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Abstract

The aim of this research is to reconstruct the 3D X-ray refractive index gradient maps by the proposed vector Radon transform and its inverse, assuming that the small-angle deviation condition is met. Theoretical analyses show that the X-ray beam can be modeled as a streamline with continuous change of direction in a row when measured in one grating period, which allows the extraction of the refraction angle signals. Experimental results show that all the 2D refraction signals of different directions can be acquired by a standard circular scanning procedure, which is typically used in the X-ray differential phase-contrast computed tomography. Furthermore, the 3D refractive index gradient maps that contain the directional density changes, can also be accurately reconstructed.

Full Text

X-ray Computed Tomography Reconstruction Algorithm for Refractive Index Gradient

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Abstract

This research aims to reconstruct three-dimensional X-ray refractive index gradient maps through a proposed vector Radon transform and its inverse, under the assumption that the small-angle deviation condition is satisfied. Theoretical analysis demonstrates that when measured within one grating period, the X-ray beam can be modeled as a streamline exhibiting continuous directional change, which enables extraction of refraction angle signals. Experimental results show that all two-dimensional refraction signals from different directions can be acquired using a standard circular scanning procedure typical of X-ray differential phase-contrast computed tomography. Furthermore, the three-dimensional refractive index gradient maps containing directional density changes can be accurately reconstructed.

Keywords: CT reconstruction, Image processing, X-ray imaging

Introduction

Generally, two approaches exist for revealing refractive information of samples from acquired differential phase contrast (DPC) signals: reconstructing the refractive index itself, or reconstructing the gradient of the refractive index. The first approach has been widely discussed due to its consistency with classical CT image reconstruction algorithms [1,2], as evidenced by numerous studies [3-12]. In contrast, the second approach has attracted limited research interest [13,14], primarily because the theoretical foundation for refractive index gradient reconstruction algorithms in X-ray DPC imaging remains underdeveloped. To address this challenge, this paper proposes, for the first time, a new vector Radon and inverse Radon analysis theory that incorporates the X-ray beam small-angle deviation condition. Based on this novel theory, we have rigorously derived an analytical algorithm for three-dimensional refractive index gradient reconstruction.

2 Model and Methodology

The flow chart of the proposed model in this paper can be summarized in Fig.1. The small-angle deviation condition constitutes a crucial foundation for establishing this new theory. The following sections present the details of each process.

Fig.1. The flow chart of the proposed model in this study.

2.1 Optical Streamline Model

According to Maxwell's electromagnetic equations, the Helmholtz equation for scalar monochromatic waves in an inhomogeneous medium can be derived, from which both the Eikonal equation and the continuity equation [15] can be obtained. Assuming an infinitely small wavelength, the differential equation of light rays in geometric optics [15,16] can be expressed as follows: is the arc

length of the beam, $\mathbf{n} = \mathbf{n}$ is the unit vector parallel to the tangent of the beam, and n is the refractive index of the medium is the optical path length, is the position where vector, the ray vector, and \mathbf{r} where shift, Substituting Eq. (2) into Eq. (1), the following results can be obtained: $\Delta\phi$ denotes the decrement of the real portion and is related to the beam phase denotes the imaginary portion and is related to the beam attenuation. $\Delta\phi = -\delta$ In Eq. (3), both the are ignored for the middle term, and the last term due to the fact that whose increment is in effect the increment of the refraction angle of the X-ray beam inside the sample, as illustrated in Fig.2. β , and the ray vector $\beta \delta$ 1 is ignored for reduced to Fig.2. Illustration of the X-ray beam passing through the object when considering the internal small refractions. The denotes the distance between the sample and the detector, and p denotes the dimension of the detector element.

Assuming that a group of parallel X-ray beams penetrates through the object with \mathbf{k} , shown in Fig.2. Based on Eq.(3), the integration of the refraction along a real X-ray beam path (no longer a straight line) is a refractive index gradient expressed as $\frac{d\phi}{ds} = -\delta$ The relation between phase and optical path $\phi = -\delta s$ is substituted into Eq.(4), one gets $\lambda = k$ Alternatively, Eq.(6) can also be rewritten as $\Phi = \Phi - \delta s = -\delta s$ Herein, the standard Cartesian coordinate system is selected with the z axis parallel to the incident direction. Due to the inhomogeneous object refractive index distribution, as illustrated in Fig.2, the exit X-ray beam would be deviated by compared to its primary incident direction. However, in the process of experimental data analysis, it is quite difficult to measure the non-zero angular deviations induced by inhomogeneous object. The reason is that the pixel size of the detector is limited.

For instance, if would be insensitive in probing such tiny beam deviations. In other words, it is difficult for the detector element with finite size to distinguish the refracted X-ray beams from the primary X-ray beams when they are falling within the same resolution unit (suppose the resolution unit size is $2p$). When this happens, essentially, the change of X-ray beam inside the x - y plane should be neglected, the arc length of the s . In order to distinguish the adjacent resolution Δs , the detection system with detector element size of can be reduced to $\Delta s < \delta s$, units, discrete imaging unit according to resolution elements are introduced into the functions of phase gradient and refraction angle, thus equation (7) can be simplified as $\Phi = \Phi - \delta s$ Please note that since the integration of $\phi = -\delta s$, δ is the path integral in front of the detector, inside the integral will not be affected by the detector and is still continuous. inside and outside the s . Thus, by Therefore, continuity and discreteness coexist integral respectively. Once the X-ray beam leaves the object, then letting $\delta = 0$, in Eq.(8), \mathbf{k} , one gets and takes account of the fact $\Phi = \Phi - \delta s = 0$ where can be reduced as are the unit vector along the x , y and z axis respectively, Eq.(8) $\Phi = -k = -\delta s$. Note that δ is defined as the small where angle deviation condition in this work, which solves the contradiction between $\mathbf{k} = e_x / \lambda + e_y / \lambda$ straight line propagation required by Radon transform and directional change propagation

caused by refraction angle. Under this condition, the refraction angle in Eq.(11) is a vector sum (integral) process, as shown in Fig.3. $\delta < \delta$, Fig.3. Diagram of refraction angle formation.

Vector Reconstruction Algorithm for Refractive Index Gradient

Assuming a parallel X-ray CT imaging geometry with the y axis being the axis of sample rotation, the analytical refractive index gradient reconstruction algorithm is discussed. Let be the object coordinate, as a result, Eq.(11) , and at the view angle between can be expressed as the Radon transformation of axis and axis, namely Φ , where operation onto Eq.(12), the is the Dirac pulse function. Afterwards, applying the inverse Radon $= - , = -\infty \sin - \delta ' , ' \cos +$ can be readily reconstructed, $\delta ' , ' \delta ' , ' \delta ' , '$ where are the unit vector along the $' + e ' ' + e ' ' = -$ axis, respectively; and $, , e + , , e \delta \sin - \cos +$ are the component of the refraction angle on the x represent the Fourier and inverse $\delta ' , ' = e ' e ' e '$ and y axis, respectively, the operators $= \Phi k$ Fourier transformation operator along x axis, correspondingly; and variable denotes the frequency counterpart of space variable $e ' = \Phi k$

DPC Imaging with Inclined Phase and Analyzer Gratings

Despite the explicit reconstruction expression of shown in Eq.(13), it is still very challenging to apply it on real experimental data analysis. This is because the refraction angle signals along the x axis and y axis are both required to apply into Eq.(13). In practice, the easiest way to obtain the two perpendicular components of refraction angle is to rotate the grating interferometry with respect to the z axis by 90 degrees. Obviously, this may bring inconvenience to the data acquisition and does not imaging demand. To overcome such difficulty, we proposed one data with the grating interferometry alternative method to acquire the inclined by degrees. In this method, is the angle between axis, axis is perpendicular to the 1D grating groove, see the proposed grating settings in Fig.4. $e + e$ axis and the fast Fig.4. Illustration of the DPC imaging with inclined phase and analyzer gratings. The variable represents the angle between axis and axis and the axis is parallel to the normal vector of the 1D grating bar.

With such special grating alignments, the refraction angle signal of object is equal to see Fig.5 (a). If rotating the object with acquired at axis by 180 degrees, as shown in Fig.5 (b), the measured refraction respect to the , as shown in , which is equal to angle signal becomes Fig.5(c). As a result, the needed bilateral refraction angle signals along two perpendicular directions can be acquired with continuous object rotation. In all, we $' - , + \pi ' , , ' , , ' , ' - ' , ' x ' , - x ' - , + \pi , , =$ By substituting the Eq.(14) and Eq.(15) back into the Eq.(13), one obtains $x ' , ' + y ' , ' x ' , ' + x ' - , + \pi , , = \delta ' , ' \delta ' , ' \delta ' , ' \delta ' , ' = e ' x ' , ' + x ' - , + \pi ' + e ' ' + e ' ' , - x ' - , + \pi e + ' = - e \delta ' \cos + ' \sin - .$

Fig.5. Scheme of the proposed fast bilateral refraction angle signal acquisition

method with . The ; (b) ; (c) the positions of both grating and object: (a) blue arrow indicates the object.

Assuming a unit vector in the object space where the 1D projection gradient of $e = e' \sin \alpha \cos \beta + e' \cos \alpha \cos \beta + e' \sin \alpha \sin \beta$ along the is the latitude, is the longitude, vector is derived to be equal to . Readily, $\delta'_{x'} = \delta'_{y'} = e' \sin \alpha \cos \beta$, $\delta'_{z'} = e' \cos \alpha \cos \beta + e' \sin \alpha \sin \beta$. Moreover, letting the vector the 2D projection gradient of $\sin \alpha$ be parallel with the normal of the selected plane, then onto this plane is $\delta'_{x'} = \delta'_{y'} = e' \sin \alpha \cos \beta$, $\delta'_{z'} = e' \cos \alpha \cos \beta + e' \sin \alpha \sin \beta$. where $e' = \frac{1}{\sqrt{\cos^2 \alpha + \sin^2 \alpha \cos^2 \beta}}$, $e' \sin \alpha \cos \beta = \frac{\sin \alpha \cos \beta}{\sqrt{\cos^2 \alpha + \sin^2 \alpha \cos^2 \beta}}$, $e' \cos \alpha \cos \beta + e' \sin \alpha \sin \beta = \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sqrt{\cos^2 \alpha + \sin^2 \alpha \cos^2 \beta}}$.

Experimental Validation

Experimental Setup

Validation experiments were performed on the Talbot interferometer system of the BL13W1 beamline at Shanghai Synchrotron Radiation Facility (SSRF). As shown in Fig.6, it consists of one 1D phase grating (π shifting, period 2.396mm) and one 1D absorption grating (period 2.400mm). The distance between the two gratings is 46.380 mm. The beam energy is 20 keV. The detector pixel size is 6.5mm. The specimen of a hamster front toe was positioned with its rotation axis along y axis with . For this Talbot interferometer on the grating interferometry inclined by SSRF, the average fringe visibility is about 0.40, and the mean detector readout is around 25,000. The projection images of object were acquired from projection angle 0 degree to 360 degrees with 0.5 degree interval. At each projection angle the phase-stepping scan was acquired by translating the absorption grating along the with a exposure time of 13 ms in 8 equidistant steps over one grating period. The intervals: refraction angle signals were extracted from two angular bilateral ranges from to 2 .

The programming language used for reconstruction is MATLAB. It takes about 62 ' ' , , seconds to reconstruct a transverse map, and the COMPUTER CPU is Intel Xeon E5- 2667*2. ranges from 0 to , and ' ' - , , for for

Fig.6. The experimental setup of the Talbot interferometer system.

Results

Experimental imaging result of distribution of is presented in the x is shown in Fig.7. The three-dimensional Cartesian coordinate system, with x plane as the coronal plane, and y plane as the plane as the transverse plane, x sagittal plane. From this scalar map of , the details of the structural information can be obtained, but the directional information about the sample is lost. To reveal the directional information of object, the vector information of will be discussed in detail in this following section.

Fig.7. Experimental imaging result of In particular, Fig.8 illustrates two special

1D projection gradients of in the coronal plane: is parallel is perpendicular to the hair. Clearly, the hairs can be easily identified $e' =$ from Fig.8(b), while hair is almost invisible in Fig.8(a), which shows the important δ' ability of the proposed vector CT algorithm when compared with the traditional scalar CT algorithms. to the hair, and the $e' = \delta'$

Fig.8. The image of 1D projection gradient of on the coronal plane with parallel (a) and perpendicular (b) component, correspondingly. to Fig.9(c), respectively.

In addition, the 2D projection gradient maps of on the coronal, sagittal, and In these transverse planes are illustrated in Fig.9(a) the color represents the signal orientation. The reconstructed projection maps, gradient variations of the refractive index can be clearly distinguished from the different color and brightness distributions. Fig.9(a) shows the 2D projection gradient . Fig.9(b) shows the 2D in the coronal plane, where . Fig.9(c) projection gradient in the sagittal plane, where $e = -e'$ shows the 2D projection gradient in the transverse plane, where $e_1 = e'$. Moreover, the hue saturation value (HSV) color map is used to depict such $e_1 = e'$ projection gradient information, including both the absolute signal strength and its $e_2 = e'$ the hue angular orientation, corresponds to the angle; the saturation is set to 1 and the value is defined as the normalized brightness of the directionality in order to fill the span $[0, 1]$, meaning that dark areas in the image correspond to areas with no gradient. For instance, the red, yellow, green, cyan, blue, magenta colors correspond to the $0, 60, 120, 180, 240, 300$ angular direction, respectively. $e_2 = e' e_1 = e' e = e'$ into one single image [17]. More specifically, $e_2 = e' e_1 = e' e = e'$

Fig.9. The 2D projection gradient in the coronal plane (a), the sagittal plane (b) and the axial plane (c).

As a consequence, the line integral process of in the Radon transform, equals to the 2D vector summation of a collection of the increment of refraction angles along the real X-ray beam propagation path, see Fig.3 and Eq.(11). However the principle of the inverse Radon transform of is to filter these 2D vector summation of each projection direction and back-project them to reconstruct the 3D refractive index , see Eq.(13) or (16). As shown in Fig.7 to Fig.9, both the Radon and gradient inverse Radon transforms can be performed on vectors in refraction angle signal based computed vector tomography.

Discussions

In other words, Based on the small angle deviation condition, the Radon transform essentially depends on the pixel size of detector. When the pixel element size reduces, the small line propagation angle deviation condition approximates to the rigorous straight condition. the ideal straight-line propagation condition is the theoretical limit of the small angle deviation condition. Therefore, the deflected X-ray beam induced by the density changes of object is made of two parts: the straight-line component and the streamline component. Obviously, the straight-line component of X-ray beam, which is measured within one reso-

lution unit, meets the requirements of Radon transform. While the streamline component with continuous change of direction, which is measured by the grating period, meets the requirements of extracting the refraction angle signals.

Conclusion and Outlook

In this study, the small angle condition was proposed to define the Radon transform of the refractive index gradient. Based on this condition, analytical reconstruction algorithm for the refractive index gradient and the corresponding data acquisition method were developed. In the verification experiment, complete projection data of refractive index gradient were collected by using grating interferometer with inclined degree θ . Further, the 3D refractive index gradient was reconstructed to verify this new CT image reconstruction theory. Experimental results show two major advantages of this proposed CT theory. First, a standard circular acquisition trajectory typically used in conventional X-ray computed tomography can measure bilateral refraction angle signals. Second, the reconstructed 3D refractive index gradient maps enhance the visualization of the direction of density changes with the flexibility.

Above all, the proposed new reconstruction algorithm has huge promising application scenarios, for example in materials testing of fibrous composites and in medical diagnosis. Besides, the theory and method established in this work can also be applied for other imaging fields, like the neutron imaging, proton imaging, electron imaging, optical imaging, and so on.

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