Thermodynamic properties at the kinetic freeze-out in the Au+Au and Cu+Cu collisions at the RHIC using the Tsallis distribution

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Date: 2023-09-07T10:57:34+00:00

Abstract

The thermodynamic properties of charged particles, such as the energy density, pressure, entropy density, particle density, and squared speed of sound at the kinetic freeze-out in the $ mathrm{Au+Au}$ collisions from the relativistic heavy ion collider (RHIC) beam energy scan program ($ sqrt{s_ text{NN}}$=7.7–200 GeV) and in the $ mathrm{Cu+Cu}$ collisions at $ sqrt{s_ text{NN}}$=62.4, 200 GeV are studied using the thermodynamically consistent Tsallis distribution. The energy density, pressure, and particle density decrease monotonically with the collision energy for the same collision centrality; These properties also decrease monotonically from the central to peripheral collisions at the same collision energy. While the scaled energy density $ varepsilon/T^4$ and scaled entropy density s/T^3 demonstrate the opposite trend with the collision energy for the same collision centrality. There is a correlation between $ varepsilon/T^4$ and s/T^3 at the same centrality. In addition, the squared speed of sound was calculated to determine that all the collision energies share nearly the same value at different collision centralities.

Full Text

Preamble

Thermodynamic properties at the kinetic freeze-out in Au+Au and Cu+Cu collisions at RHIC using the Tsallis distribution

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The thermodynamic properties of charged particles—such as energy density, pressure, entropy density, particle density, and squared speed of sound—at kinetic freeze-out in Au+Au collisions from the Relativistic Heavy Ion Collider (RHIC) beam energy scan program ($\sqrt{s_{NN}} = 7.7$–200 GeV) and in Cu+Cu collisions at $\sqrt{s_{NN}} = 62.4, 200$ GeV are studied using the thermodynamically consistent Tsallis distribution. The energy density, pressure, and particle density decrease monotonically with collision energy for the same collision centrality. These properties also decrease monotonically from central to peripheral collisions at the same collision energy. In contrast, the scaled energy density ε/T^4 and scaled entropy density s/T^3 demonstrate the opposite trend with collision energy for the same centrality. A correlation exists between ε/T^4 and s/T^3 at the same centrality. In addition, the squared speed of sound was calculated to determine that all collision energies share nearly the same value at different centralities.

Keywords: Heavy-ion collision, Tsallis distribution, Kinetic freeze-out, Energy density, Entropy density, Particle density, Squared speed of sound, Pressure

Introduction

In relativistic heavy-ion collisions, an extremely hot and dense mixture of quarks and gluons is created, known as the quark-gluon plasma (QGP) [?]. The QGP can only exist for a significantly short time and hadronizes into mesons and baryons due to color confinement. These particles interact with one another or form light nuclei and continue expanding. The system cools and reaches the chemical freeze-out point when the abundances of all particle species become fixed. The system continues evolving to reach kinetic freeze-out, where the distributions of all particles no longer change. Subsequently, particle information is recorded by detectors positioned around the collision region. With measured information such as particle multiplicities and transverse momentum (p_T) spectra, the properties of the QGP and the system can be studied at different evolution stages.

In previous experimental and theoretical studies, several statistical distributions or models based on different assumptions have been used to describe particle transverse momentum spectra and extract relevant information about the collision system. These include the Boltzmann-Gibbs (BG) distribution, Fermi-Dirac distribution, Bose-Einstein distribution, double exponential distribution, m_T-exponential distribution [?, ?], Erlang distribution [?], multi-source model

[?], blast-wave model [?], Tsallis distribution [?], and the Generalized Fokker-Planck Solution (GFPS) [?, ?], among others.

As a generalization of the BG distribution, the Tsallis distribution has recently received considerable attention [?, ?]. This is attributed to its successful application in describing particle p_T spectra in $p+p$ collisions (where transverse momentum spans two orders of magnitude and yield spans 15 orders of magnitude) presented by Wong et al. [?, ?] and in several other studies [?, ?, ?, ?] dedicated to describing particle transverse momentum spectra produced in pp, pA, and AA collisions. Cleymans et al. demonstrated the thermodynamic consistency of the Tsallis distribution. Utilizing the Tsallis distribution, Azmi et al. described the transverse momentum spectra of charged particles produced in Pb+Pb collisions at the Large Hadron Collider (LHC) and deduced the thermodynamic properties of the collision system at kinetic freeze-out. Combined with thermodynamic properties of the system at chemical freeze-out obtained by fitting particle yields using the statistical model, this can provide an evolutionary picture of thermodynamic quantities for the hadronic phase from chemical to kinetic freeze-out [?].

In this study, following Ref. [?], we systematically investigated the transverse momentum spectra of charged particles at RHIC using the thermodynamically consistent Tsallis distribution, with experimental data from the Au+Au collisions in the beam energy scan (BES) program published by the STAR Collaboration ($\sqrt{s_{NN}} = 7.7-200$ GeV) [?] and data from Cu+Cu collisions at $\sqrt{s_{NN}} = 62.4, 200$ GeV [?, ?] measured by the PHOBOS Collaboration. The nonextensive parameter q and temperature parameter T were extracted from the Tsallis distribution. Subsequently, we investigated the thermodynamic properties of charged particles at kinetic freeze-out: energy density, pressure, entropy density, particle density, and squared speed of sound. The dependence of these thermodynamic quantities on collision energy, system size, and centrality was also studied and discussed.

The remainder of this paper is organized as follows. The Tsallis distribution for charged particle transverse momentum spectra and the formulas for thermodynamic quantities are briefly introduced in Section II, along with fitting results for experimental transverse momentum spectra. The thermodynamic quantities for Au+Au and Cu+Cu collisions at different collision energies and centralities were calculated, with results discussed in Section III. A brief summary is given in Section IV.

II. Tsallis Distribution

The Tsallis distribution is a generalization of the Boltzmann-Gibbs distribution in classical thermodynamics, proposed by Tsallis [?]. Within the framework of the thermodynamically consistent Tsallis distribution, the momentum distribution of final particles produced in relativistic heavy-ion collisions can be expressed as:

$$\frac{d^{2N}}{2\pi p_T dp_T dy} = \frac{gVE}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}$$

Here, g indicates the degeneracy of particles, V is the volume, E is the energy, μ is the chemical potential, q is the Tsallis parameter, and T is the temperature parameter. Equation (1) can be expressed in terms of transverse momentum p_T, transverse mass $m_T = \sqrt{p_T^2 + m^2}$, and rapidity y as [**?**, **?**, **?**, **?**]:

$$\frac{d^{2N}}{2\pi p_T dp_T dy} = \frac{gVm_T \cosh y}{(2\pi)^3}\left[1+(q-1)\frac{m_T \cosh y-\mu}{T}\right]^{-\frac{q}{q-1}}$$

The majority of charged particles are $\pi^+(\pi^-)$ mesons, and the numbers of positive and negative pions are equal in heavy-ion collisions at RHIC and LHC, which holds even at collision energies as low as 7.7 GeV (the lowest collision energy in the BES). However, the numbers of p and $\bar{p}$ differ, leading to a nonzero baryon chemical potential. Considering that only a small portion of charged particles are baryons, assuming zero chemical potential provides a sufficient approximation. The variations due to this zero chemical potential approximation were determined to be small, and our conclusions do not depend on this approximation.

When considering only particles at mid-rapidity ($y \approx 0$), Eq. (2) reduces to:

$$\left.\frac{d^{2N}}{2\pi p_T dp_T dy}\right|_{y=0} = \frac{gVm_T}{(2\pi)^3}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}$$

In the experimental distribution of charged particles in relativistic heavy-ion collisions, pseudorapidity η is occasionally used instead of rapidity y. The conversion from rapidity to pseudorapidity is given by:

$$\frac{dy}{d\eta} = \frac{p_T \cosh\eta}{m_T \cosh^2 y}$$

According to Eqs. (3, 4), the pseudorapidity distribution of particles at mid-rapidity is:

$$\left.\frac{d_{ch}^{2N}}{2\pi p_T dp_T d\eta}\right|_{y=0} = \frac{gVm_T}{(2\pi)^3}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}$$

As indicated in Ref. [**?**], the transverse momentum spectrum of charged particles consists of three Tsallis distributions for pions, kaons, and protons, considering that the main charged particles are $\pi^+(\pi^-)$, $K^+(K^-)$, and $p(\bar{p})$, respectively, in relativistic heavy-ion collisions. Therefore, the transverse momentum distribution of charged particles at mid-rapidity can be expressed as:

$$\left.\frac{d_{ch}^{2N}}{2\pi p_T dp_T d\eta}\right|_{y=0} = \frac{2V}{(2\pi)^3}\sum_i g_i m_{T,i}\left[1+(q-1)\frac{m_{T,i}}{T}\right]^{-\frac{q}{q-1}}$$

where $i = \pi^+, K^+, p$. $m_{T,i}$ is the transverse mass of particle i in the sum of Eq. (6). The factor 2 on the right-hand side considers contributions from antiparticles, which is reasonable at the LHC because particle and antiparticle multiplicities are equal [**?**]. The degeneracy factors g of the particles are $g_{\pi^+} = 1$, $g_{K^+} = 1$, $g_p = 2$. However, experimental data demonstrates significant differences between particle and antiparticle multiplicities for kaons and protons at RHIC, particularly at lower collision energies.

Considering this, we determined effective degeneracy factors for the particles. These factors were obtained by taking half the sum of one and the multiplicity ratio between antiparticles and particles for each particle type from RHIC experimental data [**?**, **?**, **?**, **?**]. These values are listed in Table 1 .

The formulas for thermodynamic quantities at kinetic freeze-out in thermodynamically consistent Tsallis statistics are as follows [**?**, **?**]:

$$\varepsilon = 2\sum_i \int \frac{g_i d^3p}{(2\pi)^3} E_i \left[1+(q-1)\frac{E_i}{T}\right]^{-\frac{q}{q-1}}$$

$$n = 2\sum_i \int \frac{g_i d^3p}{(2\pi)^3} \left[1+(q-1)\frac{E_i}{T}\right]^{-\frac{q}{q-1}}$$

$$s = 2\sum_i \int \frac{g_i d^3p}{(2\pi)^3} \left\{\frac{E_i}{T}\left[1+(q-1)\frac{E_i}{T}\right]^{-\frac{q}{q-1}} - \frac{1}{q-1}\left[1+(q-1)\frac{E_i}{T}\right]^{1-\frac{q}{q-1}} + \frac{1}{q-1}\right\}$$

$$P = 2\sum_i \int \frac{g_i d^3p}{(2\pi)^3} \frac{p^2}{3E_i} \left[1+(q-1)\frac{E_i}{T}\right]^{-\frac{q}{q-1}}$$

$$c_s^2(T) = \left(\frac{\partial P}{\partial \varepsilon}\right)_S = \frac{(\partial P/\partial T)_V}{(\partial \varepsilon/\partial T)_V} = \frac{C_V}{C_V}$$

where $i = \pi^+, K^+, p$.

To understand the behavior of thermodynamic quantities, analytical formulas derived for massless particles with zero chemical potential in Tsallis statistics are utilized for estimation, as provided in Ref. [**?**]:

$$\varepsilon = g\frac{T^4}{(2-q)(3-2q)(4-3q)}$$

$$n = g\frac{T^3}{(2-q)(3-2q)}$$

$$s = g\frac{T^3}{(2-q)(3-2q)(4-3q)}$$

$$P = g\frac{T^4}{(2-q)(3-2q)(4-3q)}$$

where g is the particle degeneracy factor.

Prior to calculating thermodynamic quantities for Au+Au and Cu+Cu collisions at kinetic freeze-out at RHIC using Eqs. (7, 8, 9, 10, and 11), the Tsallis parameter q and temperature parameter T must be obtained. To achieve this, we fitted the transverse momentum spectra of charged particles for Au+Au and Cu+Cu collisions at $\sqrt{s_{NN}} = 200$ GeV for different collision centralities using Eq. (6). The results are presented in Fig. 1 [Figure 1: see original paper].

The Tsallis distribution describes the transverse momentum spectra of charged particles with momentum values lower than 8 GeV/c. The fit/data ratios were obtained to characterize fit quality, as shown in the bottom panels of Fig. 1, which demonstrates that most fit/data points fluctuated within 20%, with only a few data points where p_T was either close to 0 GeV/c or close to 8 GeV/c fluctuating within 30%. The corresponding χ^2/NDF values for the fits are also listed in Tables 2 and 3, respectively. The fit quality for peripheral collisions was better than for central collisions, consistent with our previous results [**?**, **?**, **?**]. The transverse momentum spectra of charged particles from Au+Au collisions in the BES program at $\sqrt{s_{NN}} = 7.7-130$ GeV and Cu+Cu collisions at $\sqrt{s_{NN}} = 62.4$ GeV were also fitted with Eq. (6), yielding similar results as shown in Fig. 1.

Tables 2 and 3 list the temperature parameter T and Tsallis parameter q obtained by fitting the transverse momentum spectra of charged particles from Au+Au collisions at $\sqrt{s_{NN}} = 7.7-200$ GeV and Cu+Cu collisions at $\sqrt{s_{NN}} = 62.4, 200$ GeV. As reported in Ref. [**?**], for a given collision energy, as collision centrality increases from most central to peripheral collisions, the temperature parameter T shows a significant decreasing trend while the Tsallis parameter q shows an increasing trend, though the absolute magnitude of the increase is very small. Similarly, for a given collision centrality, the temperature parameter T exhibits a decreasing trend as collision energy increases, whereas the Tsallis parameter q displays the opposite trend.

III. Thermodynamic Variables

In this section, the temperature parameter T and Tsallis parameter q listed in Tables 2 and 3 are used to calculate thermodynamic quantities for relativistic heavy-ion collisions within the framework of thermodynamically consistent Tsallis statistics. Errors propagated from uncertainties in the fitting parameters are also considered. Note that thermodynamic quantities are calculated for charged particles at kinetic freeze-out hereafter.

A. Energy Density

Energy densities for different centralities at various collision energies were calculated using Eq. (7). The energy densities ε in units of GeV/fm^3 are shown in Fig. 2a [Figure 2: see original paper] as a function of centrality, where 0 represents the most central collision and 1 represents the most peripheral collision. Results for Pb+Pb collisions with collision energies $\sqrt{s_{NN}} = 2760, 5020$ GeV obtained from Ref. [?] are also shown. Our results demonstrate that the energy density of the collision system decreases from central to peripheral collisions. The energy density decreases as collision energy increases for collision systems with similar size at a given centrality. The size dependence of the system can also be observed by comparing results from Cu+Cu and Au+Au collisions at the same collision energy and centrality. This may be due to the fact that the atomic number of copper is smaller than that of gold, leading to a Cu+Cu collision system with less stopping power that is more prone to expansion than Au+Au. Similar results are observed for pressure and particle density, as shown in Fig. 3a [Figure 3: see original paper] and Fig. 6 [Figure 6: see original paper], respectively.

According to these results, there is an apparent interplay between the total multiplicity of charged particles produced in collisions (associated with collision energy) and the expansion of the collision system (related to system volume). A higher collision energy results in a larger volume at kinetic freeze-out, which leads to smaller density at the same centrality, resulting in low energy density, pressure, and particle density for high collision energies [?]. The only exception was for Pb+Pb collisions at $\sqrt{s_{NN}} = 2760$ GeV at the LHC, where the total multiplicity of charged particles must increase faster than the volume from $\sqrt{s_{NN}} = 2760$ GeV to 5020 GeV.

For comparison, we determined chemical freeze-out energy density values for the collision systems analyzed in our study. Zhang and Xu [?] obtained chemical freeze-out energy densities for mid-central collisions at $\sqrt{s_{NN}} = 7.7, 14.5, 19.6, 27,$ and 39 GeV using baryon chemical potential and temperature extracted from the statistical model; the densities were 0.49, 0.62, 0.68, 0.69, and 0.69 GeV/fm^3, respectively. These energy densities during chemical freeze-out were much higher than those at kinetic freeze-out. This can be understood from Eq. (13), which indicates that the energy density of massless particles is proportional to the fourth power of temperature, and the temperature at chemical freeze-out is higher than at kinetic freeze-out. Thus, the results are consistent with the evolution of relativistic heavy-ion collision systems.

Figure 2b demonstrates the scaled energy density ε/T^4 versus centrality. Results for ε/T^4 for Pb+Pb collisions at $\sqrt{s_{NN}} = 2760, 5020$ GeV obtained from Ref. [?] are also presented. The dependence of scaled energy density on centrality appears markedly reduced because the kinetic freeze-out temperature T_{kin} strongly depends on centrality [?]. The system size dependence of scaled energy density nearly disappears when comparing Au+Au and Cu+Cu collisions at the same collision energy. In addition, the values of ε/T^4 demonstrate an increasing trend as a function of collision energy.

B. Pressure and Squared Speed of Sound

In the current analysis, pressure at kinetic freeze-out can be obtained from Eq. (10). In Fig. 3a, the pressure in units of GeV/fm^3 shows a significant and expected increase from peripheral to central collisions for a given collision energy. Pressure results for Pb+Pb collisions at $\sqrt{s_{NN}} = 2760, 5020$ GeV obtained from Ref. [?] are also shown. The pressure exhibits the same pattern as particle density, as shown in Fig. 2.

The squared speed of sound can be calculated using Eqs. (11, 12); results are shown in Fig. 3b [Figure 3: see original paper]. Parameters used to calculate the squared speed of sound for Pb+Pb collisions were obtained from Ref. [?]. The values are approximately between 0.26 and 0.275 for all collision energies and centralities. The value for a massless ideal gas is 1/3, which represents the upper limit. The values demonstrate a very small decreasing trend as collision centrality varies from central to peripheral collisions at the same collision energy.

C. Entropy Density

Entropy is a particularly important quantity in statistical mechanics. Values calculated using Eq. (9) are presented in Fig. 4 [Figure 4: see original paper], where entropy density is scaled by T^3. The s/T^3 values for Pb+Pb collisions at $\sqrt{s_{NN}} = 2760, 5020$ GeV obtained from Ref. [?] are shown in the insert. Similar to scaled energy densities in Fig. 2b, the scaled entropy density shows very weak centrality dependence for a given collision energy. No system size effect was observed for Cu+Cu and Au+Au collisions. Furthermore, the values of s/T^3 generally increase as collision energy increases.

The thermodynamic relationship $\varepsilon + P = Ts$ was also explicitly verified and holds true.

As illustrated in Fig. 5 [Figure 5: see original paper], the scaled ε/T^4 and s/T^3 are plotted for the most central collisions (0-5% or 0-6%) and most peripheral collisions (60-80%) from 7.7 to 5020 GeV as a function of $\ln(\sqrt{s_{NN}})$. The scaled ε/T^4 and s/T^3 as functions of centrality demonstrate the same trend. Data points were fitted with power-law functions, indicated by lines in the figure. The curves were similar and fitting parameters were approximately the same when collision centrality was the same. This can be understood from the massless particle limit; the analytical formulas (Eqs. (13, 15)) for ε/T^4 and s/T^3 for massless particles are proportional. Furthermore, the figure demonstrates that the difference in values between most central and peripheral collisions is subtle at high collision energies, which is reasonable because similar nuclear reaction environments are created at different centralities at higher energies.

D. Particle Density

Particle density in units of fm^{-3}, calculated as a function of centrality using Eq. (8), is shown in Fig. 6 [Figure 6: see original paper]. Particle density results for Pb+Pb collisions at $\sqrt{s_{NN}} = 2760, 5020$ GeV obtained from Ref. [?] are also plotted. The patterns of dependence of particle density on collision energy, collision system size, and centrality are the same as those indicated in Fig. 2a for energy density and Fig. 3a for pressure.

IV. Conclusion

In this study, we used the thermodynamically consistent Tsallis distribution to fit the transverse momentum spectra of charged particles from Au+Au collisions at $\sqrt{s_{NN}} = 7.7 - 200$ GeV and Cu+Cu collisions at $\sqrt{s_{NN}} = 62.4, 200$ GeV [?] at RHIC, extracting the corresponding temperature parameter T and Tsallis parameter q. The Tsallis parameter q demonstrates an increasing trend with increasing collision energy and centrality, whereas the temperature parameter T demonstrates the opposite trend.

Substituting T and q into the formulas for thermodynamic quantities of the collision system at kinetic freeze-out in the framework of Tsallis statistics, we investigated the energy density ε, scaled energy density ε/T^4, scaled entropy density s/T^3, pressure P, squared speed of sound, and particle density of charged particles. Errors propagated from uncertainties in the fitting parameters were also considered. The results indicate that energy density, pressure, and particle density exhibit decreasing trends with increasing collision energy for a given centrality. This can be explained by the interplay between the total multiplicity of charged particles produced in collisions and the volume of the collision system. These three thermodynamic quantities also demonstrate a decreasing trend with increasing centrality for a given collision energy.

The squared speed of sound obtained from different centralities was nearly constant at the same collision energy and varied only within a very small range for all collision energies. Both scaled ε/T^4 and s/T^3 increased as collision energy increased and demonstrated very weak dependence on collision centrality. For the scaled energy and entropy densities, the size dependence of the collision system disappeared. The scaled ε/T^4 and s/T^3 demonstrated similar behavior as functions of $\ln(\sqrt{s_{NN}})$ for a given centrality, which can be understood from the analytical formulas in Eqs. (13) and (15). This study complements the work in Ref. [**?**]. In future work, we will study thermodynamic quantities at chemical freeze-out and investigate their evolution in the hadronic phase from chemical to kinetic freeze-out at RHIC and LHC.

Author contributions: All authors contributed to study conception and design. Material preparation, data collection, and analysis were performed by Wei-Hao Wu, Jun-Qi Tao, Hua Zheng, Wen-Chao Zhang, Xing-Quan Liu, Li-lin Zhu, and Aldo Bonasera. The first draft was written by Wei-Hao Wu and Hua Zheng, and all authors commented on previous versions. All authors read and approved the final manuscript.

Data Availability: The data supporting this study are openly available in Science Data Bank at https://www.doi.org/10.57760/sciencedb.j00186.00239 and https://cstr.cn/31253.11.sciencedb.j00186.00239.

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Note: Figure translations are in progress. See original paper for figures.