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Full Text

Phase Field Method for Quasi-Static Hydraulic Fracture in Porous Media Under Stress Boundary Conditions Considering the Effect of Initial Stress Field

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Abstract

The phase field model (PFM) is an efficient fracture modeling method with high potential for hydraulic fracturing (HF). However, current PFMs for HF have not adequately considered the effect of in-situ stress fields, and numerical examples of porous media with stress boundary conditions have rarely been presented. The main reason is that applying remote stress on the boundaries of the calculation domain induces relatively large deformation on these stress boundaries, which is inconsistent with engineering observations. To eliminate this limitation, this paper proposes a new phase field method to describe quasi-static hydraulic fracture propagation in porous media subjected to stress boundary conditions; the new method is more aligned with engineering practice. A new energy functional that considers the effect of the initial in-situ stress field is established and subsequently used to derive the governing equations for the displacement and phase fields through a variational approach. Biot poroelasticity theory is used to couple the fluid pressure field and the displacement field, while the phase field is used for determining the fluid properties from the intact domain to the fully broken domain. Additionally, we present several 2D and 3D examples to demonstrate the effects of in-situ stress on hydraulic fracture propagation. The numerical examples indicate that under stress boundary conditions, our approach obtains correct displacement distribution and is capable of capturing complex hydraulic fracture growth patterns.

Keywords: Phase field model, Hydraulic fracture, Porous media, Stress boundary, In-situ stress, Staggered scheme

1 Introduction

Hydraulic fracture propagation in porous media is one of the most attractive and significant research topics in mechanical, geological, energy, and environmental engineering. In particular, hydraulic fracturing (HF) [1] has been widely used

to exploit oil, tight gas, and shale gas from reservoirs that were unexploitable in past decades. The primary reason for this is that the injection of highly pressurized fluid into a reservoir forms a fracture network for resource transportation. Another application of HF is the measurement of in-situ stress [2]. In addition, HF can be applied in enhanced geothermal systems to accelerate heat extraction [3]. Nevertheless, despite its advantages, HF still brings controversies because the fracturing fluid may leak and further contaminate the underground space and surface due to unfavorable fracture growth paths [4, 5]. Therefore, an accurate numerical tool is critically important for predicting complex hydraulic fracture propagation in porous media.

However, correctly modeling hydraulic fracture in porous media is challenging and full of complexity due to solid-fluid interaction, fracture network formation, and different boundary conditions. This has prompted the development of various numerical methods for modeling fracturing processes, which can be classified into two types in the continuum framework: the discrete method and the smeared method. The discrete methods introduce displacement discontinuity for fractures, and among the most popular are the extended finite element method (XFEM) [6, 7], generalized finite element method (GFEM) [8], boundary element method [9–11], phantom-node method [12, 13], and element-erosion method [14, 15]. On the other hand, in the smeared methods the displacement is continuous across a fracture, and the gradient damage model [16], the screened Poisson method [17], and the phase field model (PFM) [18–22] are the best-known.

Among all fracture modeling methods, PFMs have attracted increasing attention in recent years. In this method, an additional scalar field $\phi \in [0, 1]$ is used to reflect the extent of fracture, where $\phi = 0$ represents intact material and $\phi = 1$ indicates fully broken material (some literature [23] uses a phase field $s = 1 - \phi$, where $s = 0, 1$ represents the fully broken state and intact state, respectively). In addition, the transition zone with $\phi \in (0, 1)$ is controlled by an intrinsic length scale parameter l_0 . After being first proposed by Bourdin et al. [18], the PFM was further developed by Bourdin et al. [18], Miehe et al. [19, 20], Borden et al. [24], and Hofacker and Miehe [25, 26]. Compared with XIGA [27], XFEM [28, 29], cohesive zone model [30, 31], and continuum damage model [32], PFM offers ease of implementation. Furthermore, recent developments have shown that PFMs have high efficiency in predicting complex fracture propagation patterns, even in 3D situations, due to several reasons: (i) simulations can be performed on a fixed mesh without any remeshing or adaptive technique; (ii) complex fracture patterns such as branching and junction are automatically captured; (iii) external fracture criteria or fracture surface tracking algorithms are not required; (iv) PFMs can easily simulate fracture propagation in heterogeneous media; and (v) no penetration criteria are required for hydraulic fracture when a layer interface is encountered.

These advantages have also contributed to the development of PFMs in hydraulic fracturing. For example, in recent years many researchers have coupled

PFMs to HF and made progress [33–45]. However, the examples presented in these contributions all established only homogeneous Dirichlet boundary conditions for the displacement field. The current PFMs for HF therefore cannot adequately consider the effect of in-situ stress on fracture propagation. The main reason is that if remote stress is applied on the boundaries of the calculation domain, relatively large deformation is induced on these stress boundaries, which is inconsistent with engineering observations. In fact, in a geological environment, the displacement on the boundaries where remote stress acts should be zero before HF. It should be noted that although a recently developed PFM [46] attempted to analyze the effect of stress boundary conditions on fracture patterns, the main drawback remained unsolved because large deformation was observed on the stress boundaries.

To eliminate the limitation arising from the initial stress field, this paper proposes a new phase field method to describe quasi-static hydraulic fracture propagation in porous media subjected to stress boundary conditions. A new energy functional that fully considers the effect of the initial in-situ stress field is established and then used to derive the governing equations of strong form for the displacement and phase fields through a variational approach. Biot poro-elasticity theory is used to couple the fluid pressure field and the displacement field, while the phase field is used for determining the fluid properties from the intact domain to the fully broken domain. Additionally, several 2D and 3D examples are presented to show the effects of in-situ stress on hydraulic fracture propagation. The numerical examples indicate that under stress boundary conditions, the new PFM obtains correct displacement distribution and is capable of capturing complex hydraulic fracture growth patterns well.

This paper is organized as follows. Section 2 describes the mathematical model for the new PFM, and Section 3 shows the global numerical algorithm using a staggered scheme. In Sections 4 and 5, the 2D and 3D examples are presented to demonstrate the capability of the new model. The present work and outlook for future development are concluded in Section 6.

2.1 New Energy Functional

Let us consider a poro-elastic domain $\Omega \in \{2, 3\}$ as shown in Fig. 1 [Figure 1: see original paper], where the external and internal discontinuity boundaries are denoted as Ω_e and Γ , respectively. The porous medium is assumed homogeneous and isotropic with compressible and viscous fluid in the pores. Let $T > 0$ be the computational time interval and $u(x, t)$ be the displacement field at time $t \in [0, T]$ and position x . In addition, the displacement field must satisfy the time-dependent Dirichlet boundary conditions, $u(x, t) = g(x, t)$, on Ω_g , and the Neumann condition on Ω_h .

The stress, displacement, and fluid pressure of the porous domain are shown in Fig. 2 [Figure 2: see original paper]. Because of long-term consolidation

or other geological effects, the domain has formed an initial stress field σ_0 , initial displacement field u_0 , and fluid pressure p_0 , as shown in Fig. 2a. On the other hand, in Fig. 2b, after the domain is excavated or fractured by fluid injection, the stress, displacement, and fluid pressure are σ , $u_0 + u$, and $p_0 + p$, respectively. Here, the displacement u and pressure p are the induced relative displacement and fluid pressure by engineering activities such as HF. In this work, our presented method is only for calculating σ , u , p , and the hydraulic fracture pattern in the case where σ_0 , u_0 , and p_0 are known in advance, while how to obtain these initial fields is beyond the scope of this paper.

It should be noted that for the porous medium in the geological environment, the displacement u_0 resulting from long-term geo-stress is always ignored in stability analysis for underground engineering [47, 48], and only the displacement u caused by fracture formation or engineering excavation is calculated. That is, the porous medium Ω is subjected to an initial stress field σ_0 , and if the stress in Ω is equal to σ_0 , the displacement field must be 0. In addition, the initial fluid pressure p_0 is set as 0 for the purpose of simplicity and because the fluid pressure p results only from the relative displacement u and fluid injection [45].

If the body force is ignored, the basic idea of the previous phase field models for porous-elastic media [45] is to construct an energy functional Ψ composed of the elastic energy $\Psi(\varepsilon)$, fracture energy Ψ_f , external work W_{ext} , and the energy contribution of fluid pressure p :

$$\Psi(\mathbf{u}, p, \Gamma) = \int_{\Omega} \psi_{\varepsilon}(\varepsilon) d\Omega + \int_{\Gamma} G_c d\Gamma - \int_{\Omega} \alpha p (\nabla \cdot \mathbf{u}) d\Omega - \int_{\partial\Omega_h} \mathbf{f}_t \cdot \mathbf{u} dS \quad (1)$$

where $\psi_{\varepsilon}(\varepsilon)$ is the elastic energy density; $\alpha \in (0, 1]$ is the Biot coefficient; G_c is the critical energy release rate; and \mathbf{f}_t is the traction on the Neumann boundary. In addition, the linear strain tensor ε is given by:

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2)$$

and for an isotropic linear elastic medium, the elastic energy density $\psi_{\varepsilon}(\varepsilon)$ reads [20]:

$$\psi_{\varepsilon}(\varepsilon) = \frac{1}{2} \lambda (\text{tr } \varepsilon)^2 + \mu \text{tr}(\varepsilon^2) \quad (3)$$

where $\lambda, \mu > 0$ are the Lamé constants:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} \quad (4)$$

with E and ν being Young's modulus and Poisson's ratio, respectively.

However, as seen in Eq. (1), the previously used energy functional cannot account for the effect of the initial stress field and is therefore not suitable for modeling fracture propagation in geomaterial under stress boundary conditions. Evidence for this can be observed in Shiozawa et al. [46], where a stress boundary exhibits relatively large deformation when the fluid pressure is 0. Hence, to deal with the stress boundary and account for the effect of the initial stress field, we establish a new energy functional L for phase field modeling:

$$L(\mathbf{u}, p, \Gamma) = \int_{\Omega} \psi_{\varepsilon}(\varepsilon) d\Omega - \int_{\Omega} \sigma_0 : \varepsilon d\Omega - \int_{\Omega} \alpha p (\nabla \cdot \mathbf{u}) d\Omega + \int_{\Gamma} G_c d\Gamma - \int_{\partial\Omega_h} \mathbf{f}_t \cdot \mathbf{u} dS \quad (5)$$

where the sum of the first and second terms represents the incremental elastic energy from the state of \mathbf{u}_0 to the state of $\mathbf{u}_0 + \mathbf{u}$. It will be shown in the following sections that this incremental elastic energy drives fracture propagation in the porous medium Ω . In addition, it should be noted that by using the new energy functional (5), the fracture deflection phenomenon due to stress contrast can also be well captured by the phase field model, as demonstrated in the numerical examples presented in this paper.

2.2 Phase Field Description

It can be seen from Eqs. (1) and (5) that the energy functional contains a sharp internal surface Γ , which increases the difficulty in minimizing the energy functional when using the variational method. Therefore, to simplify numerical implementation, a phase field $\phi(x, t) \in [0, 1]$ is used to smear the sharp fracture as shown in Fig. 1b, where $\phi = 0$ and $\phi = 1$ represent intact material and fully broken material, respectively.

For a fracture in a 1D bar, the solution for the phase field is an inverse exponential function $\phi(x)$ [20]:

$$\phi(x) = \exp\left(-\frac{|x-a|}{l_0}\right) \quad (6)$$

where $x = a$ is the fracture location and l_0 denotes the intrinsic length scale parameter. The length scale l_0 is also required for 2D and 3D problems, where the crack surface density per unit volume is expressed in terms of the phase field and its gradient [20]:

$$\gamma(\phi, \nabla\phi) = \frac{1}{2l_0} \phi^2 + \frac{l_0}{2} |\nabla\phi|^2 \quad (7)$$

Note that the length scale parameter controls the width of a diffused fracture, and a larger l_0 shows lower nominal tensile strength in phase field modeling [49]. The sharp fracture can be recovered if l_0 tends to zero, which is known as Γ -convergence [20]. In addition, the length scale is assumed much larger than the pore size of the domain Ω in this study, and therefore the fracture energy in Eq. (5) is rewritten as:

$$\int_{\Gamma} G_c d\Gamma \approx \int_{\Omega} G_c \gamma d\Omega = \int_{\Omega} G_c \left[\frac{1}{2l_0} \phi^2 + \frac{l_0}{2} |\nabla \phi|^2 \right] d\Omega \quad (8)$$

The elastic energy Ψ must be decomposed into tensile and compressive parts to ensure cracks form only under tension [24]. Therefore, we follow the strain spectral decomposition of Miehe et al. [20]:

$$\varepsilon^{\pm} = \sum_{a=1}^d \langle \varepsilon_a \rangle_{\pm} \mathbf{n}_a \otimes \mathbf{n}_a \quad (9)$$

where \pm are the tensile and compressive strain tensors, respectively; \mathbf{n} are the principal strain and its direction; and the operators $\langle \cdot \rangle_{\pm}$ are defined as [20]:

$$\langle x \rangle_{\pm} = \frac{x \pm |x|}{2} \quad (10)$$

Applying the decomposed strain tensor, the tensile and compressive parts of the elastic energy density are written as:

$$\psi_{\varepsilon}^{\pm}(\varepsilon) = \frac{\lambda}{2} \langle \text{tr } \varepsilon \rangle_{\pm}^2 + \mu \text{tr} [(\varepsilon^{\pm})^2] \quad (11)$$

We follow Borden et al. [24] and assume the compressive part of the elastic energy density does not affect fracture propagation. Therefore, the elastic energy is rewritten as:

$$\int_{\Omega} \psi_{\varepsilon}(\varepsilon) d\Omega = \int_{\Omega} [g(\phi) \psi_{\varepsilon}^{+}(\varepsilon) + \psi_{\varepsilon}^{-}(\varepsilon)] d\Omega \quad (12)$$

where $g(\phi)$ is a degradation function that must satisfy $g(0) = 1$, $g(1) = 0$, and $g'(1) = 0$ [23]. Although there are many forms for $g(\phi)$, a quadratic form:

$$g(\phi) = (1 - k)(1 - \phi)^2 + k \quad (13)$$

is applied in this work, with $k = 10^{-9}$ being a stability parameter to prevent numerical singularity when $\phi = 0$.

The degradation function is also applied to the energy variation due to the initial stress field:

$$\int_{\Omega} \sigma_0 : \varepsilon d\Omega \approx \int_{\Omega} g(\phi) \sigma_0 : \varepsilon d\Omega \quad (14)$$

which means the initial stress field does not contribute to the energy functional in a fully broken region with $\phi = 0$.

2.3 Governing Equations for the Displacement

Substituting the fracture energy (8), the elastic energy (12), and Eq. (14) into Eq. (5), the energy functional can be rewritten as:

$$L = \int_{\Omega} [g(\phi) \psi_{\varepsilon}^+(\varepsilon) + \psi_{\varepsilon}^-(\varepsilon)] d\Omega + \int_{\Omega} g(\phi) \sigma_0 : \varepsilon d\Omega - \int_{\Omega} \alpha p (\nabla \cdot \mathbf{u}) d\Omega + \int_{\Omega} G_c \left[\frac{1}{2l_0} \phi^2 + \frac{l_0}{2} |\nabla \phi|^2 \right] d\Omega - \int_{\partial\Omega_h} \mathbf{f}_t \cdot \mathbf{u} dS \quad (15)$$

We then apply the variational approach [50], where fracture initiation and propagation at time $t \in [0, T]$ is a process that minimizes the energy functional L . Therefore, the first variation of the energy functional L is set to zero, yielding:

$$\delta L = \int_{\partial\Omega_h} [(\sigma_{ij}^e + g(\phi) \sigma_{0ij} - \alpha p \delta_{ij}) m_j - f_{ti}] \delta u_i dS - \int_{\Omega} (\sigma_{ij}^e + g(\phi) \sigma_{0ij} - \alpha p \delta_{ij}) \delta u_i d\Omega + \int_{\Omega} \left[g'(\phi) (\psi_{\varepsilon}^+ + \sigma_{0ij} \varepsilon_{ij}) \right] \delta \phi d\Omega \quad (16)$$

where $g(\phi) = dg(\phi)/d\phi = 2(\phi - 1)(1 - k)$ and m_j is the component of the outward-pointing normal vector of the boundary. σ_{ij}^e is the component of the effective stress tensor σ^e induced by the displacement \mathbf{u} :

$$\sigma_{ij}^e = g(\phi) \left[\lambda (\text{tr } \varepsilon)_+ \delta_{ij} + 2\mu \varepsilon_{ij}^+ \right] + \lambda (\text{tr } \varepsilon)_- \delta_{ij} + 2\mu \varepsilon_{ij}^- \quad (17)$$

Now, we define the total stress tensor σ as:

$$\sigma = \sigma^e + g(\phi) \sigma_0 - \alpha p \mathbf{I}, \quad \text{in } \Omega \times (0, T] \quad (18)$$

Combining Eqs. (17) and (18), it can be seen that in a pure tension state without fluid, if the phase field $\phi = 1$, the total stress σ is 0 in the fully broken region. In addition, because arbitrary admissible $\delta \mathbf{u}$ must satisfy Eq. (16), the governing equation emerges:

$$\nabla \cdot \sigma = \mathbf{0}, \quad \text{in } \Omega \times (0, T] \quad (19)$$

subjected to the Dirichlet boundary conditions and the Neumann boundary condition:

$$\sigma_{ij}m_j = f_{ti}, \quad \text{on } \partial\Omega_h \times (0, T] \quad (20)$$

which can be derived from the surface integral term in Eq. (16).

2.4 Governing Equations for the Phase Field

Because of the arbitrariness of $\delta\phi$, the variation in Eq. (16) results in the original governing equation for the phase field:

$$\frac{G_c}{l_0}\phi - G_c l_0 \nabla^2 \phi = -2(1-k)(\psi_\varepsilon^+ + \sigma_0 : \varepsilon), \quad \text{in } \Omega \times (0, T] \quad (21)$$

However, Eq. (21) cannot ensure the irreversibility condition $\Gamma(\mathbf{x}, s) \leq \Gamma(\mathbf{x}, t)$ ($s < t$). Therefore, a history reference field H is constructed to form a monotonically increasing phase field:

$$H(\mathbf{x}, t) = \max_{s \in [0, t]} [\psi_\varepsilon^+(\varepsilon(\mathbf{x}, s)) + \sigma_0 : \varepsilon(\mathbf{x}, s)], \quad \text{in } \Omega \times (0, T] \quad (22)$$

Replacing $\psi_\varepsilon^+ + \sigma_0 : \varepsilon$ with $H(\mathbf{x}, t)$ in Eq. (21), the strong form for the phase field is given by:

$$\frac{G_c}{l_0}\phi - G_c l_0 \nabla^2 \phi = -2(1-k)H(\mathbf{x}, t), \quad \text{in } \Omega \times (0, T] \quad (23)$$

which is subjected to the Neumann condition:

$$\nabla\phi \cdot \mathbf{n} = 0, \quad \text{on } \partial\Omega \times (0, T] \quad (24)$$

Thus, the evolution equation of the phase field (23) indicates that fracture initiation and propagation are driven by the history reference field $H(\mathbf{x}, t)$, which represents the historic high incremental elastic energy from the \mathbf{u}_0 state to the $\mathbf{u}_0 + \mathbf{u}$ state during the period $(0, t]$.

2.5 Fluid Flow in the Porous Medium

As described in Subsection 2.1, the fluid in the poro-elastic medium is assumed compressible and viscous. We calculate the fluid flow in three subdomains: the unbroken domain (reservoir domain) $\Omega_r(t)$, fractured domain $\Omega_f(t)$, and transition domain $\Omega_t(t)$. These subdomains are distinguished according to the phase field as shown in Table 1, where c_1 and c_2 are two phase field thresholds.

Table 1: Subdomain definition

Subdomain	Phase field
Reservoir domain	$\phi \leq c_1$
Transition domain	$c_1 < \phi < c_2$
Fractured domain	$\phi \geq c_2$

In this study, we calculate the hydraulic parameters in the transition domain using linear interpolation from the reservoir and fractured domains. Therefore, two indicator functions χ_r and χ_f [46] are established: $\chi_r = 1$ in the reservoir domain and $\chi_r = 0$ in the fractured domain; in contrast, $\chi_f = 0$ in the reservoir domain and $\chi_f = 1$ in the fractured domain. The indicator functions satisfy:

$$\chi_r = \frac{c_2 - \phi}{c_2 - c_1}, \quad \chi_f = \frac{\phi - c_1}{c_2 - c_1}, \quad \text{for } c_1 < \phi < c_2 \quad (25)$$

Darcy's law is used to model the fluid flow, and the mass conservation equation from Zhou et al. [45] is applied to the whole calculation domain:

$$\frac{\partial(\rho S)}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = q_m - \rho \alpha \chi_r \frac{\partial \varepsilon_{vol}}{\partial t} \quad (26)$$

where ρ , S , \mathbf{v} , $\varepsilon_{vol} = \nabla \cdot \mathbf{u}$, and q_m represent the fluid density, storage coefficient, flow velocity, volumetric strain, and fluid source term, respectively. In Eq. (26), $\rho = \rho_r \chi_r + \rho_f \chi_f$ and $\alpha = \alpha_r \chi_r + \alpha_f \chi_f$, where ρ_r and ρ_f are the fluid densities in the reservoir and fracture domains, while α_r and α_f are the Biot coefficients of the reservoir and fractured domains. Naturally, $\alpha_f = 1$ is set for the fractured domain, and therefore $\alpha = \alpha_r \chi_r + \chi_f$.

In Eq. (26), the storage coefficient S is expressed as [45]:

$$S = \varepsilon_p c + \frac{(\alpha - \varepsilon_p)(1 - \alpha)}{K_{Vr}} \quad (27)$$

where ε_p , c , and K_{Vr} are the porosity, fluid compressibility, and bulk modulus of the calculation domain, respectively. Denoting c_r and c_f as the fluid compressibility in the reservoir and fractured domains, we have $c = c_r \chi_r + c_f \chi_f$.

Gravity is not considered in this study; therefore, Darcy's velocity \mathbf{v} is calculated as:

$$\mathbf{v} = -\frac{\mathbf{K}_{eff}}{\mu_{eff}} \nabla p \quad (28)$$

where μ_{eff} denotes fluid viscosity. Similarly, $\mathbf{K}_{eff} = \mathbf{K}_r + \mathbf{K}_f$, with \mathbf{K}_r and \mathbf{K}_f being the permeability in the reservoir and fractured domains, respectively. \mathbf{K}_{eff} is the effective permeability, and $\mathbf{K}_{eff} = \mathbf{k}_r + \mathbf{k}_f$, where \mathbf{k}_r and \mathbf{k}_f are the permeabilities of the reservoir and fractured domains. Substituting Eq. (28) into Eq. (26), the governing equation for fluid flow is expressed in terms of the fluid pressure p :

$$\frac{\partial(\rho S)}{\partial t} - \nabla \cdot \left(\rho \frac{\mathbf{K}_{eff}}{\mu_{eff}} \nabla p \right) = q_m - \rho \alpha \chi_r \frac{\partial \varepsilon_{vol}}{\partial t} \quad (29)$$

which is subjected to the Dirichlet condition on Ω_D and Neumann condition on Ω_N [45]:

$$p = p_D \quad \text{on } \partial\Omega_D \quad (30)$$

$$-\mathbf{n} \cdot \rho \mathbf{v} = M_N \quad \text{on } \partial\Omega_N \quad (31)$$

where p_D and M_N are the prescribed pressure and mass flux.

3 Numerical Algorithm

We implement the proposed phase field model in the framework of the finite element method (FEM), and the FE discretization can be seen in the Appendix. In addition, a finite difference method is applied for time discretization. The overall procedure of our numerical algorithm for solving the three fields is shown in Fig. 3 [Figure 3: see original paper], where a staggered scheme is used. That is, the three fields are solved sequentially and independently in each time step.

To reduce computational effort, we implement the numerical method in the commercial software COMSOL Multiphysics, which has proven capability for multi-field problems. In all simulations, the maximum iteration number is set as 150 due to high nonlinearity resulting from the derivative σ / ϵ and the phase field, which degrades the stiffness matrix of the displacement field. In addition, a stabilization and convergence acceleration method—the Anderson acceleration technique—is applied. For more details on the COMSOL implementation, readers can refer to the previous study [51].

4 2D Examples

In this section, 2D examples of specimens subjected to internal fluid injection are presented to prove the capability of the proposed phase field method. All pre-existing cracks are established by introducing an initial history field with a relatively high H [24], and the source term in the pre-existing notches is set as $q_{\text{F}} = 10 \text{ kg}/(\text{m}^3 \cdot \text{s})$ in all examples.

4.1 Fractures from a Horizontal Notch The first example tests hydraulic fracture propagation from a horizontal notch in a square domain. The geometry and boundary conditions of this example are shown in Fig. 4 [Figure 4: see original paper] with increasing fluid volume in the initial notch. All outer boundaries of the calculation domain have a fluid pressure of $p = 0$ and zero tangential displacement to remove the effect of rigid body displacement. The parameters for calculation are listed in Table 2 .

Table 2: Basic calculation parameters

Parameter	Value	Parameter	Value
$\rho_{\text{R}}, \rho_{\text{F}}$	$1.0 \times 10^3 \text{ kg}/\text{m}^3$	k_{f}	$1 \times 10^{-8} \text{ m}^2$
$\mu_{\text{R}}, \mu_{\text{F}}$	$1 \times 10^{-3} \text{ kg}/(\text{m} \cdot \text{s})$	q_{F}	$1 \times 10^{-9} \text{ kg}/(\text{m}^3 \cdot \text{s})$
k_{r}	$8.333 \times 10^{-4} \text{ m}^2$	E	$1 \times 10^8 \text{ Pa}$
α_{R}	1		0.2
G_{c}	$1.0 \text{ J}/\text{m}^2$	l_0	0.1 m

We discretize the calculation domain with unstructured triangular elements with maximum element size $h = 0.05 \text{ m}$. Therefore, linear shape functions are used for all three physical fields. In addition, the time step $\Delta t = 0.05 \text{ s}$ is adopted for the simulation. The initial stress state of $\sigma_0 = 0.5 \text{ MPa}$, $\sigma_{y_0} = 0.5 \text{ MPa}$, and $\sigma_{z_0} = (\sigma_0 + \sigma_{y_0})$ is applied, with ν being Poisson's ratio, which can be evaluated through the well-known elasticity relationship from λ and μ . As a comparison, we also model fracture propagation using the previously developed method [45] without considering the effect of the initial stress field.

Figures 5 and 6 show the fracture propagation obtained using the proposed phase field method and that without considering the effect of the initial stress field. The figures indicate that discarding the effect of initial stress does not affect the fracture pattern for this first example; in both situations, the fracture propagates along the horizontal direction. However, the time for fracture initiation and propagation differs, and the fracture length is smaller if the initial stress field is considered in the constitutive model.

The displacement field at time $t = 0 \text{ s}$ is shown in Fig. 7 [Figure 7: see original paper], where the region of $\phi \geq 0.95$ is removed to reflect the shape of the fully broken domain. It can be observed that the proposed PFM and the method of Zhou et al. [45] achieve different initial displacement fields for the problem of a

poro-elastic domain subjected to stress boundary conditions. Fig. 7a indicates that only a small displacement appears around the center of the notch for the proposed PFM because the “excavation” or stiffness degradation of the initial notch produces displacement towards the broken domain. However, if the PFM does not consider the effect of the initial stress field, all outer boundaries have rather large displacements, as shown in Fig. 7b. In addition, Fig. 8 [Figure 8: see original paper] compares the displacements along the top and left boundaries at time $t = 0$ s. As observed, for the method of Zhou et al. [45], the stress boundary condition produces large initial displacements along the left and top boundaries, while the initial displacement on the boundaries is negligible using the proposed PFM. In summary, comparisons in Figs. 7 and 8 indicate that the proposed PFM achieves better displacement distribution compared with PFMs without considering the effect of the initial stress field, especially for poro-elastic media subjected to stress boundary conditions in geological environments.

Figure 9 [Figure 9: see original paper] shows the effect of the initial stress field on the fluid pressure-time curve. Note that data at the center of the initial notch are selected. Similar to the fracture pattern, Fig. 9 indicates that the fluid pressure is only slightly affected by the initial stress field. In this example, the time for fracture initiation is reduced if the initial stress field is not considered; therefore, the fluid pressure-time curve has an earlier drop stage and a lower maximum pressure compared with that obtained by the proposed PFM, as shown in Fig. 9.

The evolution of the displacement field obtained by the proposed PFM is shown in Fig. 10 [Figure 10: see original paper]. As expected, the maximum displacement occurs at the center of the fracture and increases with time. This phenomenon is also consistent with observations in porous media with fixed displacement boundaries [36, 45, 52], which indirectly reflects the applicability of the proposed PFM in this study.

It is well known that the hydraulic fracture pattern is highly affected by the stress contrast acting on the outer boundaries of the calculation domain. Therefore, in this example, we change the ratio of $\sigma_{y_0}/\sigma_{x_0}$ to [0.5, 1, 2, 10] with $\sigma_{x_0} = 0.5$ MPa unchanged to demonstrate the effect of stress contrast. Using the proposed PFM, the calculated fracture paths at time $t = 10$ s are shown in Fig. 11 [Figure 11: see original paper]. It can be observed that for $\sigma_{y_0}/\sigma_{x_0} = 0.5, 1,$ and $2,$ the fracture from the initial notch propagates horizontally, and the fracture length decreases as the ratio of $\sigma_{y_0}/\sigma_{x_0}$ increases. However, when $\sigma_{y_0}/\sigma_{x_0} = 10,$ the fracture deflects and propagates along the direction of the maximum in-situ stress $S\{\max\}$, which is consistent with engineering observations in hydraulic fracturing. In addition, the effect of the ratio of $\sigma_{y_0}/\sigma_{x_0}$ on the fluid pressure-time curve is shown in Fig. 12 [Figure 12: see original paper]. The maximum fluid pressure at the mid-point of the initial notch is observed to increase with increasing $\sigma_{y_0}/\sigma_{x_0}.$

Figure 13 [Figure 13: see original paper] describes the relative incremental mid-point fluid pressure and effective stress at the fracture tip under differ-

ent $\sigma_{y_0}/\sigma_{x_0}$ for the case of a horizontal initial notch. Note that the effective stress along the y-direction is used to show the effect of σ_{y_0} , and the case of $\sigma_{y_0}/\sigma_{x_0} = 0.5$ is used as the reference. As shown in Fig. 13, when the vertical stress σ_{y_0} increases linearly, the mid-point fluid pressure and effective stress at the fracture tip also increase linearly. This phenomenon indirectly verifies the feasibility and practicability of the proposed PFM. Furthermore, the incremental fluid pressure is slightly larger than the vertical stress variation, while the incremental effective stress at the fracture tip is slightly lower.

4.2 Fracture from an Inclined Notch We now consider an inclined initial notch in the calculation domain shown in Fig. 4. The notch has a length of 0.8 m and an inclination angle $\theta = 45^\circ$, while the other simulation settings are the same as those in Subsection 4.1. Stress contrast $\sigma_{x_0}/\sigma_{y_0} = [1, 2, 4, 6, 8, 10]$ is applied with the vertical stress $\sigma_{y_0} = 0.5$ MPa unchanged. Therefore, six simulations are performed to show the effect of stress contrast on fracture propagation from the inclined notch.

Using the proposed PFM, the phase field distribution for different $\sigma_{x_0}/\sigma_{y_0}$ at time $t = 6$ s is shown in Fig. 14 [Figure 14: see original paper]. As observed, the fracture propagates straight along the direction of the initial notch when $\sigma_{x_0}/\sigma_{y_0} = 1$ at time $t = 6$ s; however, if the stress ratio is larger than 1, the fracture deflects from the direction of the notch ($\theta = 45^\circ$). All simulations consistently indicate that the larger the ratio of $\sigma_{x_0}/\sigma_{y_0}$, the smaller the angle the deflected fracture makes from the direction of the maximum in-situ stress S_{\max} (σ_{x_0}) in this example. The fluid pressure-time curve for different $\sigma_{x_0}/\sigma_{y_0}$ is shown in Fig. 15 [Figure 15: see original paper]. Because the minimum in-situ stress σ_{y_0} is kept constant, the fluid pressure at the center of the notch only increases slightly with increasing stress contrast $\sigma_{x_0}/\sigma_{y_0}$.

4.3 Fracture from Two Perpendicularly Crossed Notches We also test hydraulic fracture propagation from two perpendicularly crossed notches. The notches are located in the center of the calculation domain shown in Fig. 4, and both have a length of 0.8 m. The other simulation settings are the same as those in Subsection 4.1. We only consider four cases of in-situ stress as described in Table 3, meaning that the directions of the maximum stress S_{\max} and minimum stress S_{\min} vary in the four cases.

Table 3: Four cases of boundary and in-situ stresses

Case	Horizontal stress	Vertical stress
1	$\sigma_{x_0} = 0.5$ MPa	$\sigma_{y_0} = 1$ MPa
2	$\sigma_{x_0} = 0.5$ MPa	$\sigma_{y_0} = 5$ MPa
3	$\sigma_{x_0} = 1$ MPa	$\sigma_{y_0} = 0.5$ MPa
4	$\sigma_{x_0} = 5$ MPa	$\sigma_{y_0} = 0.5$ MPa

Using the proposed PFM, the phase field distribution for different cases is shown in Fig. 16 [Figure 16: see original paper]. As observed, fractures only initiate and propagate from the notch perpendicular to the direction of the minimum stress S_{\min} , while the other notches do not grow. Note that the direction of these fractures is consistent with the direction of S_{\min} . Therefore, Fig. 16 validates the engineering observation that fractures perpendicular to the direction of the minimum in-situ stress will initiate and propagate more easily. The fluid pressure-time curves for different cases are shown in Fig. 17 [Figure 17: see original paper]. The curves for Cases 1 and 3 and those for Cases 2 and 4 are almost identical because the maximum and minimum in-situ stresses are identical.

4.4 Fracture from Two Horizontal Notches In this 2D example, hydraulic fracture from two horizontal notches is presented to further validate the proposed PFM. The geometry and boundary stress condition of this example are shown in Fig. 18a [Figure 18: see original paper], where the horizontal and vertical remote stresses are both 0.5 MPa for simplicity. The parameters and numerical settings are the same as in Subsection 4.1. The phase field at $t = 2.5$ s is shown in Fig. 18b. A symmetrical fracture pattern is observed, and fracture deflection occurs during fluid injection. This observation is in good agreement with the “stress shadowing” phenomenon in engineering practice [53].

4.5 Linearly Varying Stress In this final 2D example, we test the effect of a linearly varying stress field on hydraulic fracture propagation. The geometry of this example is the same as that in Fig. 4, while the boundary stress condition is shown in Fig. 19 [Figure 19: see original paper]. Note that in this example, gravity is applied in the vertical direction, and a linearly varying horizontal stress acts in the horizontal direction, which can be considered a combination of self-weight stress field and tectonic stress field in an underground geological environment. The parameters and numerical settings are the same as in Subsection 4.1.

The phase field evolution at different times is shown in Fig. 20 [Figure 20: see original paper]. An asymmetric fracture pattern is observed, and fracture propagation is much easier towards the upper boundary than towards the bottom. This finding can also be verified in Fig. 21 [Figure 21: see original paper], which depicts the fracture length increment at different times. Figure 21 indicates that the fracture length from the upper tip of the pre-existing notch is much larger than that from the lower tip. The downward hydraulic fracture is hampered and even cannot propagate after $t = 9$ s. Figures 20 and 21 strongly verify that the hydraulic fracture tends to propagate towards the region with lowest fracture resistance.

In summary, the 2D examples in Subsections 4.1 to 4.5 indicate the sensitivity of hydraulic fracture propagation to the stress boundary condition. Our proposed model, which incorporates the effect of the initial stress field, is feasible

and practicable for capturing the effect of remote stresses on hydraulic fracture propagation and for producing the correct displacement field.

5 3D Example

In this section, we test the performance of our method in modeling 3D hydraulic fracture propagation. The example is a 3D isotropic medium with a penny-shaped initial notch. The fluid source in the notch is set as $10 \text{ kg}/(\text{m}^3 \cdot \text{s})$. The calculation domain is a cube with dimensions of $10 \times 10 \times 10 \text{ m}^3$ and has the same center as the initial notch, the height of which is in the z -direction. The notch is parallel to the x - y plane, with radius 0.8 m and height 0.4 m.

The parameters for calculation are identical to those in Table 2, except the length scale parameter is $l_0 = 0.25 \text{ m}$ and the permeability k_f is $5.21 \times 10^{-3} \text{ m}^2$. We employ 6-node prism elements to discretize the 3D domain, while the maximum element size is set as 0.18 m to reduce computational cost. In addition, the time increment is set as 0.05 s in each simulation.

In this 3D example, we only test the influence of the stress in the z -direction σ_{z_0} . Therefore, σ_{z_0} is set as 0.5, 1, and 5 MPa, respectively, while the stresses in the other two directions are fixed at 1 MPa.

Hydraulic fracture propagation patterns in the 3D isotropic medium are shown in Fig. 22 [Figure 22: see original paper]. It can be seen from Figs. 22a–f that for $\sigma_{z_0} = 0.5 \text{ MPa}$ and 1 MPa, fractures initiate and propagate only in the x - y plane, while the area of the fractured domain increases as σ_{z_0} decreases. Figures 22g–i indicate that when σ_{z_0} is too large, fracture propagation in the x - y plane is hindered, and fractures only propagate in the x - z or y - z plane.

The fluid pressure-time curves under different σ_{z_0} are shown in Fig. 23 [Figure 23: see original paper]. The fluid pressure is observed to drop after fracture initiation, and the maximum fluid pressure increases with increasing in-situ stress σ_{z_0} . Comparing Figs. 22 and 23 indicates that the effect of in-situ stress on the 3D porous medium can be well captured by the proposed PFM in a fixed FE mesh without requiring any remeshing or adaptive techniques. The 3D simulations fully verify the strong capability of our proposed PFM for predicting complex hydraulic fracture propagation in porous media subjected to stress boundary conditions.

6 Conclusions

A new phase field model for simulating quasi-static hydraulic fracture propagation in porous media subjected to stress boundary conditions is proposed. A new energy functional is established to consider the effect of the initial in-situ

stress field. This energy functional is then used to derive the governing equations for the displacement and phase fields through a variational approach. Biot poroelasticity theory is used to couple the fluid pressure field and the displacement field, while the phase field is used for determining the fluid properties from the intact domain to the fully broken domain.

The numerical examples presented in this work verify the capability of the proposed PFM in capturing complex hydraulic fracture growth patterns in 2D and 3D. The numerical examples also indicate that under stress boundary conditions, the proposed approach can obtain correct displacement distribution and reflect the sensitivity of hydraulic fracture propagation to stress boundary conditions. In future research, the proposed PFM can be more widely applied in HF practices where stress boundaries dominate and can be used to investigate the influence of naturally-layered porous media and multi-zone HF on fracture propagation.

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Appendix: Finite Element Discretization

We first derive the weak forms of all the governing equations as:

$$\int_{\Omega} [(\sigma^e + g(\phi)\sigma_0 - \alpha p \mathbf{I}) : \delta \varepsilon] d\Omega = \int_{\partial\Omega_h} \mathbf{f}_t \cdot \delta \mathbf{u} dS \quad (\text{A1})$$

$$\int_{\Omega} [-2(1-k)H(1-\phi)\delta\phi] d\Omega + \int_{\Omega} G_c \left(l_0 \nabla \phi \cdot \nabla (\delta\phi) + \frac{1}{l_0} \phi \delta\phi \right) d\Omega = 0 \quad (\text{A2})$$

$$\int_{\Omega} \frac{\partial(\rho S)}{\partial t} \delta p d\Omega - \int_{\Omega} \rho \mathbf{v} \cdot \nabla (\delta p) d\Omega = \int_{\Omega} \left(q_m - \rho \alpha \chi_r \frac{\partial \varepsilon_{vol}}{\partial t} \right) \delta p d\Omega + \int_{\partial\Omega_N} M_N \delta p dS \quad (\text{A3})$$

In an element with n nodes, the nodal values for the three fields (\mathbf{u} , ϕ , and p) are defined as \mathbf{u}_i , ϕ_i , and p_i ($i = 1, 2, \dots, n$). The fields are then discretized as follows:

$$\mathbf{u} = \sum_{i=1}^n N_i \mathbf{u}_i, \quad \phi = \sum_{i=1}^n N_i \phi_i, \quad p = \sum_{i=1}^n N_i p_i \quad (\text{A4})$$

where N is the shape function at node i . We then derive the gradients of the three fields as:

$$\nabla \mathbf{u} = \sum_{i=1}^n \mathbf{B}_i^u \mathbf{u}_i, \quad \nabla \phi = \sum_{i=1}^n \mathbf{B}_i^\phi \phi_i, \quad \nabla p = \sum_{i=1}^n \mathbf{B}_i^p p_i \quad (\text{A5})$$

where \mathbf{B}_i^u , \mathbf{B}_i^ϕ , and \mathbf{B}_i^p are derivatives of the shape functions. For 2D problems, the components along the z -direction are removed from the above equations.

Due to the arbitrariness of the test functions, the external force $\mathbf{F}_{u,ext}$ and internal force $\mathbf{F}_{u,int}$ for the displacement field are described by:

$$\mathbf{F}_{u,ext}^i = \int_{\partial\Omega_h} N_i \mathbf{f}_t dS + \int_{\Omega} N_i \alpha \nabla p d\Omega \quad (\text{A6})$$

$$\mathbf{F}_{u,int}^i = \int_{\Omega} (\mathbf{B}_i^u)^T \sigma d\Omega - \int_{\Omega} (\mathbf{B}_i^u)^T \alpha p \mathbf{I} d\Omega - \int_{\Omega} (\mathbf{B}_i^u)^T g(\phi) \sigma_0 d\Omega \quad (\text{A7})$$

The inner force term of the phase field is also obtained by:

$$F_{\phi,int}^i = \int_{\Omega} \left[-2(1-k)(1-\phi)HN_i + G_c \left(l_0 (\mathbf{B}_i^\phi)^T \nabla \phi + \frac{1}{l_0} N_i \phi \right) \right] d\Omega \quad (\text{A8})$$

Finally, for the pressure field, the inner force $F_{p,int}$, viscous force $F_{p,vis}$, and external force $F_{p,ext}$ are given by:

$$\begin{cases} F_{p,int}^i = \int_{\Omega} N_i \frac{\partial(\rho S)}{\partial t} d\Omega \\ F_{p,vis}^i = \int_{\Omega} (\mathbf{B}_i^p)^T \rho \frac{\mathbf{K}_{eff}}{\mu_{eff}} \nabla p d\Omega \\ F_{p,ext}^i = \int_{\Omega} N_i \left(q_m - \rho \alpha \chi_r \frac{\partial \varepsilon_{vol}}{\partial t} \right) d\Omega + \int_{\partial\Omega_N} N_i M_N dS \end{cases} \quad (\text{A9})$$

Thus, the contribution of node i to the residual of the discrete equations for the three fields is written as:

$$\begin{cases} R_u^i = \mathbf{F}_{u,ext}^i - \mathbf{F}_{u,int}^i \\ R_\phi^i = -F_{\phi,int}^i \\ R_p^i = F_{p,ext}^i - F_{p,int}^i - F_{p,vis}^i \end{cases} \quad (\text{A10})$$

Because the staggered scheme is used to solve the displacement, phase field, and fluid pressure sequentially, we also adopt the Newton-Raphson approach sequentially to achieve $R_u = 0$, $R_\phi = 0$, and $R_p = 0$ for the three fields. In addition, the tangents on the element level are calculated by:

$$\begin{cases} \mathbf{K}_{uu}^{ij} = \int_{\Omega} (\mathbf{B}_i^u)^T \mathbf{D}^e (\mathbf{B}_j^u) d\Omega \\ \mathbf{K}_{pp}^{ij} = \int_{\Omega} (\mathbf{B}_i^p)^T \rho \frac{\mathbf{K}_{eff}}{\mu_{eff}} (\mathbf{B}_j^p) d\Omega \\ \mathbf{K}_{\phi\phi}^{ij} = \int_{\Omega} \left[G_c l_0 (\mathbf{B}_i^{\phi})^T \mathbf{B}_j^{\phi} + N_i \left(2(1-k)H + \frac{G_c}{l_0} \right) N_j \right] d\Omega \end{cases} \quad (\text{A11})$$

where \mathbf{D} is the elasticity matrix derived from the elasticity tensor $\mathbf{D} = \sigma / \epsilon$.

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