ChinaRxiv

AI translation · View original & related papers at
chinarxiv.org/items/chinaxiv-202310.03080

A Review of Methods for Determining Indicator Weights in Multi-Factor Evaluation: Postprint

Authors: Liu Qiuyan, Wu Xinnian

Date: 2023-10-08T00:00:00+00:00

Abstract

[Purpose/Significance] This paper provides a systematic review of common methods for determining indicator weights in multi-factor evaluation, offering a reference for researchers to reasonably select appropriate weight determination methods when addressing specific evaluation problems. [Method/Process] Through in-depth investigation of the basic principles, underlying concepts, and specific application cases of several typical subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods, this study analyzes and summarizes their respective advantages, disadvantages, and applicable scopes. [Results/Conclusions] To date, no completely universal and general-purpose method for determining indicator weights has been developed. Different indicator weight determination methods are founded on different principles and guiding concepts, resulting in variations in their applicable scopes. When conducting practical evaluations, weighting methods should be reasonably selected according to the characteristics of the evaluation object, thereby enhancing the accuracy and effectiveness of comprehensive evaluation.

Full Text

Preamble

ChinaXiv Cooperative Journal: Academic Exploration
A Review of Index Weight Determination Methods in Multi-Factor Evaluation
Liu Qiuyan[1,2], Wu Xinnian[1]

[1]Lanzhou Library of Chinese Academy of Sciences, Lanzhou 730000
[2]University of Chinese Academy of Sciences, Beijing 100049

Abstract

[**Purpose/Significance**] This paper systematically reviews common index weight determination methods in multi-factor evaluation to provide reference for researchers in selecting appropriate weighting methods for specific evaluation problems. [**Method/Process**] Through in-depth investigation of the fundamental principles, specific application cases, advantages, disadvantages, and applicable scopes of several common subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods. [**Result/Conclusion**] No completely universal weighting method exists to date. Different methods operate on different principles and guiding philosophies, resulting in varying applicable scopes. When conducting practical evaluations, weighting methods should be selected based on the characteristics of the evaluation object to improve the accuracy and effectiveness of comprehensive evaluation.

Keywords: multi-factor evaluation; index weight; weighting methods
Classification Codes: F224, G304
Citation Format: Liu Qiuyan, Wu Xinnian. Review on the weighting methods of indexes in the multi-factor evaluation [J/OL]. Knowledge Management Forum, 2017, 2(6): 500-510 [citation date]. http://www.kmf.ac.cn/p/116/.

---

1 Introduction

The determination of index weights is a critical step in multi-factor comprehensive evaluation, and whether the weights are determined reasonably directly affects the reliability and validity of evaluation results. To date, scholars both domestically and internationally have conducted extensive research on index weight determination methods in multi-factor evaluation and achieved fruitful results. A comprehensive review of these studies reveals that while numerous weighting methods exist, they can be broadly categorized into three types: subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods. Subjective weighting methods primarily rely on experts' knowledge and experience to make subjective judgments in determining index weights. Objective weighting methods calculate weights mainly through analysis of sample data. Integrated subjective-objective weighting methods combine the weights obtained from both approaches based on their respective shortcomings and advantages.

Different weighting methods employ different theoretical principles, leading to variations in their underlying models, original data requirements, and data processing approaches, which in turn produce substantially different weight distributions. Therefore, different index systems should select weighting methods appropriate to their own characteristics to ensure that the weight allocation of their indexes aligns relatively well with actual conditions.

2 Common Subjective Weighting Methods

Generally, subjective weighting methods determine weights primarily based on the knowledge, experience, or preferences of decision-makers and experts, comparing the importance of various indexes, assigning weight values, or calculating weights through computation. These methods consider the essence of weights as the quantitative representation of the relative importance of evaluation indexes to evaluation objectives. Such methods exhibit relatively strong subjectivity and randomness, but the ranking of index weight magnitudes generally aligns with the actual situation of evaluation objects. Currently, commonly used subjective weighting methods can be classified into four categories: expert estimation method, analytic hierarchy process, binomial coefficient method, and chain ratio scoring method.

2.1 Expert Estimation Method

The expert estimation method involves domain experts subjectively judging the importance of each index based on their experience and knowledge. The final weight allocation for indexes can be derived either by averaging the weights independently assigned by K experts [1], or by using frequency statistics to determine weights—i.e., grouping the K weight assignments for each index according to certain class intervals, calculating the frequency of weights in each group, and taking the midpoint of the group with the maximum frequency as the final weight value for the corresponding index [2].

As early as 1986, F. Shands et al. [3] applied the expert estimation method to determine index weights in a teacher performance evaluation system, submitting evaluation indexes to several domain experts through questionnaires for two rounds of scoring to assign weights. The resulting performance evaluation model achieved good results in practical school applications. In recent years, Chinese scholars Liu Lu et al. [4] also employed this method to determine index weights in heating system energy-saving evaluations, calculating the average weight coefficients provided by multiple domain experts to determine each index weight. When compared with comprehensive evaluation results from analytic hierarchy process and coefficient of variation methods, the expert estimation method showed the best alignment with actual system operation conditions, primarily because heating source systems involve many practical influencing factors during operation, which experts consider more comprehensively to allocate weights more reasonably.

The expert estimation method offers three main advantages: First, it fully utilizes expert experience and knowledge, enabling comprehensive consideration of various external influencing factors based on expert experience, resulting in high method reliability. Second, weight calculation relies primarily on traditional descriptive statistics, such as solving for means and statistical frequencies, making it straightforward and direct. Third, it is not constrained by the availability of sample data and can make probabilistic estimates for numerous non-technical indexes that cannot be quantitatively analyzed. However, this method also has certain limitations: First, weight allocation is completely influenced by expert experience and knowledge, and different expert compositions may produce different evaluation results, exhibiting a high degree of subjective randomness. Second, when there are many indexes, it is difficult to ensure consistency in the judgment thinking process, making it challenging to guarantee complete objectivity and rationality. Overall, this method has broad applicability and is suitable for various evaluation systems with a moderate number of indexes, particularly for practical problems lacking sample data and difficult to model mathematically, where it proves quite effective.

2.2 Analytic Hierarchy Process

The fundamental idea of the analytic hierarchy process is to decompose the indexes of a complex problem into several ordered hierarchical structures according to their interdependent relationships. Within each level, domain experts conduct pairwise comparisons of indexes based on a certain ratio scale, quantifying subjective judgments to form judgment matrices. Mathematical methods are then used to calculate the weight values of each index in the level relative to the upper level, followed by hierarchical total ranking to compute the weight coefficients of all indexes relative to the overall objective [5]. Currently, there are over 20 methods for calculating index weight coefficients from judgment matrices, including eigenvector method, least squares method, root method, and linear programming method, with different methods producing certain differences in index weight ranking.

The analytic hierarchy process is widely applied in determining index weights for various evaluation systems. For example, as early as the last century, R. Shen et al. [6] used this method to evaluate labor intensity in industry, synthesizing quantified judgments on index relative importance from dozens of domain experts to obtain judgment matrices, then using the eigenvector method to calculate index weights, which passed consistency tests. The resulting evaluation model achieved good results in practical applications. Chinese scholar Chu Chunkun et al. [7] applied it to a three-level evaluation index system for university library subject service models dominated by qualitative indexes, also achieving favorable evaluation outcomes.

The analytic hierarchy process offers three main advantages: First, it transforms qualitative judgments based on decision-makers' subjective experience and knowledge into quantitative form, organically combining qualitative and quantitative analysis to fully leverage both strengths—incorporating decision-makers' logical judgment and theoretical analysis on one hand, while employing objective deduction and precise calculation on the other, making the decision-making process highly scientific and thus producing highly credible results. Second, it hierarchically decomposes complex evaluation problems into hierarchical structures, making complex problem evaluation clearer, more explicit, and hierarchical. Third, it is not constrained by the availability of sample data and can solve practical problems that traditional optimization techniques cannot handle.

However, this method also has limitations: First, index weight determination primarily depends on expert experience and knowledge, and different expert selections may lead to variations in weight allocation results, exhibiting subjectivity, randomness, and uncertainty. Second, the judgment matrices in analytic hierarchy process are prone to serious inconsistency issues. When there are many indexes at the same level and the nine-level ratio scale method is difficult to master accurately, decision-makers can easily make contradictory and confused relative importance judgments. To address this issue, Ma Nongle et al. [8] proposed using a three-level scale method instead of the nine-level scale method to construct judgment matrices, making it easier to measure the importance degree between indexes and eliminating the need for consistency tests. However, this approach results in more concentrated weight distribution among indexes, making it difficult to distinguish weights for multiple indexes. Overall, the analytic hierarchy process has broad applicability, particularly suitable for evaluation systems with complex evaluation objectives, lacking sample data, and where domain experts have relatively clear understanding of the degree of relative importance among indexes with a moderate number of indexes.

2.3 Binomial Coefficient Method

The binomial coefficient method [9] involves K experts independently conducting pairwise comparisons of the importance of n indexes. Through repeated cyclic ratio comparisons and statistical processing, index values representing priority order are obtained. Based on these values, indexes are arranged from the middle outward to form an index priority sequence. The indexes in the sequence are then renumbered from left to right as 1, 2, …, l, …, n. According to the principle of binomial coefficients, the weight allocation for the i-th index is calculated as $w_i = C_{n-1}^{i-1}/2^{n-1}$.

The binomial coefficient method for index weight determination was initially proposed by Chinese scholar Cheng Mingxi [9] in 1983 and subsequently gained relatively wide application domestically. For example, early on, Zhao Shuli [10] applied this method to multi-index evaluation of laboratory equipment investment, first determining index priorities based on the average scores of several experts for each index, then calculating weights using binomial expansion coefficients. The resulting optimal evaluation outcome provided reliable basis for university decision-makers in equipment investment. In recent years, Liu Fuqiang et al. [11] used it to determine index weights for factors influencing excavation schedules in pumped-storage projects. Due to the numerous influencing factors in pumped-storage projects that are difficult to subjectively quantify relative weights, the method relied on domain experts to directly judge index priorities, then used binomial coefficients to calculate weights. The final evaluation results for influencing factors were consistent with those obtained using the entropy weight method.

The binomial coefficient method has four main advantages: First, it organically combines qualitative analysis with quantitative calculation, quantifying subjective experience and knowledge to increase the scientific nature and systematic organization of the evaluation process. Second, it does not require specific quantification of the degree of index importance, only needing to judge the relative magnitude between indexes, making expert judgment relatively easy and avoiding contradictory and confused judgments. Third, it employs binomial expansion for weight calculation, making the method simple and easy to operate. Fourth, it is not constrained by the availability of sample data and can solve practical problems that traditional optimization techniques cannot handle. However, the method also has certain limitations: First, weight determination primarily depends on subjective judgments based on expert experience and knowledge, exhibiting randomness and uncertainty. Second, when using binomial coefficient formulas to calculate weights for indexes of different priorities, situations may arise where weights are identical—indexes symmetrically positioned in the priority sequence will have the same calculated weight value, which may deviate from actual conditions. Third, the method only focuses on the ordinal ranking of index importance, not the degree of difference in relative importance between indexes, leading to potential deviations in weight allocation. Overall, this method has no restrictions on sample data availability and broad applicability, particularly suitable for multi-factor evaluation problems lacking precedents and quantitative weighting experience, with a moderate number of indexes.

2.4 Chain Ratio Scoring Method

The chain ratio scoring method [12] involves experts using their experience and knowledge to compare each index with its adjacent next index in terms of importance, comprehensively determining the importance ratio between adjacent indexes based on judgments from multiple experts. Using the last index as a benchmark, the comparative weights of each index are calculated in reverse order, followed by normalization processing to obtain index weights.

The chain ratio scoring method was first proposed by Chinese scholar Lu Mingsheng [12] in 1986 and has since gained relatively wide application both domestically and internationally. For example, Chen Zhigang et al. [13] applied this method to evaluate Shanghai's innovative city development stage, relying on experts to determine the chain ratio values between evaluation indexes, then performing correction and normalization to obtain index weights. The final evaluation results aligned with Shanghai's actual development status at that time. J. Xie et al. [14] also employed this method when evaluating highway emergency plans, having experts conduct top-down pairwise comparisons of indexes to determine their importance degrees, followed by benchmarking and normalization to obtain weights. The evaluation results matched real-world choices, demonstrating the method's effectiveness.

The chain ratio scoring method offers four main advantages: First, it organically combines qualitative judgment with quantitative calculation, making the evaluation process more systematic and scientific. Second, experts need to determine relatively few index importance evaluation values, making the assignment process relatively simple. Third, by sequentially determining relative importance degrees in one direction, it is not prone to judgment contradictions and does not require consistency tests as in analytic hierarchy process, effectively solving complex decision-making problems. Fourth, it is not constrained by the availability of sample data and can solve practical problems that traditional optimization techniques cannot handle. However, the method also has certain limitations: First, it demands high-level expert knowledge, requiring experts to have clear understanding of the importance of evaluation indexes and the ability to make precise quantitative comparisons between each adjacent pair; otherwise, the entire index system's weight allocation can easily produce significant deviations. Second, weight determination primarily depends on subjective experience and knowledge, exhibiting considerable uncertainty and subjective randomness. Overall, this method has no restrictions on sample data availability and broad applicability, particularly suitable for various evaluation problems where relatively accurate quantitative judgments can be made on the relative importance of adjacent evaluation indexes.

---

3 Common Objective Weighting Methods

Objective weighting methods rely on specific mathematical theories and determine index weights entirely through quantitative analysis of actual index data, ensuring absolute objectivity of weights but imposing high requirements on sample data. However, objective weighting methods ignore subjective information such as human experience, potentially producing weight allocation results that contradict actual conditions, and they depend on specific business domains, lacking universality. Currently, major objective weighting methods include: coefficient of variation method, multivariate statistical methods based on principal component analysis and factor analysis, vector similarity method, grey relational analysis method, entropy method, rough set method, and neural network method.

3.1 Coefficient of Variation Method

The principle of the coefficient of variation method is to determine index weights by calculating the degree of variation in each index's measured data. The larger the variation in an index's data, the greater its ability to distinguish evaluation objects, and thus the larger its weight allocation value [15]. The mathematical theory underlying weight determination by coefficient of variation method primarily includes standard deviation and deviation maximization—calculating the standard deviation (maximum deviation) of each index's internal data and performing normalization to obtain index weight allocation.

The coefficient of variation method has been widely applied in weighting index systems. For example, early on, Shi Guangxin et al. [16] used this method to evaluate the benefits of small watershed management, performing dimensionless processing of index sample data and calculating data standard deviations, followed by normalization to obtain index weights. The evaluation results derived from this model objectively reflected actual conditions. In recent years, H. Zheng et al. [17] applied it to determine weights for wind farm economic operation evaluation indexes, first performing consistency and dimensionless processing on large amounts of sample data from actual wind farm operation and monitoring conditions over more than a decade, then calculating standard deviations of each index's data to determine weights. Through comparative evaluation of three types of wind farms, they verified the effectiveness of the evaluation system.

The coefficient of variation method offers three advantages: First, its calculation method is relatively simple and convenient. Second, it fully utilizes sample data, objectively reflecting the discriminative power of each index and ensuring absolute objectivity of index weights. Third, the method imposes no restrictions on the number of evaluation indexes, providing broad applicability. However, the method also has certain limitations: First, evaluation results are highly correlated with sample selection; different data samples may produce different weight allocation results, and when sample size is small and lacks representativeness, the method's precision becomes very low. Second, it cannot handle outliers in sample data—if outliers appear, the method will have significant errors in weight determination. Third, it cannot reflect intrinsic relationships between indexes, only analyzing each index individually. Fourth, by relying purely on objective calculation, it cannot incorporate decision-makers' understanding of index importance and may produce results contrary to actual conditions. Therefore, this method is suitable for comprehensive evaluation systems where evaluation indexes are relatively independent, sample data are universal and relatively complete with large sample sizes, and contain no outliers.

3.2 Multivariate Statistical Methods

Multivariate statistical methods determine index weights by using multivariate statistical analysis to calculate sample data, including principal component analysis and factor analysis.

3.2.1 Principal Component Analysis The fundamental principle of principal component analysis [18] is to use dimensionality reduction to transform a set of correlated indexes into another set of uncorrelated comprehensive indexes—principal components—based on index variance contribution rates, followed by further normalization to obtain each index's weight.

Since its emergence, principal component analysis has been widely applied. For example, early on, Chinese scholar Jin Xingri et al. [19] used this method for comprehensive evaluation of industrial enterprise economic benefits, performing standardization and principal component analysis on sample data of major economic benefit indexes from Yanbian General Factory from 1990-1995, obtaining four principal components and index weights to establish an evaluation model. The final evaluation results were consistent with those from the ideal solution method. In recent years, B. Prado et al. [20] used this method when evaluating climate variables in the German city of Minas, performing principal component analysis on relevant sample data from 2008-2012 to obtain an evaluation model where one principal component explained the overall variables, with evaluation results matching actual conditions.

Principal component analysis offers three main advantages: First, it replaces numerous correlated indexes with fewer independent indexes, solving the problem of information overlap between indexes and simplifying index structure. Second, index weights are determined based on objective data through variance contribution rates of each principal component, avoiding subjective factors and being relatively objective and reasonable. Third, it imposes no specific restrictions on the number of indexes or samples, providing broad applicability. However, the method has four limitations: First, the weight calculation process is relatively complex, and results are highly correlated with sample selection. Second, it loses some sample data information—some indexes with practical significance may be eliminated, deviating from actual conditions. Third, it assumes linear relationships between indexes, but many real-world index systems with nonlinear relationships will produce deviations when using this method. Fourth, by purely relying on objective data for weight determination and ignoring subjective experience and knowledge, evaluation results may contradict actual conditions. Overall, principal component analysis is suitable for complex evaluation problems with relatively complete and representative sample data, where certain correlations exist between indexes.

3.2.2 Factor Analysis The basic idea of factor analysis [21] is similar to principal component analysis, also transforming correlated indexes into a few uncorrelated indexes, then determining index weights based on variance contribution rates of each factor. The difference is that principal component analysis linearly combines original indexes, while factor analysis decomposes original indexes into common factors shared by all indexes and special factors unique to each index, making factor representation have more explicit practical significance.

Factor analysis is widely applied in various comprehensive evaluation problems, especially in evaluating socio-economic phenomena. For example, early on, Chinese scholar Wan Jianqiang et al. [22] applied this method to evaluate listed company operating performance, performing standardization and factor analysis on 1999 annual report data from 13 representative listed companies in the building materials industry, using the first four mutually independent comprehensive factors to replace the original 11 indexes for comprehensive evaluation, with results consistent with actual economic rankings of each enterprise. In recent years, A. Bai et al. [23] used it for national economic ranking assessment, applying factor analysis to 15 economic indicators from 20 countries using IMF datasets, using three comprehensive factors to explain and represent all indexes for comprehensive evaluation. The final calculated rankings were almost identical to world rankings, confirming the method's feasibility.

The advantages and disadvantages of factor analysis in index weight determination are similar to those of principal component analysis. However, since the number of factors is smaller than the number of original indexes (whereas the number of principal components can equal the number of original indexes), factor analysis generally loses more information than principal component analysis, and its precision is typically lower than principal component analysis. The calculation process is also more complex, and factor analysis strictly requires correlations between indexes in the evaluation system. Nevertheless, factor analysis can explicitly explain the specific content of original indexes and the reasons for correlations between indexes, enabling deeper understanding of index content. Overall, factor analysis is more suitable for complex evaluation problems requiring deeper analysis of socio-economic phenomena and related evaluation objects, with large amounts of representative complete data samples and significant correlations between indexes.

3.3 Vector Similarity Method

The basic principle of the vector similarity method [24] is to use each index's sample data to form feature vectors, then solve for similarity with a reference vector composed of ideal values for all indexes. The magnitude of vector similarity reflects each index's contribution to the system achieving optimal performance, which is then normalized to obtain index weights.

The vector similarity method for index weight determination was initially proposed by Chinese scholar Jiao Liming et al. [24] and subsequently applied by many domestic scholars in comprehensive evaluations. For example, Jiao Liming et al. [25] used this method to evaluate the effectiveness of air defense brigade systems, extracting six representative sample data sets, performing dimensionless processing, calculating similarity between each index's data vector and the standardized ideal reference vector, and normalizing to obtain index weights for effective system effectiveness evaluation. Xie Ping et al. [26] used it to evaluate lake eutrophication, taking measured water quality data from 30 lakes nationwide as samples, performing dimensionless processing on vectors composed of each evaluation index's sample data and ideal reference vectors for each level, calculating vector similarity, normalizing to obtain index weights, with evaluation results highly consistent with previous evaluations using fuzzy methods and stochastic methods.

The vector similarity method offers three main advantages: First, calculation is simple and easy to operate. By using the ideal reference vector composed of all indexes (which becomes a unit vector with all element values equal to 1 after dimensionless processing), it cleverly reduces computational steps. Second, results are easy to understand, as the method considers the relationship with the optimal solution and has strong practical applicability. Third, it fully utilizes sample data without human factor interference, ensuring strong objectivity. Fourth, it imposes no specific restrictions on the number of indexes or samples, providing broad applicability. However, the method also has certain limitations: First, the method's precision is affected by data samples—when sample size is small and unrepresentative, the method's evaluation precision is very low. Second, it cannot solve the problem of information repetition caused by correlations between indexes, easily resulting in inflated weights for some correlated indexes due to repeated calculation. Third, by purely using objective data for weight calculation, it ignores subjective experience and knowledge, potentially producing results contrary to actual conditions. Overall, this method has broad applicability and is more suitable for comprehensive evaluation systems with relatively complete, representative, and typical sample data, and relatively independent indexes.

3.4 Grey Relational Analysis Method

The basic idea of grey relational analysis [27] is to combine data comparison with geometric curve trend changes to calculate weights, using the magnitude of relational degree between each scheme and the optimal scheme to determine index weights. Specifically, the method calculates each index's contribution to the entire system achieving optimal performance through grey relational judgment matrices and relational coefficients between each scheme and the ideal scheme, then normalizes to obtain index weights.

Grey relational analysis is widely applied in practical decision-making problems across many disciplines. As early as the 1980s, Chinese scholar Ma Zhiying et al. [28] applied this method to cotton variety evaluation, using 1986 cotton variety characteristic data from the Yellow River cotton disease resistance region as samples, performing dimensionless processing, calculating grey relational coefficients between each variety and the ideal variety, normalizing to obtain index weights, with final evaluation results consistent with fuzzy comprehensive evaluation results. Later, C. Ho [29] applied it to bank operational performance evaluation, using financial documents from three Taiwanese banks as data samples, following similar steps to calculate index weights, and establishing an evaluation model to assess the three banks, with results consistent with financial statement analysis.

Grey relational analysis offers four main advantages: First, the calculation process is relatively simple, considering the relationship with the ideal decision scheme, with intuitive and understandable results. Second, it does not require large samples—only a small amount of representative data is needed, with no restrictions on the number of indexes. Third, the method has certain fault tolerance because relational degree calculation uses the two-pole maximum and minimum differences, weakening inaccuracies caused by partial data missing or human errors, making analysis results relatively reasonable. Fourth, it relies on sample data for weight calculation, avoiding human factor interference and ensuring strong objectivity. The method also has certain limitations: First, the discrimination coefficient value in grey relational coefficient calculation is subjectively determined by humans without fixed standards, and different values affect final weight allocation, reducing credibility. Second, the method's accuracy is affected by samples—different sample selections may lead to different final evaluation results. Third, it cannot solve the problem of information overlap caused by correlations between indexes. Fourth, without considering subjective experience and knowledge, evaluation results may contradict actual conditions. Overall, this method has broad applicability, no restrictions on index or sample numbers, and is more suitable for comprehensive evaluation systems with relatively large sample sizes, complete sample data information, universality, and relatively independent indexes.

3.5 Entropy Method

The basic idea of the entropy method [30] is to reflect the degree of distinction of indexes to evaluation objects from the perspective of index disorder—i.e., index entropy. The smaller an index's entropy value, the more ordered its sample data, the greater the differences among sample data, the stronger its ability to distinguish evaluation objects, and the larger its corresponding weight. This method first calculates each index's entropy value using an entropy function, then normalizes the entropy values into index weights.

Since its proposal, the entropy method has been widely applied in evaluating problems across many fields. For example, early on, Zhu Shunquan et al. [31] used this method to evaluate the financial status of listed companies, taking 15 evaluation index data from 20 companies in the "China Securities Journal" for 2000 as samples, performing dimensionless processing, calculating each index's entropy value, and normalizing to obtain index weights. The comprehensive evaluation values obtained through simple weighting were reasonable. Later, A. Gorgij et al. [32] used it in groundwater quality assessment, evaluating 21 groundwater samples from the Azarshahr Plain in Iran in 2016 through similar entropy method calculation steps to determine index weights and further using a comprehensive evaluation model to determine water quality grades for each sample, with results consistent with spatial autocorrelation coefficient evaluations.

The entropy method has three advantages in determining index weights: First, the calculation process is relatively simple, determining index weights from the perspective of index distinction degree to evaluation objects, with intuitive and understandable results and strong practical applicability. Second, it completely relies on sample data for weight calculation, avoiding subjective factor interference and ensuring strong objectivity. Third, the method imposes no restrictions on the number of indexes, providing broad applicability. However, the method also has certain limitations: First, its precision is affected by data samples—

different sample selections may produce different weight allocation results, and it has high requirements for sample data completeness and sample size. Second, it cannot reflect correlations between indexes or solve information overlap problems. Third, it cannot incorporate decision-makers' understanding of index importance and may produce results contrary to actual conditions. Therefore, this method is suitable for comprehensive evaluation systems with relatively large sample sizes, complete and representative sample data, and relatively independent indexes.

3.6 Rough Set Method

The basic idea of using rough set method [33] for index weight determination is to first classify evaluation objects according to all indexes in the system, then remove one index at a time and consider the degree of change in object classification compared with the original classification. Index importance is proportional to the degree of change. The concept of attribute (index) importance in rough sets can be summarized into algebraic representation definitions and information representation definitions. Further, various detailed weighting methods are derived based on equivalence relations, dominance relations, and tolerance relations among indexes. The differences among rough set weighting methods based on equivalence, dominance, and tolerance relations mainly lie in the criteria for object classification: equivalence relations divide all evaluation objects according to equivalence thinking, dominance relations divide objects according to their dominance degree on condition attribute sets, and tolerance relations divide objects according to differences between objects. Equivalence relations require discrete data, dominance relations can handle continuous data, and tolerance relations are mainly used for handling missing sample data.

Rough set method is widely applied in determining index weights for various evaluation problems. For example, W. C. Ip et al. [34] used it for water quality evaluation, taking water quality measured data from the Han River basin in April-May 1992-1997 as condition attribute set samples and using equal-weight aggregated averages of each index value across time periods as decision attributes. After discretizing sample data, they calculated index importance using algebraic rough set methods based on equivalence relations and normalized to obtain index weights. The final comprehensive water quality evaluation grades for each time period matched actual conditions. Zou Bin et al. [35] used it to evaluate energy consumption in East China, taking energy consumption data for 8 East China provinces (municipalities) from the 2011 China Energy Statistical Yearbook as samples, performing dominance class division on attribute sets (without requiring sample data discretization), and calculating index weights based on algebraic attribute importance definitions. The evaluation results were relatively consistent with those from grey relational analysis weighting.

The advantages of rough set method in index weighting are mainly reflected in five aspects: First, it can handle extensive data types, including discrete and continuous data, with effective processing of continuous data reducing information loss problems caused by data discretization. Second, it has strong fault tolerance and can handle missing sample data, effectively solving weighting problems under data missing conditions. Third, information representation importance can compensate for the defect that index weight values may be zero in algebraic representation importance, improving method precision. Fourth, the method imposes no restrictions on the number of indexes, providing broad applicability. Fifth, it completely relies on objective data for weight calculation, avoiding subjective factor interference. However, the method also has certain limitations: First, its precision is affected by data samples—different sample selections may produce different decision results. Second, it cannot solve the problem of information overlap caused by correlations between indexes. Third, as an objective weighting method without requiring prior information, calculation results may contradict decision-makers' understanding. Overall, rough set method has the widest application scope among objective weighting methods, suitable for multi-attribute decision-making problems with relatively universal and large-volume data samples and relatively independent indexes.

3.7 Neural Network Method

The most commonly used neural network method is the BP neural network algorithm. The basic idea for weighting [36] is to perform nonlinear parallel learning training on large amounts of data samples according to certain learning mechanisms. Specifically, it involves continuous iterative adjustment based on error precision requirements for the difference between output data and known sample output data to obtain connection weight matrices from input layer to hidden layer that meet requirements. The absolute values of connection weights from each input layer node to all hidden layer nodes are summed and normalized to obtain index weights.

Neural network methods are increasingly used for index weight determination. For example, early on, J. Ch et al. [37] used this method for e-government website evaluation, taking relevant data from 20 e-government websites in Ningbo as training samples (including 11 evaluation indexes and expert evaluation results), training to calculate index weights, and applying the evaluation model to evaluate another 10 e-government websites, with results relatively consistent with expert assessments. Later, S. Silva et al. [38] applied it to evaluate extra virgin olive oil stability, with training samples consisting of 18 types of extra virgin olive oil stored under dark and light conditions (including 11 evaluation indexes and experimental data). After training to calculate index weights, the evaluation model was applied to 10 groups of olive oil stability assessments outside the training samples, with assessment results showing over 90% consistency with experimentally displayed test classifications, indicating high accuracy.

Neural network methods have three main advantages: First, they can handle nonlinear complex system evaluation problems and perform dynamic evaluation, with strong processing capabilities for system analysis. Second, through learning and training from samples, they can obtain relative importance information that is reasonable, scientific, and practically validated, ensuring objectivity and practicality of index weights. Third, they impose no restrictions on the number of evaluation indexes, providing broad application scope. However, the method also has certain limitations: First, being completely based on training samples, it has high requirements for samples—training samples must be correct and cover a wide range (emphasizing broad coverage types rather than necessarily large quantity) to ensure reliable results. Second, it largely ignores subjective experience and knowledge, and evaluation results may contradict decision-makers' subjective preferences. Overall, neural network methods have strong processing capabilities for complex system evaluation problems and are more suitable for determining index weights in various complex systems with relatively large sample data volumes, broad coverage, and relatively independent indexes.

4 Common Integrated Weighting Methods

Integrated weighting methods are comprehensive approaches that combine subjective and objective weighting methods according to different preference coefficients to determine index weights. Based on the information representation of expert experience and decision-makers' subjective intentions in subjective weighting methods, and the information representation of intrinsic relationships between indexes and evaluation objects in objective weighting methods, integrated weighting methods effectively combine both through certain mathematical operations to achieve complementary advantages. Currently, integrated weighting methods based on different principles have various forms, but they can be broadly categorized into four types: integrated weighting methods based on additive or multiplicative synthesis normalization, integrated weighting methods based on deviation square sum, integrated weighting methods based on game theory, and integrated weighting methods based on target optimization.

Integrated weighting methods based on additive or multiplicative synthesis normalization directly add or multiply the index weights obtained from subjective and objective weighting methods with equal preference, followed by normalization to obtain comprehensive weights for each index. For example, L. Yang et al. [39] used this method in supply chain risk assessment, equally weighting and multiplying the subjective weights determined by analytic hierarchy process with the objective weights determined by coefficient of variation method, then normalizing to obtain comprehensive index weights, achieving high evaluation accuracy.

Integrated weighting methods based on deviation square sum solve for subjective and objective weight allocation coefficients that maximize the total deviation square sum among comprehensive evaluation values of decision schemes—i.e., making comprehensive evaluation values of schemes as dispersed as possible—from the perspective of scheme distinguishability. For example, B. Meng et al. [40] used this method in bank credit risk assessment, calculating optimal adjustment coefficients for subjective and objective weights based on the principle of maximizing overall deviation square sum of evaluation values for different objects from both weighting methods, normalizing to obtain comprehensive index weights, with evaluation results showing higher accuracy compared to single subjective or objective weighting methods.

Integrated weighting methods based on game theory seek compromise or consistency between different subjective and objective weights, maintaining original information from both as much as possible, and solving for weight allocation coefficients that minimize deviation from subjective and objective weights. For example, C. Lu et al. [41] used this method's philosophy in evaluating educational informatization development levels, calculating and allocating weight coefficients between index subjective weights determined by analytic hierarchy process and objective weights determined by coefficient of variation method, normalizing to obtain comprehensive integrated weights. This model achieved good results in evaluating educational informatization development levels in 11 regions of Suzhou.

Integrated weighting methods based on target optimization solve for subjective and objective weight coefficients based on the principle of optimal comprehensive decision results, including two specific solution methods: maximizing comprehensive target values and maximizing deviation from negative ideal solutions. For example, J. Yan et al. [42] used this method in learning city evaluation, calculating allocation weights for index subjective weights obtained through expert estimation and objective weights obtained through entropy method based on the principle of maximizing overall comprehensive evaluation values, and used this model to accurately evaluate four learning cities.

Overall, various integrated weighting methods have certain theoretical foundations and are solved through mathematical concepts such as linear equations and matrix operations. Some methods have simple integration calculations, while others bring greater computational load to the evaluation process. However, there is no absolute superiority among methods, and currently there is no consistent conclusion on which integrated weighting method to choose for which evaluation problem. Compared with single subjective or objective weighting methods, integrated weighting methods produce relatively more scientific and reasonable evaluation results, but they may also have significant random deviations leading to results that do not match actual conditions. They cannot completely replace single weighting methods, and a rational understanding is needed when selecting weighting methods in practical problem research.

---

5 Conclusion

This paper focuses on analyzing the basic ideas and principles of specific weight calculation methods among common subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods, comparing their advantages and disadvantages, and clarifying their respective applicable scopes. The study finds that each weighting method considers problems from different perspectives, each having certain advantages and defects in weight calculation, with correspondingly large differences in applicable scopes. Therefore, when selecting weighting methods for multi-factor evaluation indexes, it is necessary to rationally understand and grasp the advantages and disadvantages of each method and conduct specific analysis for specific problems. The appropriate weighting method should be selected according to the actual characteristics of the evaluation object and problem, such as whether sample data are available, whether samples are representative, and whether correlations exist between indexes, to ensure relative scientificity and rationality of evaluation results.

References

[1] Xie Jijian, Liu Chengping. Fuzzy mathematics methods and their applications [M]. Wuhan: Huazhong University of Science and Technology Press, 2006: 149-179.
[2] Zhen Changqing. Weight determination methods in multi-objective decision-making [J]. Systems Engineering—Theory & Practice, 1987, 7(2): 16-24.
[3] SHANDS F, LEVARY R R. Weighting the importance of various teacher behaviors by use of the delphi method[J]. Education, 1986, 106(3): 306-320.
[4] Liu Lu, Dai Xin. Research on weight determination methods for heating system energy-saving evaluation indexes [J]. Building Energy Efficiency, 2015, 43(7): 112-115.
[5] Shi Zhenwu, Zhao Min. Using analytic hierarchy process to determine index weights [J]. Science Technology and Industry, 2008, 8(2): 23-25.
[6] SHEN R, MENG X, YU Y. Analytic hierarchy process applied to synthetically evaluate the labour intensity of jobs[J]. Ergonmics, 1990, 33(7): 867-874.
[7] Chu Chunkun, Sun Siqin, Han Fengtan. University library subject service evaluation model based on analytic hierarchy process [J]. Journal of Academic Libraries, 2014(6): 86-90.
[8] Ma Nongle, Zhao Zhongji. Analysis of determining weight coefficients based on analytic hierarchy process and its improvements [J]. Water Conservancy Science and Technology and Economy, 2006, 12(11): 732-733, 736.
[9] Cheng Mingxi. Binomial coefficient weighted sum method for handling multi-objective decision problems [J]. Systems Engineering—Theory & Practice, 1983(4): 23-26.
[10] Zhao Shuli. Calculation method for weights in laboratory equipment investment evaluation [J]. Petroleum Education, 1994, 33(S1): 8-12.
[11] Liu Fuqiang, Lu Zhihang, Lü Chengxin, et al. Analysis of influencing factors for pumped-storage project excavation schedule based on combined weighting method [J]. Hydropower and New Energy, 2017(6): 40-43, 72.
[12] Lu Mingsheng. Weight coefficients in multi-objective decision-making [J].

Systems Engineering—Theory & Practice, 1986(4): 77-78.
[13] Chen Zhigang, Zhou Dan. Evaluation of Shanghai's innovative city stage based on chain ratio multiplication scoring method [J]. Consume Guide, 2008(16): 175.
[14] XIE J, QIN Y, M X, et al. The fuzzy comprehensive evaluation of the highway emergency plan based on G1 method[C]//3rd IEEE international conference on computer science and information technology (ICCSIT). Chengdu: IEEE, 2010: 313-316.
[15] Xue Huiqin. Research on index weight determination methods in multi-attribute decision-making [D]. Lanzhou: Northwest Normal University, 2008: 10-43.
[16] Shi Guangxin, Wang Qichang, Liu Jianqiang. Application of coefficient of variation method in small watershed management benefit evaluation [J]. Bulletin of Soil and Water Conservation, 2000, 20(6): 47.
[17] ZHENG H P, XUE M, HAN Y, et al. Application of the variation coefficient method to comprehensive evaluation of wind farms[J/OL]. Applied mechanics & materials, 2014[2017-06-25]. https://www.scientific.net/AMM.488-489.489.256.
[18] Su Weihua. Research on multi-index comprehensive evaluation theory and methods [D]. Xiamen: Xiamen University, 2000: 12-38.
[19] Jin Xingri, Yin Xijie, Xu Hunan. Application of principal component analysis in comprehensive evaluation of industrial enterprise economic benefits [J]. Application of Statistics and Management, 1997, 16(6): 15-18.
[20] PRADO B Q D, FERNANDES H R, ARAUJO T G, et al. Evaluation of climatological variables in Uberlandia (MG) by principal component analysis[J]. Engenharia sanitaria e ambiental, 1990, 21(2): 407-413.
[21] HARTE J M, KOELE P, ENGELEN B V. Estimation of attribute weights in a multi-attribute choice situation[J]. Acta psychological, 1996, 93(1/3): 37-55.
[22] Wan Jianqiang, Wen Zhou. Application of factor analysis in listed company operating performance evaluation [J]. Forecasting, 2001, 20(5): 39-42.
[23] BAI A, HIRA S, DESHPANDE P S. An application of factor analysis in the evaluation of country economic rank[C]//11th international conference on data mining and warehousing(ICDMW). Bangalore: Elsevier, 2015: 311-318.
[24] Jiao Liming, Yang Jianli. A new method for determining index weights [J]. Command Control & Simulation, 2006, 28(1): 94-97, 101.
[25] Jiao Liming, Feng Shili, Deng Changjiang. C~3I system effectiveness evaluation based on vector similarity weighting method [J]. Fire Control & Command Control, 2005, 30(S1): 49-51.
[26] Xie Ping, Xiao Chan, Lei Hongfu, et al. Lake eutrophication evaluation method based on vector similarity principle and its verification [J]. Journal of Safety and Environment, 2008, 8(4): 93-96.
[27] Zhou Bin. Objective multi-objective decision-making method for weight determination by grey relational degree [J]. Journal of Kunming University of Science and Technology (Science and Technology), 2003, 28(5): 159-161.
[28] Ma Zhiying, Shi Shuyan. Preliminary application of grey relational analysis in cotton variety evaluation [J]. Journal of Hebei Agricultural University,

1987(4): 121-128.
[29] HO C T. Measuring bank operations performance: an approach based on grey relation analysis[J]. Journal of the Operational Research Society, 2006, 57(4): 337-349.
[30] Zhang Wenquan, Zhang Shiying, Jiang Liqin. Decision evaluation model based on entropy and its application [J]. Journal of Systems Engineering, 1995, 10(3): 69-74.
[31] Zhu Shunquan, Zhang Yaoting. Entropy-based fuzzy comprehensive evaluation model for listed company financial status [J]. Journal of Shanxi University of Finance and Economics, 2002(5): 101-103.
[32] GORGIJ A D, KISI O, MOGHADDAM A A, et al. Groundwater quality ranking for drinking purposes, using the entropy method and the spatial autocorrelation index[J]. Environmental earth sciences, 2017, 76(7): 261-270.
[33] Zhong Jiaming, Li Dingfang. Research on optimization method for attribute weight determination based on rough set theory [J]. Computer Engineering and Applications, 2008, 44(20): 51-53.
[34] IP W C, HU B Q, WONG H, et al. Applications of rough set theory to river environment quality evaluation in China[J]. Water resourcess, 2007, 34(4): 459-470.
[35] Zou Bin, Li Xia, Wen Jiawu. Energy consumption evaluation analysis based on dominance relation grey rough set [J]. Natural Science Journal of Harbin Normal University, 2013, 29(1): 23-26.
[36] Sun Huijun, Wang Xinhua. Using artificial neural network to determine evaluation index weights [J]. Journal of Shandong University of Science and Technology (Natural Science), 2001, 20(3): 78-79.
[37] JU C H, LIANG Y, LIU D S. A kind of E-government website evaluation method based on neural network[C]//IEEE international conference on e-business engineering. Shanghai: IEEE, 2006: 407-413.
[38] SILVA S F, ANJOS C A, CAVALCANTI R N, et al. Evaluation of extra virgin olive oil stability by artificial neural network[J]. Food chemistry, 2015, 179(7): 35-43.
[39] YANG L Q, LIANG X D, DENG FM, et al. Risk assessment of supply chain based on combination weighting method and grey theory[C]//IEEE international conference on industrial engineering and engineering management(IEEM). Singapore: IEEE, 2015: 895-899.
[40] LU C, LI L, WU D. Application of combination weighting method to weight calculation in performance evaluation of ICT[C]//15th IEEE international conference on advanced learning Technologies(ICALT). Hualien: IEEE, 2015: 258-260.
[41] MENG B, CHI G T. New combined weighting model based on maximizing the difference in evaluation results and its application[J]. Mathematical problems in engineering, 2015(8): 1-9.
[42] JIANYUAN Y, HAILING G, LEI Z. Research of the evaluation index system of learning city–based on the data of China's four municipalities[C]//Workshop on electrical engineering and automation. Beijing: Springer, 2011: 491-495.

Author Contributions: Liu Qiuyan was responsible for data collection, analysis, and paper writing; Wu Xinnian proposed the paper topic and framework, and revised and improved the paper.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.