Study on Chaotic Characteristics of the One-Dimensional Logistic Model

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Abstract

The one-dimensional Logistic model is ubiquitous in social and natural evolution processes, representing a highly representative class of simple nonlinear models. This work primarily investigates the nonlinear characteristics and chaotic properties of the one-dimensional Logistic model through numerical simulation, and subsequently analyzes the general features of chaos. In the one-dimensional Logistic model, when the environmental parameter assumes different values, the model exhibits a wide variety of markedly different evolutionary behaviors. It transitions from completely predictable periodic behavior to completely unpredictable chaotic behavior, displaying chaotic phenomena such as period-doubling bifurcation and sensitivity to initial conditions.

Full Text

Study on Chaotic Characteristics of the One-Dimensional Logistic Model

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Abstract

One-dimensional Logistic models are widely encountered in both social and natural evolutionary processes, representing a particularly representative class of simple nonlinear models. This work primarily investigates the nonlinear characteristics and chaotic properties of the one-dimensional Logistic model through numerical simulation, thereby analyzing the general features of chaos. In the one-dimensional Logistic model, when environmental parameters take different values, the model exhibits many distinct evolutionary behaviors across generations. It transforms from a completely predictable periodic behavior to a completely unpredictable chaotic behavior, demonstrating chaotic phenomena of period-doubling bifurcation and sensitivity to initial values.

Keywords: Logistic model; Chaotic characteristics; Numerical analysis

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1 Introduction

In nature, the evolution of population numbers over time under constraints of specific space and natural resources often contains extremely disordered evolutionary patterns. Similarly, chaotic situations also exist in social models. This was proposed in a highly influential review published in 1976 by mathematical ecologist R. May in the British journal *Nature*, representing one of the earliest examples of the transition to chaos through period-doubling bifurcation. The Logistic model was originally proposed by mathematical biologist Pierre-Francois Verhulst in 1838 as a world population growth model. Since its inception, its applications have expanded from biological population models of population growth to many fields, widely used in biology, medicine, economics, and management.

Yu et al. [1] obtained sufficient conditions under which all solutions of the equation [2] are related to the fractional-order logistic equation. Oruganti et al. [3] considered a reaction-diffusion equation that models constant-yield harvesting of spatially heterogeneous populations satisfying logistic growth. Reference [4] studied a diffusive logistic equation with a heterogeneous environment and free boundary, formulated to investigate the spread of invasive species, where the free boundary represents the expansion front. The main contribution of Izadi et al. [5] is obtaining approximate solutions for fractional-order nonlinear Logistic equations by developing a collocation method based on fractional-order Bessel and Legendre functions. Population dynamics models, including predator-prey problems and logistic equations, have been generalized using fractional operators for Caputo-Fabrizio derivatives [6]. Other influential works include those by Gopalsamy et al. [7,8], Cambel [9], and Ahmad [10], as well as other studies [11,12]. This paper analyzes in detail the evolutionary characteristics of the Logistic model and its chaotic properties sensitive to initial values, providing a detailed analysis of the dynamic characteristics of the Logistic model as a chaotic model.

2 Introduction to the One-Dimensional Logistic Model

The one-dimensional Logistic equation is a widely used relationship describing insect population models:

$$x(k+1) = a \cdot x(k) \cdot (1 - x(k))$$

where $x(k)$ represents the population quantity in generation k, with $x \in (0, 1)$, and $a \in (0, 4)$ is the environmental parameter describing the survival pressure on the population.

3.1 Numerical Analysis of Different Parameter Values in the One-Dimensional Logistic Model

[Figure 1: see original paper] shows the relationship between population number and population generation under different environmental parameters. Figure 1(a) describes population evolution when the environmental parameter a is in the range (0,1), where the population quantity shows an extinction trend, indicating that the population cannot adapt to the external environment at all or that the external environmental pressure is too great. Figure 1(b) describes the state when the environmental parameter a is in the range (1,2), where the population requires an initial low point before returning to normal, similar to the bacterial growth curve in fermentation engineering. Figure 1(c) describes the evolutionary state of the population when the environmental parameter a is in the range (3, 3.4495), where the population quantity exhibits intense oscillatory decline with increasing generations, specifically manifesting as a periodic variation process between the production and consumption of survival resources required for population reproduction, leading to oscillations in population numbers. Figure 1(d) shows the situation when a is in the range (3.4496, 3.5441), where the population change trend still exhibits oscillations but with longer oscillation periods. Figures 1(e) and 1(f) describe the evolutionary state of the population when the environmental parameter a is in the range (3.5441, 4), where the population quantity exhibits intense oscillations with increasing generations, but these oscillations lose their original periodicity and become irregular and chaotic. In the subsequent bifurcation diagram, we will see that the system is in a completely chaotic state at this point.

3.2 Numerical Analysis of Chaotic Characteristics in the One-Dimensional Logistic Model

[Figure 2: see original paper] depicts the complete process of the system' s transition to chaos, namely the phenomenon of period-doubling bifurcation. The source of the system' s uncertainty also results from these period-doubling bifurcations. In the chaotic region, we can still see windows composed of periodic systems. In the enlarged chaotic region, Figure 2 expresses the detailed characteristics of chaos, namely period-doubling bifurcation.

3.3 Sensitivity Analysis of Initial Values for Chaotic Characteristics in the One-Dimensional Logistic Model

We have previously obtained a description of the chaotic behavior of the one-dimensional Logistic model. We know that chaos generally exhibits sensitivity to initial values. Therefore, we now examine the sensitivity characteristics of the one-dimensional Logistic model through multiple iterations. We take the environmental parameter in the chaotic range $a = 3.8$, which clearly shows the system is in a chaotic state from Figure 2. If several graphs are plotted on the same coordinate axis, we obtain Figure 3.

[Figure 3: see original paper] shows the evolution diagram of the initial-value-sensitive one-dimensional Logistic model with environmental parameter $a = 3.8$. We observe that when the initial values differ very slightly and the number of generations is not very large, the population evolution quantities are identical, with the first few lines almost overlapping. However, as the number of evolutionary generations increases, the overlapping curves begin to separate and evolve along completely different paths, demonstrating obvious sensitivity to initial values. This indicates that there indeed exist essential chaotic properties in the one-dimensional Logistic model.

From the perspective of numerical simulation, we studied the evolutionary behavior in the nonlinear one-dimensional Logistic model. When the environmental parameter increases to a certain extent, there is actually no obvious boundary for the onset of chaotic behavior; instead, the system continuously undergoes period-doubling bifurcations, eventually becoming completely unpredictable chaos. For these chaotic regions, all deterministic ideas are completely inapplicable. However, through analysis of the model' s behavior, we find that even when the system is in a completely chaotic region, ordered regions still exist. In the bifurcation diagram, this appears as several separated blank windows in the chaotic region, where the system' s behavior is similarly predictable and periodic or quasi-periodic. But as the environmental parameter changes, the system' s behavior becomes unpredictable and chaotic again.

In a chaotic region, the system exhibits very obvious sensitivity to initial values. For very small differences, the system will eventually become completely different under sufficiently long evolutionary generations. Very small differences in initial values will lead to very different behaviors, demonstrating the general characteristics of chaos.

The analysis shows that as a representative nonlinear model, the Logistic model has behavioral characteristics completely different from linear models. Nonlinear models have rich evolutionary behaviors and chaotic properties that do not appear in linear systems. The conclusions of this work are primarily obtained through numerical simulation. How to study these characteristics of nonlinear models through analytical means remains a significant challenge. The analytical description of unpredictable chaotic characteristics is also a major theoretical problem that awaits future research.

[1] JS Yu, Bo Zhang, ZC Wang 1994 APPLICABLEANALYSIS, 53,1-2:117-124
[2] Ahmed MA El-Sayed, AEM El-Mesiry, HAA El-Saka 2007 APPL.MATH.LETT. 20,7: 817-823
[3] Shobha Oruganti, Junping Shi, Ratnasingham Shivaji 2002 TRANSACTIONS OF THEAMERICANMATHEMATICALSOCIETY, 354: 3601-3619
[4] Mingxin Wang 2014 arXiv:1404.4565
[5] Mohammad Izadi, HM Srivastava 2021 CHAOS SOLITONS & FRACTALS, 145,
[6] H Jafari, RM Ganji, NS Nkomo, YP Lv 2021 RESULTSINPHYSICS, 27, 104456.
[7] K Gopalsamy, BG Zhang 1988 DYNAMICSANDSTABILITYOFSYSTEMS, 2,3-4:
[8] K Gopalsamy. Stability and Oscillations in Delay Differential Equations of Population Dynamics[M]. 1992, ISBN: 978-94-015-7920-9
[9] Ali Bulent Cambel. Applied Chaos Theory: A Paradigm for Complexity[M].1992, ISBN:0121559408
[10] Shair Ahmad 1993 Proceedings of the American mathematical society, 117, 1
[12] Daqing Jiang, Ningzhong Shi, 2005 JOURNALOFMATHEMATICALANALYSIS ANDAPPLICATIONS, 303,1: 164-172

Note: Figure translations are in progress. See original paper for figures.