (Shuttle Radar Topography Mission DEM) 90 m resolution elevation data on the Geographic Data Cloud platform of the Chinese Academy of Sciences' Network

Information Center[1], which is projected onto the WGS84 (World Geodetic System 84) Earth-fixed reference ellipsoid. First, five sets of ballistic trajectories were designed using established ballistic design software. Then, the trial-and-error method with high-precision numerical extrapolation was used to obtain the precise intersections between these five trajectories and the Earth's surface as the true values for launch and landing points. Their orbital elements, ranges, and intersection elevations are listed in Table 1 . The range in this paper refers to the arc length of the missile's trajectory in inertial space outside the Earth. The orbital element format used is the first-type singularity-free elements commonly employed in satellite orbit determination: in addition to a, i, and Ω, the other three elements are $\xi = e\cos\omega$, $\eta = -e\sin\omega$, and $\lambda = M + \omega$, where M is the mean anomaly.

In practical applications, target identification and early warning information generation are part of real-time processing during radar tracking. Therefore, simulating the radar real-time processing procedure to examine the method's computational efficiency and accuracy changes is the best approach. We designed experiments to simulate calculations for all five sets of orbital elements with different ranges and elevations in Table 1. Following the method in [6], we first selected appropriate observation station locations based on the ballistic elements and launch/landing points, then generated simulated observation data using these stations to track the targets. Random errors of 0.1° in angle and 50 m in range were added to these simulated observations. Subsequently, existing laboratory programs were used to perform filter initialization with the first 30 seconds of data, followed by Kalman filtering on the simulated data at one-second intervals after 30 seconds to update the target's orbital elements.

Each time the orbital elements were updated through filtering, we used both the original method and the new method proposed in this paper to generate the positions and times of the two intersection points, observing the error variations in the calculated intersection times and positions from both methods. The convergence accuracy of the original method was set to 0.01 m, the elevation iteration convergence accuracy ε_2 of the new method was set to 1 m, and the maximum elevation iteration number $k_{\max}$ was set to 5. The errors in intersection time and position were calculated as:

$$\Delta\text{Time} = t_c - t_s$$

$$\Delta R = \|\mathbf{R}_c - \mathbf{R}_s\|$$

where t_c and t_s are the calculated and true times, respectively, and $\mathbf{R}_c$ and $\mathbf{R}_s$ are the calculated and true position vectors in the Earth-fixed coordinate system.

Regarding computational efficiency changes, the increased computational load of the new method mainly comes from the additional elevation iteration calculations. Therefore, in evaluating the new method's efficiency, we primarily observe the number of added elevation iterations. Assuming there are l instances of early warning information generation during the filtering process that involve m elevation iterations ($m = 1, 2, \cdots$), the value of m can be used to evaluate the efficiency change of the new method compared to the original. In fact, DEM reading speed is also an important factor affecting computational efficiency, but it involves database reading technology beyond the scope of this paper. Therefore, we only assume that reading the DEM is as fast as reading memory variables without further elaboration.

Table 2 lists the number of added elevation iterations for each test case during early warning information generation. The table shows that for the five test cases, 270 early warning information generation calculations were performed for both launch and landing points. In most cases, convergence is achieved after 2 elevation iterations. Only for the launch point calculations of Cases 1 and 2, which are located in high-elevation, steep-slope mountainous areas, were there more instances requiring over 2 elevation iterations, with 72 and 99 non-convergent cases respectively. The landing point of Case 1 is located in Bolivia's Uyuni Salt Flat, where elevation changes are minimal. Therefore, despite its high elevation of 3799 m, convergence is generally achieved after 2 iterations. For Case 5, where both launch and landing points have very low elevations, elevation iteration basically converges in 1 iteration throughout the process.

Figures 1 [Figure 1: see original paper], 3, 5, 7, and 9 show the variation of launch and landing point position errors with radar tracking time for the five test cases, while Figures 2 [Figure 2: see original paper], 4, 6, 8, and 10 show the variation of launch and landing point time errors with radar tracking time. Figures 1 through 10 [Figure 10: see original paper] reveal that during the initial filtering stage, due to large orbit determination errors, both the new and original methods produce large time and position errors, with no significant difference between them. The largest time and position errors occur in Case 1, reaching magnitudes of hundreds of seconds and five hundred kilometers, respectively, while the smallest errors in Case 5 still reach tens of seconds and hundreds of kilometers. After 3.5–4 minutes of filtering, the intersection time and position errors stabilize. At this point, it can be observed that for the first four cases and the launch point of the fifth case, the new method yields better accuracy in both intersection time and position than the original method. The higher the elevation of the intersection point, the more significant the accuracy improvement brought by the new method.

For Case 1, the new method achieves an intersection time error of approximately 0.5 s and a position error of about 3.8 km, whereas the original method yields errors of approximately 2 s and 10 km, respectively. For Cases 2–4, the new method's position and time errors are on the order of hundreds of meters and approximately 0.01 s, respectively, both superior to the original method's errors on the order of kilometers and approximately 0.02 s. For Case 5, where the launch and landing points are close to the reference ellipsoid, the difference between the two methods is minimal. For the launch point at 75 m elevation, the new method's accuracy is only slightly better than the original method, while for the landing point at only -38 m elevation, Figures 9 [Figure 9: see original paper] and 10 show the calculation result curves overlapping, indicating nearly identical accuracy between the two methods.

These results demonstrate that the greater the elevation of the intersection point, the more the new method improves the calculation accuracy of intersection position and time compared to the original method. For trajectories with very low intersection elevations, the difference between the two methods is not significant. In most cases in this experiment, each early warning information generation only adds 1–2 elevation iteration calculations. When the intersection elevation is high and the slope is steep, the number of iterations increases and non-convergence cases become more frequent, but the accuracy of intersection position and time still shows substantial improvement. This shows that after adding elevation iteration, the new method can largely eliminate elevation-induced errors with only a small increase in iteration count. Compared to other cases, Case 1's accuracy with the new method is also relatively low. This is because its main error source is the longer extrapolation time due to its longer range, which amplifies orbit determination errors over time—an error component that clearly cannot be eliminated through elevation iteration.

5 Conclusions and Discussion

Based on the research in [6], this paper proposes an early warning information generation method supported by the elevation information database DEM. Simulation results demonstrate that the method has the following characteristics:

1. Compared to the original method, this method only adds a small number of elevation iteration calculations and includes truncation and optimal selection measures for non-convergent cases. Therefore, it maintains high computational speed, robustness, and stability, enabling rapid provision of ballistic missile early warning information for missile defense decision-making.

2. During real-time radar tracking of ballistic missiles, this method improves the prediction accuracy of intersection position and time compared to the original method. The higher the elevation of the intersection point, the more significant the accuracy improvement. Typically, after 3.5–4 minutes of filtering, relatively accurate launch and landing point information can be provided, with improvements of up to 6 km in position and 1.5 s in time. For ballistic targets with smaller launch and landing point elevations, the improvement in calculation accuracy diminishes.

Additionally, the simulation experiments designed in this paper show that during the first 2–3 minutes of observation, both the original and new methods produce low accuracy, with the new method even occasionally yielding lower accuracy than the original method. Therefore, in practical applications, one might consider using only the original method for target identification and early warning information generation before the filtering reaches steady state, and then adding elevation iteration calculations after steady state is achieved. This approach can reduce multiple elevation iterations and DEM database reading operations during the tracking process, thereby further improving computational speed and efficiency without affecting overall early warning information accuracy.

It should be noted that in actual situations, the force process during the target's boost phase is difficult to obtain, so even using this method, the calculated launch point information will still have significant errors. Landing point calculation is different, as ballistic design generally requires accurate arrival at the landing point without considering atmospheric dynamic effects. The control adjustments in this phase primarily aim to enable the target to hit the designated location accurately. Therefore, using this method to improve landing point position and time accuracy has significant application value.

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Note: Figure translations are in progress. See original paper for figures.