Bayesian Neural Network Methods for Nuclear Beta Decay Lifetimes: Postprint

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Abstract

β-decay lifetime is one of the fundamental physical properties of atomic nuclei and plays an important role in both nuclear physics and nuclear astrophysics. This paper employs the Bayesian Neural Network (BNN) method to predict the β-decay lifetimes of atomic nuclei and their uncertainties. The study finds that introducing β-decay energy and physical quantities related to nuclear pairing effects in the neural network input layer, and using the logarithm of β-decay lifetime as the network output, can significantly improve the learning accuracy. For nuclei with lifetimes less than 1 s, the prediction accuracy is approximately 0.2 orders of magnitude, achieving accuracy comparable to that obtained when using the BNN method to learn the logarithmic difference between experimental and theoretical values of β-decay lifetimes. When extrapolated to unknown nuclear regions, the predicted β-decay lifetimes agree well with results from other theoretical models within the error range, especially for nuclei with Z 50.

Full Text

Preamble

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Studies of Nuclear β-Decay Half-Lives with Bayesian Neural Network Approach

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Abstract

β-decay half-life is one of the fundamental physical properties of atomic nuclei and plays an important role in both nuclear physics and nuclear astrophysics. This paper employs the Bayesian Neural Network (BNN) method to predict nuclear β-decay half-lives and their uncertainties. The study finds that incorporating β-decay energy and physical quantities related to nuclear pairing effects into the neural network input layer, while using the logarithm of β-decay half-life as the network output, can significantly improve learning accuracy. For nuclei with half-lives shorter than 1 s, the prediction accuracy is approximately 0.2 orders of magnitude, achieving precision comparable to that obtained by the BNN method learning the logarithmic difference between experimental and theoretical β-decay half-lives. When extrapolated to unknown nuclear regions, the predicted β-decay half-lives agree well with results from other theoretical models within error bounds, particularly for nuclei with Z < 50.

Keywords: β-decay half-lives, neural network, Bayesian method

Abstract (English)

[Background] β-decay half-life is one of the fundamental physical properties of unstable nuclei and plays an important role in nuclear physics and astrophysics. [Purpose] This study aimed to provide accurate nuclear β-decay half-life predictions and reasonable uncertainties associated with the predictions. [Methods] Nuclear β-decay half-lives were studied based on the Bayesian neural network (BNN) approach. Three types of neural networks with x = (Z, N), x = (Z, N, Qβ), and x = (Z, N, δ, Qβ) were constructed as inputs to explore the influence of the input on the prediction. The posterior distributions were sampled using the Markov chain Monte Carlo algorithm. The mathematical expectations and standard deviations of the neural network predictions on the posterior distributions were used as the predicted values and errors of the BNN approach. [Results] The learning accuracy can be significantly improved by incorporating the β-decay energy and physical quantity related to the nuclear pair effect into the neural network input layer and then using the logarithm of β-decay half-life as the network output. For nuclei with half-lives of less than 1 s, the prediction accuracy is approximately 0.2 orders of magnitude, which is similar to that afforded by the BNN method by learning the differences between the logarithms of the experimental half-lives and theoretical results. [Conclusions] The Bayesian neural network can accurately predict β-decay half-lives. When extrapolated to the unknown nuclear region, the predicted β-decay half-lives agree with the results of other theoretical models within errors, especially for nuclei with Z < 50.

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1. Introduction

The origin of heavy elements in the universe is a hot topic of common interest in nuclear physics and nuclear astrophysics [1,2]. The rapid neutron capture process (r-process) synthesizes about half of the elements heavier than iron through successive neutron captures and β decays. β-decay is a key process in the r-process, where β-decay half-lives determine the timescale of the r-process and have become a research hotspot in recent years [3,4]. Although significant progress has been made in measuring β-decay half-lives [5,6], the decay half-lives of many nuclei on the r-process path remain unmeasurable, particularly for nuclei near N = 126. Therefore, theoretical predictions of β-decay half-lives are of great importance.

Theoretical models for studying nuclear β-decay half-lives mainly include phenomenological formulas [7,8], Gross Theory (GT) [9-12], and the Quasiparticle Random Phase Approximation (QRPA) [13-22]. The shell model can be successfully applied to describe β-decay half-lives of light nuclei or those near magic numbers. However, due to the excessively large configuration space, the shell model cannot be used to study medium and heavy mass nuclei far from magic numbers. Phenomenological formulas, Gross Theory, and QRPA methods can be used to describe β-decay half-lives of most nuclei in the nuclear chart, yet their prediction accuracy still cannot meet the needs of r-process studies. When extrapolated to unknown nuclear regions, the predictions of various theories still show large deviations, especially in the heavy nuclear region with N = 126 [23]. Due to the complexity of strong interactions and nuclear many-body calculations [24,25], precise theoretical predictions of nuclear β-decay half-lives are quite difficult. Consequently, accurate prediction of nuclear β-decay half-lives remains an important but unresolved problem in nuclear physics.

In recent years, machine learning has achieved remarkable success and become one of the most popular and fastest-growing directions in science and technology [26]. The combination of machine learning and physics is an emerging interdisciplinary frontier that has attracted widespread attention. It has been applied in particle physics [27,28], condensed matter physics [29,30], and astrophysics [31,32], providing a powerful tool for physical research. Machine learning is powerful in extracting relevant features from complex nonlinear systems and can be used to solve complex physical problems that are difficult or currently impossible to address with traditional methods. In nuclear physics, machine learning methods have also been widely used to study various nuclear properties, such as nuclear masses [34-38], charge radii [36,37], α decay [38,39], low-lying spectra [40,41], and nuclear β-decay half-lives [42,43].

The Bayesian Neural Network (BNN) method can automatically avoid overfitting by introducing priors and can quantify the uncertainty of model predictions, making it an important tool for studying nuclear properties. In recent years, BNN has been successfully used to study nuclear masses [44-48], charge radii [49], low-lying excitation spectra [50], and nuclear fission yields [51]. Compared with traditional nuclear theoretical models, these machine learning methods can generally achieve higher prediction accuracy.

Based on the BNN method, reference [52] learned the logarithmic difference between experimental β-decay half-lives and theoretical model half-lives, achieving the highest prediction accuracy at that time. For nuclei with half-lives shorter than 1 s, the accuracy was about 0.2 orders of magnitude, reproducing experimental data within a factor of about 1.6. Different from reference [52], this paper uses the Bayesian neural network method to directly learn nuclear β-decay half-lives, verifies the accuracy of direct machine learning prediction of β-decay half-lives, and analyzes the predictive capability of the BNN method by comparing with experimental data and other nuclear model results, providing a reference for directly learning nuclear properties with the BNN method.

2. Methods

In Bayesian methods, model parameters ω are described using probability distributions. First, based on prior experience, a prior distribution p(ω) is introduced to describe the possible values of ω. Assuming the experimental dataset D = {(x_1, t_1), (x_2, t_2), ..., (x_N, t_N)} is known, the prior distribution p(ω) is updated according to Bayes' theorem:

$$p(\omega|D) = \frac{p(D|\omega)p(\omega)}{\int p(D|\omega)p(\omega)d\omega}$$

where x_n, t_n (n = 1, 2, 3, ..., N) are the input and output data, respectively, and N is the number of experimental data points; p(D|ω) is the likelihood function containing information about parameters ω obtained from observations; p(ω|D) is the probability distribution of parameters ω given data D, called the posterior distribution; p(D) is the distribution of experimental data, which serves as a normalization constant ensuring the posterior distribution is a valid probability density with integral 1 over the entire space.

The likelihood function p(D|ω) is typically taken as a Gaussian distribution, i.e., p(D|ω) exp(- ^{2}/2), where the ^{2} function is expressed as:

$$\chi^2 = \sum_{n=1}^{N} \left[\frac{y(x, \omega) - t_n}{\Delta t_n} \right]^2$$

Here, the standard deviation parameter Δt_n is the noise error associated with the nth data point. Similar to reference [44], the inverse of its square is set as a gamma distribution. After specifying p(ω) and p(D|ω), this paper uses the Markov chain Monte Carlo method to sample the posterior distribution p(ω|D).

In the BNN approach, the function y(x, ω) is described by a neural network. For a single hidden layer neural network, its representation is:

$$y(x,\omega) = a + \sum_{j=1}^{H} b_j \tanh\left(c_j + \sum_{i=1}^{I} d_{ji} x_i\right)$$

where x = {x_i}; ω = {a, b_j, c_j, d_{ji}}; H is the number of neurons in the hidden layer; I is the number of input variables. The total number of parameters in this neural network is (2 + I) × H + 1. Since model parameters are described as probability distributions, the BNN method can naturally provide uncertainty estimates for predictions.

3. Results and Discussion

Figure 2 [Figure 2: see original paper] shows the β-decay half-lives and errors for Sn isotopes predicted by BNN-I2, BNN-I3, and BNN-I4 approaches [5]. By incorporating the physical quantity δ related to nuclear pairing effects, the BNN-I4 predictions for β-decay half-lives are more similar to results from other theoretical models (Figure 3 [Figure 3: see original paper]), suggesting that introducing δ helps the neural network better describe β-decay half-lives of nuclei in unknown regions. Therefore, only predictions based on the BNN-I4 method are presented below.

To further investigate the predictive capability of the BNN-I4 method, Figures 3 and 4 [Figure 4: see original paper] compare the BNN-I4 approach with Relativistic Hartree-Bogoliubov (RHB)+QRPA [17], Finite-Range Droplet Model (FRDM)+QRPA [54], Hartree-Fock-Bogoliubov model with Skyrme force (SHFB)+Finite-Amplitude Method (FAM) [55], SHFB+QRPA [56], and WS4+GT [57] models, using Ni, Sn, and Pb isotopic chains and N = 50, 82, 126 isotonic chains as examples. In known nuclear regions, BNN-I4 predictions agree well with experimental data, while other theoretical models show relatively larger deviations. For lighter nuclei such as Ni isotopic chains and N = 50 isotonic chains, theoretical models generally overestimate experimental half-lives. Therefore, for lighter nuclei, BNN-I4 predicts significantly shorter half-lives for nuclei far from the stability line compared to other theoretical models. However, as nuclear mass increases, BNN-I4 predictions for nuclei far from the stability line become very close to results from other microscopic theoretical models. Additionally, the uncertainty of BNN-I4 predictions gradually increases with distance from known nuclear regions. The study also finds that predicted half-lives for the Ni isotopic chain suddenly increase in the N = 56–58 region, similar to results in reference [52]. However, since the uncertainties of BNN-predicted half-lives for these nuclei are large, future measurements of half-lives for nuclei near this region will help confirm whether this phenomenon is real.

Figure 5 [Figure 5: see original paper] shows the logarithmic difference between experimental data and BNN-I4 predicted β-decay half-lives on the nuclear chart. Nuclei with large deviations from experiments are mainly concentrated near the stability line, i.e., nuclei with longer half-lives. Calculations of half-lives for these nuclei are very sensitive to model parameters and decay energy $Q\beta$. Therefore, similar to other nuclear models, the BNN-I4 method also describes half-lives of nuclei near the stability line relatively poorly. Fortunately, half-lives of these nuclei can be measured directly by experiments. For short-lived nuclei far from the stability line, deviations of neural network predictions for β-decay half-lives are within 0.5 orders of magnitude. Additionally, BNN-I4 predictions for β-decay half-lives of nuclei with $Z < 50$ show larger deviations from experimental values, possibly due to insufficient experimental data for β-decay half-lives in this nuclear region.

To compare differences between BNN-I4 predicted β-decay half-lives and theoretical model predictions, Figure 6 [Figure 6: see original paper] shows the comparison between BNN-I4 predictions and WS4+GT model predictions. Deviations between BNN-I4 predictions and WS4+GT model predictions are generally within 1 order of magnitude. Specifically, in the region $Z < 50$, especially far from the stability line, BNN-I4 predictions agree well with WS4+GT results, consistent with conclusions from Figures 3 and 4. Although BNN-I4 predictions show relatively large deviations from model predictions in nuclear regions far from the stability line between $Z = 20$–50, its high prediction accuracy in known nuclear regions suggests that its predictions in unknown regions may have high credibility. Furthermore, using the BNN method to calculate β-decay half-lives on a large scale can provide nuclear physics inputs for r-process studies, thus being of great significance for understanding the origin of heavy elements in the universe.

4. Conclusion

In summary, this paper employs a machine learning method based on Bayesian neural networks to accurately predict nuclear β-decay half-lives by directly learning from experimental values and provides reasonable uncertainty estimates. The study finds that introducing β-decay energy $Q\beta$ and the physical quantity δ related to nuclear pairing effects into the input layer, while using the logarithm of β-decay half-life as the output, can significantly improve the learning accuracy of the neural network method. When extrapolated to unknown nuclear regions, its predictions are closer to results from other microscopic theoretical models, especially for nuclei with $Z < 50$. However, for light-mass nuclei, although the predicted half-lives in unknown regions are shorter than those from nuclear theoretical models, the predictions in known regions still maintain high accuracy. Future work will focus on developing machine learning methods that incorporate more physical effects or constraints to improve predictive capability for nuclear β-decay half-lives, with emphasis on describing β-decay half-lives of light-mass nuclei, to provide more precise nuclear physics inputs for nucleosynthesis studies.

Author Contributions

All authors contributed to the research conception and design. Material preparation, data collection, and numerical calculations were performed by LI Weifeng, ZHANG Xiaoyan, and NIU Zhongming. The initial draft was written by LI Weifeng. ZHANG Xiaoyan and NIU Zhongming revised the initial draft. All authors read and approved the final manuscript.

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Note: Figure translations are in progress. See original paper for figures.