Introduction to the Space Debris Multi-Source Data Fusion Orbit Determination Software SPODFMD (Postprint)

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Abstract

SPODFMD (Space debris Precise Orbit Determination Fusing Multi-source Data) is a numerical cataloging and orbit determination software independently developed by the Satellite Precision Orbit Determination Team of Purple Mountain Observatory, aimed at addressing multi-source data fusion challenges in domestic space debris monitoring networks. During the initial software design phase, comprehensive investigations were conducted on mainstream domestic space debris monitoring equipment. Considering practical factors such as the complexity of space debris orbital and physical characteristics, the computational efficiency requirements for updating large quantities of debris orbits, and the robustness demands for engineering applications, the orbit determination algorithm integrates proprietary technologies including a singularity-free fast algorithm for Earth's gravitational potential and its partial derivatives, an analytical expression method for the singularity-free DTM94 (Drag Temperature Model) atmospheric density model and its partial derivatives, a precise and rapid computation algorithm for dense ephemerides of high-eccentricity orbits, and a robust adaptive weighting method. In conjunction with software engineering design theory, the software has currently achieved arbitrary fusion of data from 14 types of mainstream equipment, attaining second-level orbit determination speed in typical scenarios, and is applicable to full-orbit debris across high, medium, and low Earth orbits with various eccentricity characteristics, while being free from computational singularities and polar singularities. Preliminary testing has confirmed the multi-source capability, high efficiency, universality, and robustness of the SPODFMD software.

Full Text

Introduction to Orbit Determination Software SPODFMD for Multi-source Data Fusing of Space Debris

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ABSTRACT

SPODFMD (Space debris Precise Orbit Determination Fusing Multi-source Data) is a data-fusion orbit determination software developed for space debris cataloging. Created by the Satellite Precise Orbit Determination (SPOD) team at Purple Mountain Observatory (PMO), it is characterized by its integration of pure numerical integrators. During initial development, the team thoroughly investigated domestic space debris monitoring equipment, fully understood the orbital and physical complexities of numerous space debris objects, recognized the critical requirement for high computational efficiency when updating orbits for large debris populations, and carefully considered robustness requirements for engineering applications. SPODFMD integrates several proprietary algorithms, including a rapid non-singular algorithm for computing Earth's gravitational potential and its derivatives, analytical non-singular expressions for the DTM94 thermosphere model and its derivatives, an accurate and efficient method for calculating dense ephemerides of high-eccentricity orbits, and a robust adaptive weighting method. By applying software engineering principles to integrate these advanced algorithms, SPODFMD can freely fuse observation data from 14 types of equipment, achieving second-level computational efficiency in most typical orbit determination scenarios. The software demonstrates consistent performance for objects in GEO (Geosynchronous Equatorial Orbit), MEO (Medium Earth Orbit), LEO (Low Earth Orbit), and HEO (Highly Elliptical Orbit) without computational or polar singularities. Testing has verified that SPODFMD is multi-source, efficient, general-purpose, and robust.

Key words: astronomical instrumentation, space debris, multi-source data, data-fusion orbit determination, orbit determination accuracy, computation efficiency

Introduction

Orbital information of space debris is a key focus of space situational awareness. The catalog database formed from this information underpins applications such as space resource development and utilization, collision warning for space assets, and space debris removal, and its importance is widely recognized. Over the past two decades, the number of dedicated, dual-use, and available space debris monitoring devices worldwide has grown substantially, forming multiple monitoring networks represented by the U.S. Space Surveillance Network. The main types of equipment are radio radar, optical telescopes, and laser radar, and the probability of the same target being detected or tracked by multiple devices has increased significantly.

Utilizing data from multi-station, multi-type equipment for catalog orbit determination is one of the problems to be solved in networked observation. Because multi-station networking improves measurement geometry, increases observation frequency, and enables the combination of high-precision data, fusion orbit determination achieves higher accuracy than single-station orbit determination.

Orbit computation in cataloging work differs from that in general applications, requiring not only sufficient accuracy to precisely grasp the motion state of space debris but also high computational efficiency. The minimum requirement for computational efficiency in cataloging work is that the orbit computation processing time must not exceed the data acquisition time of automatically operating equipment, to avoid creating a bottleneck effect from data backlog that would make the cataloging process unsustainable. Taking a catalog of 20,000 targets with one orbit update calculation per day as an example, the average processing time per target must be no greater than 4.32 seconds, representing a very high requirement for computational efficiency in catalog orbit determination.

Currently, analytical methods are widely used for orbit computation in cataloging work. For example, the United States employs the SGP4 (Simplified General Perturbation)/SDP4 (Simplified Deep Space Perturbation) analytical models, while domestic cataloging work typically uses methods such as the quasi-mean orbital elements method. Although analytical methods adequately address the timeliness requirements of cataloging work, with the development of observation technology, their model accuracy cannot match current observation precision (radar ranging accuracy better than 20 m, optical angle measurement accuracy better than 10), and thus cannot achieve ideal orbit determination accuracy, limiting the improvement of related application levels. Therefore, numerical methods have potential application value in cataloging work. On the other hand, the continuous improvement of modern computer processing speeds has also made it possible to apply numerical methods to catalog processing.

When applying numerical methods to catalog orbit determination, the processing approach of "one satellite, one solution" used in artificial satellite orbit determination should be transformed, focusing on solving problems of poor generality and timeliness. Specific research directions include expanding data types, improving computation speed, applicability to all orbit types, and meeting the computational robustness requirements brought by massive data.

The authors have conducted extensive beneficial explorations in the following areas: investigating domestic mainstream space monitoring equipment (equipment types, installation methods, data types, etc.), surveying the current status and improvement requirements of space debris data processing (accuracy and speed, etc.), developing optimization algorithms that balance accuracy and efficiency for key time-consuming steps in numerical orbit improvement computation, performing non-singular processing for polar and mathematical singularities, and researching adaptive weighting methods for prior accuracy distortion in multi-source data. The above investigation results and research findings form the foundation of the SPODFMD (Space debris Precise Orbit Determination Fusing Multi-source Data) software architecture and orbit determination algorithms. This paper will first introduce the software's functions and characteristics, then describe the key technologies integrated into the software, and finally test the software using multi-source data and summarize its performance based on the test results.

2. Software Functions and Features

SPODFMD software meets the fusion requirements for data from most mainstream space monitoring equipment and possesses multiple characteristics.

2.2 Software Characteristics

Multi-source Data: The software supports data input and arbitrary fusion from 14 types of equipment, with equipment types listed in the appendix.

High Efficiency: The software's efficiency has been optimized in both orbit determination algorithms and programming techniques. Algorithmically, on one hand, it employs an integrate-first-then-interpolate strategy to replace point-by-point integration. The number of interpolation base points integrated per orbital period increases from 25 to a maximum of 80 with increasing eccentricity, and the actual number of constructed base points only needs to cover the ephemeris points. Since interpolation is highly efficient, the ephemeris computation time is approximately equal to the integration time of the interpolation base points, saving significant time compared to point-by-point integration of dense ephemeris. On the other hand, the most computationally complex term in a single integration is the calculation of Earth's gravity and its partial derivatives, with its time consumption growing quadratically with degree. The Métris coefficient transformation method can solve this problem: whether computing the gravitational potential or its partial derivatives of any order, the computation can be converted to a summation of spherical harmonic series, with the coefficients of the spherical harmonic terms calculable in advance through a one-time transformation, again saving computation time. Additionally, when computing partial derivatives of atmospheric density with respect to Earth-fixed rectangular coordinates, analytical expressions are used, which are more efficient than finite difference differentiation that requires multiple atmospheric density calculations. In terms of programming techniques, the software employs efficient sorting methods, search algorithms, sliding window methods, etc., to further improve computational efficiency.

Generality: The software is applicable to space debris in all orbital regimes (low, medium, and high orbits) with various eccentricities. The generality for eccentricity is mainly achieved through the non-uniform interpolation base point construction method and multiple integrators. The non-uniform interpolation base point construction method was proposed to address the imbalance in ephemeris accuracy between perigee and apogee when using uniform base points for high-eccentricity orbits, but is actually applicable to any eccentricity orbit and can be combined with any single-step integrator. The software provides four integrators and three combined integrators to accommodate variations in orbital eccentricity. Among them, the AC (Adams-Cowell) integrator is suitable for orbits with eccentricity less than 0.1, while RKF (Runge-Kutta-Fehlberg), RKNF (Runge-Kutta-Nystrom-Fehlberg), and Everhart are applicable to any eccentricity orbit. The generality for orbital altitude is mainly achieved through the orbit determination process. In addition to the six orbital elements, high-orbit targets simultaneously adjust physical parameters related to solar radiation pressure, low-orbit targets adjust parameters related to atmospheric drag, and medium-orbit targets adjust both atmospheric and solar radiation pressure parameters. This differentiation by orbital altitude of space debris achieves full coverage of orbital heights.

Robustness: The software's robustness is mainly manifested in its non-singular design. The software integrates two algorithms for Earth's gravitational potential and its partial derivatives: the Balmino and Métris coefficient transformation methods. Their common feature is that both have undergone non-singular transformations, ensuring that computing gravitational forces and partial derivatives at points above the poles does not cause definitional or computational singularities. The DTM94 (Drag Temperature Model) atmospheric density model also possesses the conditions for non-singular transformation. After undergoing the same processing as the Earth's gravitational potential, it also achieves the effect of eliminating singularities, further enhancing the software's robustness.

Modularity: The software adopts a modular design, with all functions that are independent and have reuse requirements managed as modules. This brings flexibility to program design. For example, the dynamic model has 24 related modules for arbitrary combinations of four configuration options: orbit determination/prediction, low/medium/high orbit, Balmino/Métris, and 1st/2nd-order integrators. For instance, "orbit determination + low orbit + Métris + 2nd-order integrator" corresponds to a separate module, making program modification and invocation very convenient and facilitating the implementation of more complex functions.

Flexibility: The software's flexibility is an extension of the multi-source data concept. The 14 types of equipment supported by the software are logical device concepts. Logical devices can correspond one-to-one with actual devices or be completely different from actual devices, with equipment types set flexibly according to data characteristics and fusion requirements. For example, for a phased array radar whose ranging accuracy is much higher than its angular accuracy, if only ranging data is used for fusion with data from other high-precision angular measurement devices, the software can be invoked by simply setting the phased array radar's equipment type to laser radar. Similar scenarios include treating a laser ranging and angular measurement device as a single ground-based radar with three data elements for input. The logical device concept makes data fusion scenarios more versatile and user-friendly.

Intelligence: Intelligence is reflected in the software's handling of physical parameter adjustment. In addition to the six orbital elements, the software attempts to simultaneously adjust physical parameters related to the target during orbit determination. If iteration diverges or the adjusted values are unreasonable, it automatically fixes these physical parameters and restarts the adjustment, with the entire process requiring no manual intervention.

Complete Information Feedback: All stages of software processing provide necessary feedback information. Through 17 types of feedback information, issues such as unreasonable inputs, integration anomalies, and stack exceptions can be traced, meeting engineering application requirements for software.

3. Key Technologies

The key technologies integrated into the SPODFMD software mainly include: a non-singular fast calculation method for Earth's gravitational potential and its derivatives, analytical expressions for the non-singular DTM94 atmospheric density model and its derivatives, a fast and accurate calculation method for dense ephemeris of high-eccentricity orbits, and a robust adaptive weighting method. Although these algorithms were proposed to solve specific problems, during development they all considered the requirements of orbit determination for efficiency, accuracy, generality, and robustness. Their integration optimizes the orbit determination process from various perspectives simultaneously, ultimately determining the effectiveness of the SPODFMD software.

3.1 Fast and Accurate Calculation Method for Dense Ephemeris of High-Eccentricity Orbits

Orbit computation in cataloging work generally involves the generation and calculation of dense ephemeris. When ephemeris points are dense, the step size of point-by-point integration cannot be sufficiently extended, and the number of calculations of the integration right-hand function increases dramatically, severely reducing the computational efficiency of numerical methods. An effective technical approach to avoid this problem is to use interpolation methods. Currently, various interpolation methods for generating dense ephemeris adopt a fixed-step division of interpolation base points. This approach is suitable for near-circular orbits with very gentle changes in motion characteristics but is not applicable to high-eccentricity orbits with more dramatic motion characteristic changes frequently encountered in cataloging work. It can easily cause an imbalance where ephemeris near perigee has low interpolation accuracy due to sparse interpolation base points, while ephemeris near apogee has excessively high accuracy due to dense interpolation base points.

This imbalance can be improved by increasing the number of interpolation base points, but this correspondingly increases numerical integration time, which can reduce computational efficiency. To balance both the overall ephemeris calculation accuracy and computational efficiency for high-eccentricity orbits, a non-uniform interpolation base point construction method [1–3] is proposed. Specifically, interpolation base points are constructed sequentially starting from an initial base point with known ephemeris. The base point to be constructed is calculated from the ephemeris of the previous base point, with the base point's ephemeris obtained through numerical integration. Base point construction and numerical integration calculations alternate until all base points covering the interpolation points are constructed.

The construction method for non-uniform interpolation base points is shown in equation (1), where the base point interval is a function of the space object's geocentric distance. This method can be used in coordination with general single-step integration methods in the ephemeris calculation process, and achieves the highest overall efficiency when combined with Hermite interpolation polynomials.

$$\Delta T_l = \beta \tilde{a}^{1+\gamma} \frac{P_t}{N}$$

where ΔT_l is the interval between the l-th interpolation base point and the $(l+1)$-th base point to be constructed; a is the orbital semi-major axis; γ is the time transformation parameter; β is the orbital period transformation factor; R_l is the geocentric distance calculated using the ephemeris of the l-th base point; P_t is the orbital period calculated with time variable t; and N is the number of interpolation base points in one orbital period. Additionally, γ and N are constants predetermined before base point construction, with the optimal universal value of γ determined from numerical experiments being 0.3, and β determined by the orbital eccentricity e and the value of γ, also being constant during the base point construction process.

3.2 Non-singular Fast Calculation Method for Earth's Gravitational Potential and Its Derivatives

Earth's gravitational potential is typically expressed as a function of spherical coordinates, which introduces computational singularities manifested in two ways: (1) the longitude definition is ambiguous for points above the poles; (2) when solving for partial derivatives of the gravitational potential with respect to Earth-fixed rectangular coordinates, the cosine of latitude appears in the denominator, causing division by zero for points above the poles. From a physical perspective, Earth's external gravitational potential is continuously distributed without local singularities. The above singularities are not essential but mathematical singularities that can be eliminated through appropriate mathematical methods.

Pines [4] first introduced a non-singular expression form of Earth's gravitational potential, converting the spherical harmonic functions of Earth's gravitational potential expressed by Legendre polynomials into expressions using Helmholtz polynomials, and introduced geocentric distance and direction cosines to replace spherical coordinates. Balmino et al. [5] further organized and improved upon Pines' [4] results. Their basic idea is to use the chain rule to transform the first and second-order partial derivatives of the gravitational potential with respect to Earth-fixed coordinates into partial derivatives with respect to geocentric distance and direction cosines. The transformed partial derivatives do not produce latitude cosine factors in the denominator and are therefore non-singular.

Hotine [6] discovered that partial derivatives of any spherical harmonic series with respect to rectangular coordinates can also be expressed as a spherical harmonic series, thereby transforming the computation of partial derivatives of Earth's gravitational potential with respect to Earth-fixed rectangular coordinates into a coefficient transformation. Bettadpur [7] conducted further research and improvements. Métris et al. [8] absorbed the conclusions of Bettadpur [7] and Pines [4], summed the Cunningham [9] results for all individual solid spherical harmonic terms in the gravitational potential expansion, and merged and reorganized the solid harmonic functions in the sum by degree and order to obtain a non-singular compact formula for the partial derivatives of Earth's gravitational potential with respect to Earth-fixed rectangular coordinates. The derivative formula is completely similar in form to the original gravitational potential expression, with transformation relationships between its harmonic coefficients and some variables and the original coefficients and variables. The quantities requiring transformation are: $\mu \rightarrow \mu'$, $N_1 \rightarrow N_1'$, $N_2 \rightarrow N_2'$, $C_{nm} \rightarrow C_{nm}^{(i,j,k)}$, $S_{nm} \rightarrow S_{nm}^{(i,j,k)}$, and $L \rightarrow L'$. The specific definitions and transformation methods for these variables are given in equations (2)–(3) below.

SPODFMD integrates both the Balmino and Métris calculation methods. The Balmino method integrated in the software unifies the separately treated Earth central gravity term, zonal harmonic term, and tesseral harmonic term in the original formula, providing concise expressions convenient for computer programming. Its drawback is the need for recursive calculation of Helmholtz polynomials and their first and second-order partial derivatives, which affects computational efficiency. The Métris method is actually a generalization of the Hotine algorithm and is the only method capable of achieving high-order partial derivatives above second order for Earth's gravitational potential to date. Whether computing the gravitational potential or its partial derivatives of any order, its computational form is unified as a summation of spherical harmonic series, differing only in the coefficients of the spherical harmonic terms. Since these coefficients can be obtained in advance through corresponding transformations, computation time is greatly saved. When changes in Earth's gravitational potential harmonic coefficients are not considered (e.g., ignoring tidal effects), at least 30% of computation time can be saved [10].

Let U be the gravitational potential or its partial derivatives of any order with respect to Earth-fixed rectangular coordinates, with the form:

$$\sum_{n=N_1}^{N_2} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n (C_{nm}\gamma_m + S_{nm}\sigma_m)H_{nm}, \quad N_2 \geq N_1 \geq L \geq 0$$

In the original form, μ is the Earth's gravitational constant; n and m are the degree and order of the truncated spherical harmonic expansion, with N_1 and N_2 being the lower and upper limits of n; L is the order of partial derivatives with respect to the z-coordinate, which is zero in the original form; C_{nm} and S_{nm} are normalized spherical harmonic coefficients; H_{nm} is the normalized Helmholtz polynomial; and $\gamma_m = \cos^m \phi \cos m\lambda$, $\sigma_m = \cos^m \phi \sin m\lambda$, where λ and ϕ are the longitude and latitude of the target point, respectively.

Then the formula for the partial derivative of U with respect to rectangular coordinates (x, y, z) of any order (i, j, k) is:

$$\frac{\partial^{i+j+k}}{\partial x^i \partial y^j \partial z^k} U = \mu' \sum_{n=N_1'}^{N_2'} \sum_{m=0}^{n-L'} \left(\frac{a}{r}\right)^n \left(C_{nm}^{(i,j,k)}\gamma_m + S_{nm}^{(i,j,k)}\sigma_m\right) H_{nm},$$

where $\mu' = \mu$, $N_2' = N_2 + i + j + k$, $N_1' = N_1 + i + j + k$, $L' = L + k$, and the transformation formulas for $C_{nm}^{(i,j,k)}$ and $S_{nm}^{(i,j,k)}$ and the meanings of the symbols are given in reference [9].

3.3 Adaptive Weighting Method

When fusing multi-type data with different measurement accuracies, the allocation of weights must be carefully considered. The common practice is to weight according to the prior accuracy of the data, but distortion in prior accuracy can easily degrade orbit improvement quality. Considering that accuracy distortion is a common phenomenon, a robust adaptive weighting method is proposed. Adaptive weights are obtained through a process that uses residuals from a fixed-weight orbit improvement convergence to reallocate weights until convergence. Its characteristics are: (1) Adaptive weighting orbit determination uses the converged result from fixed-weight orbit improvement as the initial orbital elements, so it defaults to executing fixed-weight orbit determination first before starting. This avoids over-reliance on the accuracy of initial orbital elements when using adaptive weights directly (adaptive weights are determined from residuals between initial orbit predictions and data values), a dependency that significantly increases the probability of orbit improvement deviating from true values and may even cause divergence. (2) During adaptive weighting orbit determination, observation data is no longer rejected. Instead, all data retained after outlier rejection in the final iteration of fixed-weight orbit improvement participates in adaptive orbit determination. This approach both corrects the weights of data with accuracy distortion to some extent and ensures the robustness of adaptive iteration. Typically, adaptive weighting orbit determination requires only a small number of iterations to converge, not significantly increasing computation time.

3.4 Non-singular DTM94 Atmospheric Density Model and Its Analytical Derivative Expressions

In catalog orbit determination for low-orbit targets, atmospheric drag perturbation must be considered. For low-orbit targets with particularly low altitudes or large area-to-mass ratios, atmospheric drag perturbation is comparable to the Earth's main zonal harmonic J_2 perturbation. Therefore, to ensure convergence of the orbit determination iteration process, partial derivatives of atmospheric drag with respect to the target's position and velocity vectors must also be computed. SPODFMD employs the DTM94 thermospheric model [11], which is a three-dimensional model with high accuracy. However, because it also uses spherical coordinates as basic variables, the model itself and its partial derivatives with respect to Earth-fixed rectangular coordinates also have polar singularities. On the other hand, currently, except for the Harris-Priester atmospheric model which has mature analytical expressions for partial derivatives, other atmospheric models are too complex for differentiation, and using finite differences to compute partial derivatives is a common alternative. However, when the model is discontinuous or non-differentiable, finite difference calculations produce distorted results.

Given that Legendre polynomials exist in the original DTM94 model, the method used by Pines [4] to eliminate singularities in the spherical harmonic expression of Earth's gravitational potential is similarly adopted. The target's geocentric distance and direction cosines are used to replace spherical coordinates as the basic variables of the DTM94 atmospheric model. The transformed atmospheric density model is non-singular and makes it possible to seek analytical expressions for atmospheric density with respect to the space target's Earth-fixed rectangular coordinates.

Drawing upon the chain rule proposed by Pines [4], an expression formula for atmospheric density ρ with respect to Earth-fixed rectangular coordinates is proposed [12]:

$$\frac{\partial \rho}{\partial X_j} = \sum_{k=1}^{3} \frac{\partial \rho}{\partial \theta_k} \frac{\partial \theta_k}{\partial X_j},$$

where $j = 1, 2, 3$, $\rho = \rho(x, y, z)$ is a continuously differentiable function of x, y, z, with $X_1 = x$, $X_2 = y$, $X_3 = z$, and $r, \theta_1, \theta_2, \theta_3$ are the target's geocentric distance and direction cosines on the X, Y, Z axes of the Earth-fixed coordinate system, respectively. Compared with the finite difference method for obtaining partial derivatives, analytical expressions have advantages in accuracy, robustness, and efficiency.

4. Testing and Analysis

Multi-source observation data has two meanings: first, data from multiple stations, and second, multiple data types. Satisfying either condition can be called fusion

Note: Figure translations are in progress. See original paper for figures.

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