Effect of Isoscalar Pairing Force on Spin-Isospin Transitions in 42Ca (Postprint)

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Date: 2023-09-04T00:00:00+00:00

Abstract

This study investigates the influence of a Gaussian isoscalar pairing force on β^--direction Gamow-Teller (GT) and spin-dipole (SD) transitions in the nucleus 42Ca using the relativistic quasiparticle random phase approximation (QRPA) model. The results indicate that the isoscalar pairing force is essential for restoring SU(4) symmetry and thereby reproducing the experimentally observed low-energy super GT state in 42Ca. The isoscalar pairing force mixes spin-flip transition components into the low-energy GT state, which enhances the collectivity of this state and substantially increases its transition strength. Additionally, due to the attractive nature of the isoscalar pairing force, the excitation energy of the low-energy GT state is reduced. For SD transitions, the isoscalar pairing force exhibits negligible effects on both excitation energy and transition strength.

Full Text

Introduction

Pairing is a ubiquitous phenomenon in many-body correlated systems. In atomic nuclei composed of protons and neutrons interacting via the strong force [?, ?], nucleon-nucleon pairing interactions can be classified into isovector ($T = 1$) and isoscalar ($T = 0$) channels. Isovector pairing interactions exist between proton-proton, neutron-neutron, and proton-neutron pairs, where proton-proton and neutron-neutron pairing [?] are crucial for describing bulk properties of open-shell nuclei [?, ?] and weakly bound systems [?, ?, ?]. Due to the charge independence of nuclear forces [?], the isovector proton-neutron pairing should equal the proton-proton or neutron-neutron pairing [?]. Isoscalar pairing interactions exist exclusively between protons and neutrons, and currently no experimental observable can directly probe this interaction. For systems with unequal proton and neutron numbers ($N \neq Z$), proton-neutron pairing typically does not contribute to the nuclear ground state because protons and neutrons occupy different single-particle orbitals at the Fermi surface. However, for nuclear spin-isospin excitations, even in $N \neq Z$ systems, proton-neutron pairing interactions must be considered [?].

Nuclear spin-isospin excitations provide effective constraints on nuclear force information. The primary modes of nuclear spin-isospin excitation include Gamow-Teller (GT) and spin-dipole (SD) excitations. Experimentally, Fujita et al. [?, ?] discovered through charge-exchange reactions on ^{42}Ca that most of the GT transition strength concentrates in the lowest excited state of ^{42}Sc at 0.6 MeV, revealing the existence of low-energy GT phonon excitations. This reflects the restoration of SU(4) symmetry in ^{42}Ca, and this low-energy excited state is termed a super GT state. To explain this phenomenon, Bai et al. [?] employed the self-consistent Skyrme Hartree-Fock-Bogoliubov plus quasiparticle random phase approximation (SHFB+QRPA) model, demonstrating that introducing isoscalar pairing can reproduce the low-energy super GT state in ^{42}Sc. Sun et al. [?] reached the same conclusion through large-scale shell model calculations, namely that isoscalar pairing is essential for forming the 1^+ state in ^{42}Sc. For SD transitions in nuclei, studies on isoscalar pairing remain scarce. Yoshida et al. [?] investigated the effect of zero-range isoscalar pairing on SD transition strength distributions using a three-body model, showing that isoscalar pairing influences configurations with high orbital angular momentum and identical principal quantum numbers in SD transitions.

The Random Phase Approximation (RPA) theory can study nearly all nuclei across the nuclear chart except for a few light nuclei and has been widely applied to nuclear collective excitations [?, ?, ?, ?, ?, ?, ?]. In RPA theory, self-consistency requires using the same energy density functional to describe both nuclear ground states and excited states. Self-consistency is crucial for restoring symmetries broken by the mean-field approximation, eliminating spurious states from physical excitations, and extrapolating theoretical results to unexplored regions [?, ?]. Depending on the density functional, RPA theory can be categorized as non-relativistic RPA [?, ?, ?, ?] or relativistic RPA [?, ?, ?]. Covariant density functional theory has achieved tremendous success in studying both nuclear ground states and excited states [?, ?, ?, ?, ?]. Based on the relativistic Hartree-Bogoliubov (RHB) or relativistic Hartree-Fock-Bogoliubov (RHFB) models, proton-neutron QRPA models [?, ?] have been established to investigate the effects of isoscalar pairing on GT transitions [?, ?, ?, ?] and β decay [?]. However, the influence of isoscalar pairing on SD transitions and the low-energy super GT state in $N \neq Z$ nuclei remains unstudied. Therefore, this work employs the RHB+QRPA model to investigate the effects of isoscalar pairing on β^--direction GT (GT^-) and SD (SD^-) transitions in ^{42}Ca.

1. Theory and Methods

1.1 QRPA Equation

The relativistic QRPA equation can be derived from the time-dependent RHB model through the small-amplitude approximation [?], which is equivalent to the linear Bethe-Salpeter equation [?]. The linear Bethe-Salpeter equation is an integral equation in momentum or coordinate space. Since QRPA equations are typically written in the canonical single-particle basis, the integral becomes a sum over canonical basis indices. For spherical even-even nuclei, the angular-momentum-coupled proton-neutron relativistic QRPA (PNQRPA) equation can be written as:

$$\begin{pmatrix} A_J^{pn,p'n'} & B_J^{pn,p'n'} \\ -B_J^{pn,p'n'} & -A_J^{pn,p'n'} \end{pmatrix} \begin{pmatrix} X_{\nu J}^{pn} \\ Y_{\nu J}^{pn} \end{pmatrix} = E_\nu \begin{pmatrix} X_{\nu J}^{p'n'} \\ Y_{\nu J}^{p'n'} \end{pmatrix}$$

where p and n denote proton and neutron states in the canonical basis, respectively; $u_{p,n}$ and $v_{p,n}$ are the unoccupied and occupied amplitudes of single-particle orbitals in the canonical basis; $H_{pp'}$ and $H_{nn'}$ are composed of the single-nucleon Hamiltonian h_D and the pairing field, with the form ($\kappa\kappa' = pp'$ or nn'):

$$H_{\kappa\kappa'} = (u_\kappa u_{\kappa'} - v_\kappa v_{\kappa'})h_{\kappa\kappa'} - (u_\kappa v_{\kappa'} + v_\kappa u_{\kappa'})\Delta_{\kappa\kappa'}$$

The matrix elements $A_J^{pn,p'n'}$ and $B_J^{pn,p'n'}$ in the canonical basis are expressed as:

$$A_J^{pn,p'n'} = H_{pp'}\delta_{nn'} + H_{nn'}\delta_{pp'} + (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})V_J^{PH} + (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'})V_J^{PP}$$

$$B_J^{pn,p'n'} = (u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'})V_J^{PH} + (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'})V_J^{PP}$$

where V_J^{PH} and V_J^{PP} are the particle-hole and particle-particle residual interaction matrix elements between protons and neutrons, respectively.

To ensure completeness of the QRPA model, the configuration space includes not only proton-neutron pairs from the Fermi sea but also pairs composed of fully or partially occupied states from the Fermi sea and negative-energy states from the Dirac sea. Including configurations with negative-energy states from the Dirac sea is essential for satisfying the Ikeda sum rule [?, ?]. In the PN-QRPA equation, E_ν is the eigenenergy of the excited state $|\nu J\rangle$, while $X_{\nu J}^{pn}$ and $Y_{\nu J}^{pn}$ are the two-quasiparticle transition amplitudes for the excited state

$|\nu J\rangle$. The matrix $A_J^{pn,p'n'}$ represents the particle-hole residual interaction matrix elements between protons and neutrons, obtained by differentiating the single-particle Hamiltonian with respect to the proton-neutron density ρ_{pn}. In RHB calculations, when Fock terms are omitted, the contribution from pion exchange vanishes due to parity conservation. However, pion contributions are crucial for spin-related excitations and are therefore included in QRPA calculations. To eliminate divergences arising from pion introduction, a zero-range Landau-Migdal term must be included, with strength g' as a free parameter determined by reproducing the GT resonance energy in ^{208}Pb. For the DD-ME2 interaction [**?**], $g' = 0.52$. $V_{pn,p'n'}^{PP}$ represents the particle-particle residual interaction matrix elements between protons and neutrons. For the isovector channel ($T = 1$), the same pairing interaction form as in the RHB model is adopted, namely the Gogny pairing force with the D1S parameter set [**?**]. For the isoscalar channel ($T = 0$), we employ a Gaussian-type pairing interaction [**?**]:

$$V_{12} = \sum_{j=1}^{2} g_j e^{-(r_1-r_2)^2/\mu_j^2} \hat{P}_{S=1,T=0}$$

where $\mu_1 = 1.2$ fm, $\mu_2 = 0.7$ fm, $g_1 = 1$, $g_2 = -2$; $\hat{P}_{S=1,T=0}$ is the projection operator onto proton-neutron coupled states with quantum numbers $S = 1$ and $T = 0$; V_0 denotes the strength of the isoscalar pairing force. Since canonical single-particle wavefunctions are typically expressed in coordinate space, calculating residual interaction matrix elements requires integration over coordinates, which consists of a product of radial and angular integrals.

1.2 Transition Strengths

By solving the QRPA equation, the transition strength induced by the spin-isospin operator $\hat{T}_J$ can be expressed as:

$$B_\nu = \left| \sum_{pn} \langle p \| \hat{T}_J \| n \rangle (X_{\nu J}^{pn} u_p v_n + Y_{\nu J}^{pn} v_p u_n) \right|^2$$

To obtain a continuous response function $R(E)$, we can introduce Lorentzian smoothing of the discrete transition strengths B_ν:

$$R(E) = \sum_\nu \frac{1}{\pi} \frac{\Gamma/2}{(E - E_\nu)^2 + \Gamma^2/4} B_\nu$$

where Γ is the smoothing width, taken as 1 MeV in calculations. For β^--direction transitions, the transition amplitude corresponding to a specific configuration can be defined as [**?**]:

$$A_{pn} = \langle p \| \hat{T}_J \| n \rangle (X^{pn}_{\nu J} u_p v_n + Y^{pn}_{\nu J} v_p u_n)$$

For a given nuclear excited state, this transition amplitude can be used to evaluate the contribution from a specific proton-neutron configuration.

For GT^- and SD^- transitions, the corresponding spin-isospin operators are [?]:

$$\hat{T}_{GT^-} = \sum_i \sigma(i) \tau_-(i)$$

$$\hat{T}_{SD^-} = \sum_i r_i [\sigma(i) \otimes Y_1(i)]_{J=0,1,2} \tau_-(i)$$

Using the QRPA model, we calculated the GT and SD transition strength distributions in ^{42}Ca. For solving the ground-state RHB equation, we employed a spherical harmonic oscillator basis with the major shell number taken as 20. In the QRPA model, for two-quasiparticle configurations from both the Fermi sea and Dirac sea, the maximum energy cutoff was set at 2000 MeV, and the minimum cutoff for the occupation factor product $u_p v_n$ or $u_n v_p$ was taken as 0.01.

2. Results and Discussion

2.1 GT Transitions

[Figure 1: see original paper] presents the GT^- transition strength distributions in ^{42}Ca for different isoscalar pairing strengths. These results were obtained from RHB+QRPA calculations using the DD-ME2 interaction, with isoscalar pairing strengths of $V_0 = 0$ MeV, 100 MeV, 200 MeV, and 250 MeV. When $V_0 = 0$ MeV, two GT excited states appear at energies of 8.12 MeV and 15.46 MeV, labeled as A and B in the figure. At $V_0 = 100$ MeV, a GT excited state emerges at 13.62 MeV (labeled C), while the strength of state B decreases significantly and state A's strength increases, with its excitation energy shifting to lower energies. As the isoscalar pairing strength increases further to 200 MeV, the strength of GT state A continues to rise and its excitation energy moves to even lower energies. For GT state C, its strength decreases and its excitation energy also shifts lower. State B shows minimal changes in both strength and excitation energy. When $V_0 = 250$ MeV, state A's strength continues to increase and its excitation energy continues to decrease, while state C nearly disappears. The figure also shows the experimentally observed low-energy super GT state [?, ?]. At $V_0 = 200$ MeV, the GT^- strength primarily concentrates in state A, in good agreement with experiment, demonstrating that introducing isoscalar pairing can reproduce the experimental low-energy super GT state and restore SU(4) symmetry in ^{42}Ca. However, the calculated energy is lower than the experimental value, possibly due to the choice of particle-hole interaction.

To understand the evolution of the GT^- transition strength distribution in ^{42}Ca with isoscalar pairing strength, we analyzed the configurations of each excited state. In [Figure 1: see original paper], the dominant two-quasiparticle configurations for GT states A, B, and C are $(\nu 1f_{7/2}, \pi 1f_{7/2})$, $(\nu 1f_{7/2}, \pi 1f_{5/2})$, and $(\nu 1f_{5/2}, \pi 1f_{7/2})$, respectively. According to the equations, the contributions from particle-hole and particle-particle residual interactions are related to the coefficients $c_{ph} = v_p u_n v_{p'} u_{n'} + u_p v_n u_{p'} v_{n'}$ and $c_{pp} = u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}$ in front of the matrix elements. presents the coefficients c_{ph} and c_{pp} for two-body matrix elements composed of configurations $(\nu 1f_{7/2}, \pi 1f_{7/2})$, $(\nu 1f_{7/2}, \pi 1f_{5/2})$, and $(\nu 1f_{5/2}, \pi 1f_{7/2})$. The table shows that for the dominant two-quasiparticle configuration $(\nu 1f_{5/2}, \pi 1f_{7/2})$ of GT state C, the coefficient c_{pp} is much larger than c_{ph}, indicating that this configuration is primarily affected by particle-particle residual interactions rather than particle-hole interactions. When isoscalar pairing is absent ($V_0 = 0$ MeV), the QRPA matrix non-diagonal elements related to configuration $(\nu 1f_{5/2}, \pi 1f_{7/2})$ are very small, so this configuration does not mix with others. Since the corresponding occupation factor product $u_p v_n$ is only 0.08, the transition strength contributed by this configuration is very small, making the transition strength of GT state C nearly zero.

At $V_0 = 100$ MeV, the QRPA matrix non-diagonal elements related to the two-quasiparticle configuration $(\nu 1f_{5/2}, \pi 1f_{7/2})$ increase, enhancing the mixing between GT state C's main component $(\nu 1f_{5/2}, \pi 1f_{7/2})$ and state B's main component $(\nu 1f_{7/2}, \pi 1f_{5/2})$. This mixing increases state C's transition strength while decreasing state B's strength. Additionally, the attractive nature of isoscalar pairing makes the diagonal elements of the particle-particle residual interaction matrix negative, shifting states A and B to lower energies. However, for state B, the QRPA matrix non-diagonal elements formed by configurations $(\nu 1f_{7/2}, \pi 1f_{5/2})$ and $(\nu 1f_{5/2}, \pi 1f_{7/2})$ are positive, largely offsetting the reduction in diagonal elements, resulting in minimal energy shift for state B.

When $V_0 = 200$ MeV, the mixing proportion of configuration $(\nu 1f_{7/2}, \pi 1f_{5/2})$ in state A continues to increase, further enhancing its transition strength. The particle-particle residual interaction matrix elements decrease further, lowering state A's excitation energy. Meanwhile, state C further mixes with state A's main configuration $(\nu 1f_{7/2}, \pi 1f_{7/2})$. Since the transition amplitude contributed by this configuration has opposite phase to that from state B's main component $(\nu 1f_{7/2}, \pi 1f_{5/2})$, state C's transition strength decreases. The attractive nature of isoscalar pairing also reduces state C's excitation energy. At $V_0 = 250$ MeV, for the same reasons, state A's transition strength becomes even larger while state C's becomes smaller, and both shift to lower excitation energies.

To examine model dependence, we performed the same calculations using the relativistic point-coupling interaction PC-PK1. The results show that the effect of isoscalar pairing on GT states in ^{42}Ca is consistent with the DD-ME2 interaction. The differences lie in the excitation energies and transition strengths of the high- and low-energy peaks of GT states in ^{42}Ca, which are related to the model dependence of residual interaction matrix elements.

2.2 SD Transitions

To investigate the effect of isoscalar pairing on SD^- transitions in ^{42}Ca, [Figure 2: see original paper] presents the strength distributions for 0^-, 1^-, and 2^- transitions at different isoscalar pairing strengths. These results were also obtained from RHB+QRPA calculations using the DD-ME2 interaction, with isoscalar pairing strengths of $V_0 = 0$ MeV, 150 MeV, 300 MeV, and 500 MeV. The figure shows that as the isoscalar pairing strength increases, the strength distributions for 0^-, 1^-, and 2^- transitions change only slightly. This occurs because the particle-particle residual interaction matrix elements formed by the dominant transition configurations in SD^- transitions of ^{42}Ca are relatively small. Consequently, varying the isoscalar pairing strength produces minimal changes in QRPA matrix elements, resulting in insignificant variations in both the strengths and excitation energies of peaks in the SD^- transitions. The main configurations for the dominant peaks in 0^- and 1^- transitions are $(\nu 1d_{5/2}, \pi 1f_{5/2})$, while for 2^- transitions it is $(\nu 1d_{5/2}, \pi 1f_{7/2})$. Only when the isoscalar pairing strength becomes very large (500 MeV) do 0^- and 1^- transitions exhibit some degree of configuration mixing, producing new low-intensity peaks in the low-energy region. The main component of this peak in 0^- transitions is $(\nu 1f_{7/2}, \pi 2g_{7/2})$, while in 1^- transitions it consists of $(\nu 1f_{7/2}, \pi 1g_{7/2})$ and $(\nu 1f_{7/2}, \pi 2g_{9/2})$.

3. Conclusion

Using the RHB+QRPA model, we have investigated the effects of Gaussian-type isoscalar pairing on GT^- and SD^- transitions in ^{42}Ca. For GT^- transitions, increasing isoscalar pairing strength induces mixing between the main components of different GT excited states, leading to enhancement of the low-energy peak and reduction of the high-energy peak. Due to the attractive nature of isoscalar pairing, the GT^- transition strength distribution shifts toward lower excitation energies, with noticeable shifts for GT excited states dominated by configurations $(\nu 1f_{7/2}, \pi 1f_{7/2})$ and $(\nu 1f_{5/2}, \pi 1f_{7/2})$. At $V_0 = 200$ MeV, the GT^- transitions show good agreement with experiment, confirming the important role of isoscalar pairing in restoring SU(4) symmetry. For SD^- transitions, because the particle-particle residual interaction matrix elements formed by the main configurations of peaks in 0^-, 1^-, and 2^- transitions are relatively small, changes in isoscalar pairing strength do not significantly affect the SD^- transition strength distribution.

Author Contributions: Guo Liang performed the theoretical calculations and wrote the manuscript; Niu Yifei reviewed and revised the manuscript.

Acknowledgments: This work was supported by the National Key Research and Development Program (No. 2021YFA1601500) and the National Natural Science Foundation of China (No. 12075104).

References

[References section preserved exactly as in original]