Design and optimization of diffraction-limited storage ring lattices based on many-objective evolutionary algorithms

Authors: Yin, Hexing, Guan, Jiabao, Tian, Shunqiang, Wang, Jike, Wang, Jike

Date: 2023-08-11T00:00:00+00:00

Abstract

Multi-objective evolutionary algorithms (MOEAs) are typically used to optimize two or three objectives in the accelerator field and perform well. However, the performance of these algorithms may severely deteriorate when the optimization objectives for an accelerator [A1] are equal to or greater than four. Recently, many-objective evolutionary algorithms (MaOEAs) that can solve problems with four or more optimization objectives have received extensive attention. In this study, two diffraction-limited storage ring (DLSR) lattices of the ESRF-EBS [A2] type with different energies were designed and optimized using three MaOEAs and a widely used MOEA. The initial population[A3] was found to have a significant impact on the performance of the algorithms and was carefully studied. The performances of the four algorithms were compared, and the results demonstrated that the grid-based evolutionary algorithm (GrEA) had the best performance. MaOEAs were applied in many-objective optimization of DLSR lattices for the first time, and lattices with natural emittances of 116 pm rad and 23 pm rad were obtained at energies of 2 GeV and 6 GeV, respectively, both with reasonable dynamic aperture and local momentum aperture (LMA). This work provides a valuable reference for future multi-objective optimization of DLSRs.

Full Text

Design and Optimization of Diffraction-Limited Storage Ring Lattices Based on Many-Objective Evolutionary Algorithms

He-Xing Yin[1], Jia-Bao Guan[1], Shun-Qiang Tian[2], and Ji-Ke Wang[1],*

[1]Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shanghai 201210, China
[2]School of Physics and Technology, Wuhan University, Wuhan 430072, China

*Corresponding author. E-mail address: jike.Wang@whu.edu.cn

Abstract

Multi-objective evolutionary algorithms (MOEAs) are typically employed to optimize two or three objectives in accelerator physics and have demonstrated good performance. However, their performance may degrade severely when the number of optimization objectives for an accelerator reaches four or more. Recently, many-objective evolutionary algorithms (MaOEAs) capable of solving problems with four or more objectives have attracted extensive attention. In this study, two diffraction-limited storage ring (DLSR) lattices of the ESRF-EBS type with different energies were designed and optimized using three MaOEAs and a widely used MOEA. The initial population was found to significantly impact algorithm performance and was therefore carefully investigated. The four algorithms were compared, and the results demonstrated that the grid-based evolutionary algorithm (GrEA) exhibited the best performance. For the first time, MaOEAs were applied to many-objective optimization of DLSR lattices, yielding lattices with natural emittances of 116 pm • rad and 23 pm • rad at energies of 2 GeV and 6 GeV, respectively, both with reasonable dynamic aperture and local momentum aperture (LMA). This work provides a valuable reference for future multi-objective optimization of DLSRs.

Keywords: storage ring lattices, many-objective evolutionary algorithms, GrEA algorithm, NSGA-III, MOEA/D

1. Introduction

Fourth-generation synchrotron radiation light sources, also known as DLSRs [1], have been constructed worldwide. Their natural emittance is approximately one to two orders of magnitude lower than that of third-generation light sources, typically on the order of tens or hundreds of picometers, approaching the X-ray diffraction limit. DLSRs employ a multibend achromat (MBA) structure with strong focusing magnets, which significantly reduces emittance but results in high natural chromaticity. High-strength sextupoles can effectively correct chromaticity but introduce large nonlinear forces that may deteriorate beam lifetime and dynamic aperture [2]. Accelerator optimization requires satisfying multiple objectives simultaneously, such as achieving low emittance, good nonlinear performance, and excellent overall performance, which presents an enormous challenge [3].

The optimization of a DLSR lattice is a nonlinear and intricate problem. Previously, the global scan of all stable settings (GLASS) [4] was used to explore the global linear properties of storage rings and provide a systematic method for finding stable optics. However, compared to third-generation light sources, DLSRs contain a remarkably larger number of magnets, making GLASS computationally expensive. Over the last two decades, MOEAs have been widely applied to accelerator optimization [5-9] to determine a set of reasonable solutions for each objective. Among these algorithms, multi-objective particle swarm optimization (MOPSO) [10] and multi-objective genetic algorithm (MOGA) [11] are the most commonly used. Moreover, the High Energy Photon Source (HEPS) [12] study demonstrated that rationally combining MOGA and MOPSO for lattice optimization can yield better results than using either algorithm alone [13].

Recent studies on MOEAs have indicated that classical evolutionary algorithms, such as NSGA-II [14] and SPEA2 [15], become ineffective for many-objective optimization problems (MaOPs) with four or more objectives due to decreased selection pressure toward the Pareto front [16, 17]. When the dimensionality of the objective space increases to four or more, most solutions in a population become non-dominated in early generations [18], degrading algorithm performance to nearly random search levels. Since the number of non-dominated solutions increases exponentially with the number of objectives, the effectiveness of NSGA-II's crowding distance method [19] and other diversity-preserving selection mechanisms diminishes, causing premature convergence to local optima [20].

MaOEAs have received considerable attention in recent years [21], with various methods proposed to enhance evolutionary algorithm performance on MaOPs. These approaches can be roughly classified into five categories [22]: Pareto-dominance modification, indicator-based, decomposition-based, grid-based, and diversity-emphasis methods. Leveraging these approaches, several MaOEAs can effectively solve MaOPs. In the accelerator field, the Shanghai soft X-ray free-electron laser (SXFEL) has applied NSGA-III [23], an MaOEA, to optimize the overcompression mode in the linac for producing large-bandwidth XFEL pulses [24]. Using decomposition, NSGA-III establishes a well-distributed hyperplane and associates all solutions with reference points on this hyperplane to enhance selection pressure and maintain diversity for solving MaOPs. The SXFEL experiment demonstrated that NSGA-III significantly outperforms NSGA-II for more than three objectives. However, NSGA-III's performance varies across different test problems and is even worse than NSGA-II for some cases [25], necessitating further investigation of MaOEA performance in accelerator applications.

MOGA performance depends on the distribution of the initial population [13] and requires a good initial distribution to converge to the true global optima [26]. A stable periodic solution for a lattice is called a stable solution, and a stable solution satisfying constraints is called a feasible solution. DLSRs use many magnets, making it difficult to find stable or feasible solutions through random combinations. In this study, the 2 GeV ring had 28 variables; varying each once produces 2^{28} magnet combinations. On average, only 10-15 stable solutions are obtained from 100,000 randomly generated solutions, with only an extremely small fraction being feasible. Assuming the proportion of feasible solutions in the early optimization stage is very small, these solutions and their feasible offspring will dominate all remaining infeasible solutions. Eventually, all population solutions will be generated from these few solutions, reducing population diversity and causing convergence to local optima.

This study performs many-objective optimization of DLSR lattices for the first time and compares the performance of four different algorithms. Two DLSR lattices and their optimization strategies are described in Section 2. Section 3 discusses the effects of initial solutions obtained through three different methods on evolutionary algorithms. Section 4 presents and analyzes multi-objective optimization results for the 2 GeV ring using four algorithms. Section 5 presents multi-objective optimization results for the 6 GeV ring. Conclusions are summarized in Section 6.

2.1 Designed Lattices

To evaluate algorithm performance on DLSR lattice MaOPs, two lattices were designed with energies of 2 GeV and 6 GeV. The magnetic arrangement of one cell of the 2 GeV ring lattice is shown in Fig. 1 [Figure 1: see original paper] (top). The lattice was designed with a circumference of approximately 300 m and an H7BA focusing structure [27], comprising 12 cells and providing 12 identical straight sections longer than 5.3 m. In addition to the H7BA structure, the focusing cell comprehensively employs reverse bends (RBs) [28] and longitudinal gradient bends (LGBs) [29] with transverse focusing to minimize beam emittance. The focusing structure follows the ESRF-EBS scheme [27], where the space between two pairs of dipoles (first and second, sixth and seventh) is expanded to correct natural beam chromaticity with weakened sextupoles due to the dispersion bump. Each cell's main magnets include seven combined transverse and longitudinal gradient dipoles, ten quadrupoles, four RBs, and six sextupoles. As shown in Fig. 1 (bottom), the 6 GeV ring features an additional focusing quadrupole adjacent to the purple sextupole compared to the 2 GeV ring, as higher-energy storage rings require stronger focusing forces. The 6 GeV ring lattice has 48 cells with an approximate circumference of 1350 m.

The 2 GeV and 6 GeV rings have 28 and 29 variables, respectively. For the 2 GeV ring, the maximum dipole field strength is 1.2 T; the maximum gradient of independent quadrupoles is 60 T/m; and the maximum gradient of combined magnets is 40 T/m. The corresponding maximum limits for the 6 GeV ring are 1.5 T, 80 T/m, and 60 T/m, respectively. As shown in Table 1 , K_1 is the gradient of independent quadrupoles combined in RBs; K_2 is the gradient combined in dipoles; RBA is the RB angle; LA is the longitudinal gradient distribution coefficient for each dipole (the first and second dipoles share the same coefficient). Assuming the angles of four different dipoles in a cell are A, B, C, and D, MB is the angular distribution coefficient, where the ratios a:b:c:d

= A:B:C:D. LB is each dipole's length, and LD is the deviation of distances between different dipoles (these distances are fixed initially). Table 1 lists the variables for both DLSR lattices and their ranges.

2.2 Algorithms

The benchmark algorithm used in this study was NSGA-II, and the three MaOEAs were GrEA [30], NSGA-III, and MOEA/D [31]. Algorithm flowcharts are shown in Fig. 2 [Figure 2: see original paper] and are described below.

NSGA-II is a non-dominated sorting algorithm based on crowding distance. Its basic framework proceeds as follows: First, for a given initial population Q with fixed size N, an offspring population S of the same size is generated through selection, crossover, and mutation. Then, N non-dominated solutions are selected from the combined Q and S populations based on non-dominated sorting [23] and crowding distance to form the new population Q. These steps repeat until convergence or maximum iteration count is reached.

GrEA is a grid-based algorithm for MaOPs with a framework similar to NSGA-II. It introduces a grid mechanism to enhance algorithm convergence and diversity. The non-dominated sorting method in NSGA-II is replaced by grid dominance and grid difference, which effectively strengthen convergence pressure to determine the Pareto front. Additionally, the fitness assignment process is revised through three grid-based criteria: grid ranking (GR), grid crowding distance (GCD), and grid coordinate point distance (GCPD).

NSGA-III is an improved NSGA-II version that solves MaOPs by modifying the crowding distance operator. All NSGA-III steps except the crowding distance operator are identical to NSGA-II. Instead of crowding distance, NSGA-III: first generates many well-distributed reference points using Das and Dennis's systematic approach [32] to maintain diversity; then associates each population member with its nearest reference line after standardization; and finally gives higher priority to non-dominated solutions close to reference lines.

MOEA/D uses uniformly distributed weight vectors to decompose a multi/many-objective problem into a series of single-objective subproblems that are optimized simultaneously. Each subproblem is optimized and updated with information from its neighborhood, determined by the distance between individual weight vectors. Several aggregation methods, such as weighted sum (WS) [33], Tchebycheff [33], and penalty-based boundary intersection (PBI) [34], are used to aggregate individual objective values in MOEA/D.

Three metrics—SumMin, MinSum, and Range—were used to evaluate MaOEA performance [35]. For MaOPs where each objective's optimal value is the minimum, SumMin is the sum of each objective's minimum values in the population, characterizing convergence to the Pareto front edge. MinSum is the minimum value of the sum of individual objectives in the population, characterizing convergence to the Pareto front center. Smaller values indicate better convergence. Range is the sum of each objective's value range in the population, characterizing population diversity. Larger values indicate better diversity. For clarity, all three metric values were divided by the number of objectives.

Inverted generational distance (IGD) [36] and hypervolume (HV) [37] are widely used metrics for comparing MaOEA performance. However, IGD calculation requires knowledge of the true Pareto front, so it was not used in this study. HV measures combined convergence and diversity performance, with larger HV indicating better performance. In this study, HV was the primary metric for evaluating algorithm performance, with the other three metrics serving as supplements.

2.3 Strategy

To achieve high performance and good stability, natural emittance, dynamic aperture, LMA, and brilliance at the central bending magnet and straight section center were selected as optimization objectives for all multi-objective optimizations. Although emittance and brilliance are not independent objectives, they can be used simultaneously because brilliance depends not only on emittance but also on beam optics at the source point. A lattice with larger emittance may have higher brilliance if it has smaller optical functions at the source point. Moreover, several light sources have implemented or proposed replacing the central bending magnet with a superbend to obtain higher-energy photons, such as DIAMOND [38], ALS-U [39], SLS-II [40], Sirius [41], and WHPS [42]. Therefore, using brilliance at the central bending magnet as an objective is reasonable. All calculations and simulations in the following experiments were performed using AT [43].

The two most important storage ring characteristics are brilliance and coherence, and a very effective way to improve them is reducing natural emittance. Therefore, natural emittance was used as an optimization objective. However, a lattice with extremely small emittance may be impractical if its dynamic aperture and LMA are too small for beam injection and lifetime. The dynamic aperture area without energy deviation was used as an optimization objective, denoted as DA. Sufficient LMA is typically required for desired beam lifetime. The LMA determined by transverse beam dynamics tends to be small at locations with large dispersion invariants, such as the H7BA cell arc. In practice, LMA is usually limited by transverse dynamics only in the arc and by the RF bucket height elsewhere. It tends to be largest at the straight section center and smallest at the maximum dispersion position. The LMA at these two positions was obtained through 1000-turn tracking, and their average was used as the LMA optimization objective, denoted as δ [26].

Light from bending magnets can have higher energy, and its brilliance [44] is given by:

$$B = \frac{F}{\sigma_x \sigma_y \sigma'_x \sigma'_y}$$

where F is the peak photon flux, σ_x and σ_y represent bunch sizes in horizontal and vertical directions, and σ'_x and σ'_y are beam divergences. In this study, the central dipole brilliance (BriB) was used as an objective, measured when the photon frequency equaled the characteristic frequency, beam current was 200 mA, and the coupling ratio (vertical to horizontal emittance) was 10%. An insertion device at the straight section center generates synchrotron radiation with brilliance [44]:

$$B = \frac{F}{4\pi^2 \sigma_x \sigma_y \sigma'_x \sigma'_y} \cdot \frac{1}{1 + \frac{1}{2}K^2}$$

where K is the undulator parameter. BriU was generated by a 200 cm long undulator with 25 mm period and $K = 1.5$ at the straight section center under 200 mA current and 10% coupling ratio. By changing variable signs, all objectives were transformed to minimization problems during optimization. The objectives are denoted as ϵ, DA, δ, BriB, and BriU in the text.

Bending magnets inevitably produce dispersion that separates off-energy orbits from the ideal trajectory. Since high-frequency accelerating cavities and insertion devices are mounted in straight sections introducing energy deviations, straight section dispersion was matched to zero during multi-objective optimization to minimize coupling between energy variations and transverse oscillations. Before calculating dynamic aperture and LMA, horizontal and vertical chromaticities were corrected to +1 if the momentum compaction factor was positive, otherwise to -1. In addition to power supply limitations on variables, the following constraints were imposed: (1) natural emittance and tune values must be real numbers to ensure solution stability; (2) maximum β-functions in horizontal and vertical directions must not exceed 30 m.

In the experiments below, simulated binary crossover and polynomial mutation were used as genetic operators with identical settings for all algorithms. For each optimization, the initial population size was 1000. Specific algorithm parameters were set as follows: GrEA used nine grids per dimension; NSGA-III's number of reference points equaled the population size; MOEA/D used the Tchebycheff aggregation function; and neighborhood size was set to 1/10 of the population size (i.e., 100).

3. Optimization Results with Different Initial Populations

Initial multi-objective optimization of the 2 GeV ring showed that most solutions in the first 100 generations of each algorithm were infeasible, with the final optimized natural emittance minimum exceeding 250 pm·rad—much larger than expected. Almost no feasible or stable solutions existed in the initial population. Since this optimization involved 28 variables, the probability of randomly generating a stable solution within the variable ranges was nearly zero, forcing the algorithms to search for stable and feasible solutions in early stages. Once one or several feasible solutions were found, only those dominating other solutions determined the search direction, causing the population to become trapped in local optima.

This section examines three approaches for generating the initial population. The first approach generated initial feasible solutions through transport line design: one storage ring cell was treated as a transport line with initial beam optical functions input at the starting point. Since the starting point was the straight section center (a symmetry point), β-functions were set to suitably small values and dispersion to zero. As the transport line did not require periodic solutions, stability was not a concern. Each variable was then adjusted individually until the subsequent beam optical distribution met three criteria: beam optics inside dipoles satisfied low-emittance requirements; dispersion bump distribution was appropriate; and output beam optical functions equaled or approximated the initial functions. After obtaining the first feasible solution, sufficient solutions were seeded into the initial population.

The second approach generated the initial population by seeding a certain number of feasible solutions. Many solutions were randomly generated within variable ranges, and all satisfying Section 2 constraints were considered feasible. However, only 63 feasible solutions were found among 200 million randomly generated solutions, requiring 23 hours and 45 minutes on 200 Xeon 2.4 GHz CPUs at the Wuhan University Supercomputing Center. The initial population was then seeded with these feasible solutions to save time and computational resources. The third approach directly searched for and selected sufficiently stable solutions as the initial population. Twenty million solutions were randomly generated within appropriate variable ranges, and all stable solutions were identified. A total of 2365 stable solutions were found in 2.4 hours using the same CPUs, with 1000 randomly selected as the initial population.

To investigate initial population effects on algorithms, natural emittance and dispersion were used as optimization objectives with NSGA-II and a maximum of 300 iterations. As shown in Fig. 3 [Figure 3: see original paper], the third approach's results at the 100th generation were much worse than the other two, with rare solutions on the Pareto front. The third approach's initial solutions were randomly generated stable solutions, most of which did not satisfy the given constraints, leading to early-stage feasible solution searches and poor population diversity. Across generations, the first approach consistently produced the best

Pareto front, followed by the second approach. The first approach performed best because its initial solution was seeded with a low-emittance, zero-dispersion solution, though it may not perform equally well for optimizations with more than three objectives.

Table 2 shows diversity and convergence metrics for the final generation across different approaches. The first and second approaches produced different initial populations for NSGA-II multi-objective optimization with 250 maximum iterations. The algorithm converged in the final iteration. Table 2 data show results after normalizing each objective value between zero and one (applied to all subsequent results). The second approach achieved SumMin = 0, indicating smaller values for each objective and its MinSum. However, the second approach's Range and HV were larger. Figure 4 [Figure 4: see original paper] shows the second approach had a smaller lower bound and broader distribution, consistent with the metric results. Similar effects were observed for other algorithms.

These results demonstrate that the second approach outperforms the first in multi-objective optimization. Although the first approach performed best in two-objective optimization, it seeded the initial population with only one feasible solution, restricting the variable space around this solution and making it smaller than the second approach's. The second approach produced a larger initial population variable space, yielding better multi-objective optimization results. When many variables create a large variable space, randomly generated stable solutions are unlikely to be feasible, reducing population diversity and trapping algorithms in local optima. Finding global optimal solutions requires spending sufficient time identifying additional initial feasible solutions for large variable space problems. Therefore, the second approach was adopted for subsequent experiments. Recently, machine learning techniques have been used as simulation substitutes in optimization algorithms [45, 46]. If applied here, machine learning could significantly reduce the computational resources required for the second approach and potentially yield better results when incorporated into evolutionary algorithm iterations.

4.1 Results

This section describes many-objective optimization results for the 2 GeV ring lattice using four evolutionary algorithms. The maximum iteration count was set to 250, as all algorithms converged by the 250th iteration. Table 3 shows algorithm performance ranking by HV as: GrEA > NSGA-III > NSGA-II > MOEA/D. Thus, GrEA demonstrated the best combined convergence and diversity performance. The ascending order of SumMin and MinSum and descending order of Range followed the same ranking. As shown in Fig. 5 [Figure 5: see original paper], GrEA had the smallest lower bound for all objectives and the widest distribution; NSGA-III was second; MOEA/D had a very high lower bound with solutions distributed in a narrow region. This indicates GrEA performed best, followed by NSGA-III, NSGA-II, and MOEA/D.

Table 3. Diversity and convergence metrics for the final generation across four algorithms.

Algorithm	SumMin	MinSum	Range	HV
NSGA-II				
NSGA-III				
MOEA/D				

Figure 6 [Figure 6: see original paper] illustrates Pareto front projections obtained by the four algorithms in the final generation. Overall, NSGA-III's performance was close to but slightly worse than GrEA's. When emittance was below 100 pm · rad, all solutions had dynamic apertures too small for long-term particle survival. In the preferred emittance interval of 100-150 pm · rad (more relevant for practical requirements), GrEA achieved maximum dynamic aperture, LMA, and BriB. GrEA's BriU was close to NSGA-III's and larger than the other two algorithms' values. When emittance was between 250-350 pm · rad, NSGA-II's dynamic aperture was significantly smaller than GrEA's and NSGA-III's, though its BriU was high. This suggests NSGA-II failed to maintain diversity in this emittance interval, resulting in very small β-functions at the straight section center for solutions with small dynamic aperture and high BriU. GrEA could obtain both large dynamic aperture and high BriU in this interval, indicating better diversity maintenance. MOEA/D solutions clustered in a small region, showing poor diversity maintenance and causing the algorithm to fall into local optima. The objective space results confirm GrEA performed best, NSGA-III was slightly inferior, and MOEA/D performed worst, consistent with the performance metrics.

Because increasing objectives exponentially increases non-dominated solutions, all population solutions from approximately the 30th generation onward were non-dominated. Selecting the best or preferred solution from many alternatives is challenging. This study used feature extraction to determine preferred solutions. Features extracted and sorted by ascending natural emittance included: natural emittance, the smaller of positive or negative maximum horizontal dynamic aperture, maximum vertical dynamic aperture, minimum LMA in the entire ring and at the straight section center, and brilliance at the straight section center and central bending magnet. Feature extraction enables direct observation of each solution's information for easy selection. With sufficient dynamic aperture and LMA, solutions with smaller emittance and higher brilliance are preferred. Two competing solutions were selected from GrEA's final generation via feature extraction. Figure 7 [Figure 7: see original paper] shows beam optics for these two lattices with natural emittances of 116.70 pm · rad and 124.00 pm · rad, respectively. Figure 8 [Figure 8: see original paper] shows the first lattice's horizontal dynamic aperture reached 7.5 mm and vertical aperture 3.7 mm without energy deviation, while the second lattice achieved 7.8 mm and 6.8 mm, respectively. Both lattices provided adequate dynamic aperture for particles with $±$2% energy deviation. Figure 9 [Figure 9: see original paper] shows both lattices had sufficiently large LMAs at the straight section center, with minimum LMAs in one cell exceeding 2%.

4.2 Discussion

Diversity represents exploration space size—greater diversity yields larger exploration space and higher probability of finding better solutions, leading to better convergence. MOEA/D divides the population into T neighborhoods, where solutions only crossover and update within their neighborhoods. If one solution outperforms another in a problem, the inferior solution is replaced. Since the 2 GeV ring optimization has strict constraints, only a small fraction of the initial population is feasible. This causes feasible solutions to replace all infeasible solutions in early evolutionary stages, significantly reducing the number of unique solutions in the population. As shown in Fig. 10 [Figure 10: see original paper], MOEA/D's unique solution count was small in early generations, fluctuating between 400-600 after approximately 50 generations, while the other three algorithms maintained close to 1000 unique solutions across all generations. Therefore, MOEA/D's small unique solution count leads to significantly reduced diversity, severely degrading its performance.

NSGA-III uses the achievement scalarization function (ASF) [47] to calculate extreme points for each objective axis and forms a hyperplane through these points, as illustrated on the left side of Fig. 11 [Figure 11: see original paper]. All solutions are normalized by dividing each objective axis's intercept formed by this hyperplane. After normalization, uniformly distributed reference points on the hyperplane (intersecting each objective axis at intercept 1) are connected to the origin to form reference lines. The right side of Fig. 11 shows reference line generation in a two-dimensional objective space. NSGA-III balances diversity and convergence based on solution-reference line relationships, so its ability to find a suitable hyperplane directly determines performance.

As DLSR lattice optimization objectives increase, the objective space shape becomes more complex, causing ASF to encounter negative intercept problems and hyperplane determination failures, as noted by Deb et al. [48]. This study used the maximum of the non-dominated front (MNDF) [48] instead of ASF to determine the hyperplane. MNDF determines the hyperplane by using each Pareto front objective's maximum value as the intercept for each objective axis. Despite using MNDF, NSGA-III did not outperform GrEA, likely because the MNDF hyperplane prevented NSGA-III from operating at full capacity. Identifying a hyperplane in a complex objective space that perfectly enables NSGA-III's mechanism is difficult and requires further research.

GrEA divides the hyperspace into a specified number of grids. Solutions closest to the ideal point or to the grid endpoint nearest the ideal point receive higher priority to enhance selection pressure. To increase diversity, solutions in sparsely populated grids receive higher priority. For example, as shown in Fig. 12 [Figure 12: see original paper], the objective space is divided into five segments per dimension with the ideal point at the square's lower left corner. All five solutions are non-dominated. Solutions A and B have grid coordinates (0, 4) and (1, 4) with grid distances from the ideal point (GR) of 4 and 5 (0+4=4, 1+4=5). Since A's GR is smaller than B's, A receives higher priority. Solutions B and C share the same grid with GR=5, but B receives higher priority for being closer to the endpoint. For diversity, E receives higher priority than A because more solutions exist near A's grid.

Instead of finding a suitable hyperplane in complex objective space, using uniformly distributed grids to achieve good convergence-diversity balance is highly effective and robust. However, this leads to GrEA's converged solutions being distributed near vertices of different grids close to the ideal point. Specifically, the solution distribution in hyperspace forms a series of independent blocks rather than a uniform surface. As shown in Figs. 6 and 13, the Pareto front consisted of many inhomogeneous blocks. GrEA exhibits good parallelism and can be easily applied to massive parallel computations. GrEA's computation time is negligible compared to complex physical simulations. The most impactful GrEA parameter is the number of divisions per dimension, recommended to be set to nine for unknown problems or slightly higher/lower when achieving a good Pareto front approximation is too difficult [30].

---

5. Optimization Results for the 6 GeV Ring

Section 4 demonstrated GrEA's outstanding performance. This section applies GrEA to multi-objective optimization of the 6 GeV ring. The second approach generated an initial population of 1000, with a maximum of 250 iterations. As shown in Fig. 13 [Figure 13: see original paper], the Pareto front began changing slightly and converged by the 200th generation. After convergence, solutions were uniformly distributed in individual blocks, consistent with GrEA's principle.

Two representative lattices with similar emittance were selected (Fig. 14 [Figure 14: see original paper]). The first lattice had smaller β-functions at both the straight section center and central dipole, yielding higher brilliance at these locations but smaller dynamic aperture. As shown in Fig. 15 [Figure 15: see original paper], the first lattice's horizontal dynamic aperture reached 2.1 mm and vertical aperture 2.1 mm, while the second lattice achieved 4.5 mm and 4.4 mm, respectively. Both provided adequate dynamic aperture for particles with $±$2% energy deviation. Figure 16 [Figure 16: see original paper] shows both lattices had good LMAs, with minimum LMAs in one cell exceeding 2%.

6. Conclusion

Two ESRF-EBS-type lattices with different energies were designed for experiments. Initial population distribution affected algorithm performance. For MaOPs with strict constraints and no preferences, increasing randomly generated feasible solution count can enhance population diversity and potentially improve algorithm performance. Three MaOEAs and the widely used MOEA NSGA-II were applied to multi-objective optimization of the 2 GeV ring. GrEA performed best due to its efficient and robust MaOP-solving capability. Because of the objective space's complex shape, NSGA-III's ASF failed to find a suitable hyperplane in this optimization. After substituting MNDF for ASF, NSGA-III outperformed NSGA-II but was slightly inferior to GrEA. MOEA/D could not handle constraints well, showing inability to maintain diversity and falling into local optima, performing even worse than NSGA-II. Representative lattices with good attributes were selected from GrEA results via feature extraction. GrEA was then applied to many-objective optimization of the 6 GeV ring, demonstrating its effectiveness on two lattices with different beam optics at the straight section center and central dipole.

This study is the first to apply MaOEAs to multi-objective optimization of DLSR lattices, demonstrating GrEA's robustness and excellent performance. Solutions with promising properties satisfying all objectives can be obtained for lattices with different energies. This optimization strategy can be easily applied to other DLSRs under construction or other accelerator aspects with more than three objectives.

Data Availability Statement

The data supporting this study's findings are openly available in Science Data Bank at https://www.doi.org/10.57760/sciencedb.j00186.00174 and https://cstr.cn/31253.11.sciencedb.j00186.00174.

Author Contributions

All authors contributed to study conception and design. He-Xing Yin performed the experiments. Jia-Bao Guan performed part of the experiments. Shun-Qiang Tian guided some physical aspects. Ji-Ke Wang contributed to study conception and key manuscript revisions. He-Xing Yin wrote the first draft, and all authors commented on previous versions and approved the final manuscript.

References

1. X.-Z. Liu, S.-Q. Tian, X. Wu, et al., Intra-beam scattering and beam lifetime in a candidate lattice of the soft X-ray diffraction-limited storage ring for the upgraded SSRF, Nucl. Sci. Tech., 32, 83 (2021). https://doi.org/10.1007/s41365-021-00913-y

2. W. Gao, L. Wang, W. Li, Simultaneous optimization of beam emittance and dynamic aperture for electron storage ring using genetic algorithm, Phys. Rev. ST Accel. Beams, 14, 094001 (2011). https://doi.org/10.1103/PhysRevSTAB.14.094001

3. R. Husain, A. Ghodke, Constrained multi-objective optimization of storage ring lattices, Nucl. Instrum. Methods Phys. Res., Sect. A (2018). https://doi.org/10.1016/j.nima.2017.11.077

4. D.S. Robin, W. Wan, F. Sannibale, et al., Global analysis of all linear stable settings of a storage ring lattice, Phys. Rev. ST Accel. Beams, (2008). https://doi.org/10.1103/PhysRevSTAB.11.024002

5. L. Yang, D. Robin, F. Sannibale, et al., Global optimization of an accelerator lattice using multiobjective genetic algorithms, Nucl. Instrum. Methods Phys. Res., Sect. A, 609, 50-57 (2009). https://doi.org/10.1016/j.nima.2009.08.026

6. Y. Li, L. Yang, Multi-objective dynamic aperture optimization for storage rings, Int. J. Mod. Phys. A, 31, 1644019 (2016). https://doi.org/10.1142/S0217751X1644019X

7. Y. Lu, S.C. Leemann, C. Sun, et al., Demonstration of machine learning-enhanced multi-objective optimization of ultrahigh-brightness lattices for 4th-generation synchrotron light sources, Nucl. Instrum. Methods Phys. Res., Sect. A, 1050, (2023). https://doi.org/10.1016/j.nima.2023.168192

8. Z.-Y. Dai, Y.-C. Nie, Z. Hui, et al., Design of S-band photoinjector with high bunch charge and low emittance based on multi-objective genetic algorithm, Nucl. Sci. Tech., 34, 41 (2023). https://doi.org/10.1007/s41365-023-01183-6

9. Y. Zhao, Y. Jiao, S. Wang, Design study of APS-U-type hybrid-MBA lattice for mid-energy DLSR, Nucl. Sci. Tech., 32, 71 (2021). https://doi.org/10.1007/s41365-021-00902-1

10. Z.-H. Bai, L. Wang, Q.-K. Jia, et al., Lattice study for the HLS-storage ring, Chin. Phys. C, 37, 047004 (2013). https://doi.org/10.1088/1674-1137/37/4/047004

11. S.-C. Wang, D.-Y. He, C. Meng, et al., Development and simulation of a gridded thermionic cathode electron gun for a high-energy photon source, Nucl. Sci. Tech., 34, 39 (2023). https://doi.org/10.1007/s41365-023-01195-2

12. Y. Jiao, G. Xu, X.-H. Cui, et al., The HEPS project, J. Synchrotron Radiat., 25, 1611-1618 (2018). https://doi.org/10.1107/S1600577518012110

13. Y. Jiao, G. Xu, Optimizing the lattice design of a diffraction-limited storage ring with a rational combination of particle swarm and genetic algorithms, Chin. Phys. C, 41, 027001 (2017). https://doi.org/10.1088/1674-1137/41/2/027001

14. K. Deb, A. Pratap, S. Agarwal, et al., A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans. Evol. Comput., 6, 182-197 (2002). https://doi.org/10.1109/4235.996017

15. E. Zitzler, M. Laumanns, L. Thiele, SPEA2: Improving the strength Pareto evolutionary algorithm, TIK-report, 103, (2001). https://doi.org/10.1145/2851613.2851667

16. Z. He, G.G. Yen, J. Zhang, Fuzzy-based Pareto optimality for many-objective evolutionary algorithms, IEEE Trans. Evol. Comput., (2013). https://doi.org/10.1109/TEVC.2013.2258025

17. M. Elarbi, S. Bechikh, A. Gupta, et al., A new decomposition-based NSGA-II for many-objective optimization, IEEE Trans. Syst. Man Cybern.: Syst., (2017). https://doi.org/10.1109/TSMC.2017.2654301

18. B. Li, J. Li, K. Tang, et al., Many-objective evolutionary algorithms: A survey, ACM Computing Surveys (CSUR), 48, 1-35 (2015). https://doi.org/10.1145/2792984

19. C.R. Raquel, P.C. Naval Jr, An effective use of crowding distance in multiobjective particle swarm optimization, in Proceedings of the 7th Annual Conference on Genetic and Evolutionary Computation, 257-264 (2005). https://doi.org/10.1145/1068009.1068047

20. C. von Lücken, C. Brizuela, B. Barán, An overview on evolutionary algorithms for many-objective optimization problems, Wiley Interdiscip. Rev.: Data Min. Knowl. Discovery, 9, e1267 (2019). https://doi.org/10.1002/widm.1267

21. S. Bechikh, M. Elarbi, L.B. Said, Many-objective optimization using evolutionary algorithms: A survey, in: Recent Advances in Evolutionary Multi-Objective Optimization, (Springer, 2017), pp. 105-137. https://doi.org/10.1007/978-3-319-42978-6_4

22. Y. Zhou, J. Wang, J. Chen, et al., Ensemble of many-objective evolutionary algorithms for many-objective problems, Soft Computing, 21, 2407-2419 (2017). https://doi.org/10.1007/s00500-015-2037-1

23. K. Deb, H. Jain, An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: solving problems with box constraints, IEEE Trans. Evol. Comput., 18, 577-601 (2013). https://doi.org/10.1109/TEVC.2013.2281535

24. J. Yan, H. Deng, Generation of large-bandwidth x-ray free electron laser with evolutionary many-objective optimization algorithm, Phys. Rev. Accel. Beams, (2019). https://doi.org/10.1103/PhysRevAccelBeams.22.020703

25. H. Ishibuchi, R. Imada, Y. Setoguchi, et al., Performance comparison of NSGA-II and NSGA-III on various many-objective test problems, in 2016 IEEE Congress on Evolutionary Computation (CEC), 3045-3052 (2016). https://doi.org/10.1109/CEC.2016.7744174

26. X. Huang, J. Safranek, Nonlinear dynamics optimization with particle swarm and genetic algorithms for SPEAR3 emittance upgrade, Nucl. Instrum. Methods Phys. Res., Sect. A, 757, 48-53 (2014). https://doi.org/10.1016/j.nima.2014.04.078

27. P. Raimondi, ESRF-EBS: The extremely brilliant source project, Synchrotron Radiat. News, 29, 8-15 (2016). https://doi.org/10.1080/08940886.2016.1244462

28. A. Streun, The anti-bend cell for ultralow emittance storage ring lattices, Nucl. Instrum. Methods Phys. Res., Sect. A, 737, 148-154 (2014). https://doi.org/10.1016/j.nima.2013.11.064

29. A. Streun, A. Wrulich, Compact low emittance light sources based on longitudinal gradient bending magnets, Nucl. Instrum. Methods Phys. Res., Sect. A, (2015). https://doi.org/10.1016/j.nima.2014.10.002

30. S. Yang, M. Li, X. Liu, et al., A grid-based evolutionary algorithm for many-objective optimization, IEEE Trans. Evol. Comput., 17, 721-736 (2013). https://doi.org/10.1109/TEVC.2012.2227145

31. Q. Zhang, H. Li, MOEA/D: A multiobjective evolutionary algorithm based on decomposition, IEEE Trans. Evol. Comput., 11, 712-731 (2007). https://doi.org/10.1109/TEVC.2007.892759

32. I. Das, J.E. Dennis, Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems, SIAM J. Optim., 8, 631-657 (1998). https://doi.org/10.1137/S1052623496307510

33. K. Miettinen, J. Hakanen, D. Podkopaev, Interactive nonlinear multiobjective optimization methods, in: Multiple Criteria Decision Analysis, (Springer, 2016), pp. 927-976. https://doi.org/10.1007/978-1-4939-3094-4_{22}

34. M. Ming, R. Wang, Y. Zha, et al., Pareto adaptive penalty-based boundary intersection method for multi-objective optimization, Sci., (2017). https://doi.org/10.1016/j.ins.2017.05.012

35. H. Ishibuchi, N. Tsukamoto, Y. Nojima, Evolutionary many-objective optimization: A short review, in 2008 IEEE Congress on Evolutionary Computation, 2419-2426 (2008). https://doi.org/10.1109/CEC.2008.4631121

36. H. Ishibuchi, H. Masuda, Y. Tanigaki, et al., Modified distance calculation in generational distance and inverted generational distance, in Interna-

tional Conference on Evolutionary Multi-Criterion Optimization, 110-125 (2015). https://doi.org/10.1007/978-3-319-15892-1_8

37. S. Jiang, J. Zhang, Y.-S. Ong, et al., A simple and fast hypervolume indicator-based multiobjective evolutionary algorithm, IEEE Trans. Cybern., (2014). https://doi.org/10.1109/TCYB.2014.2367526
38. R. Bartolini, G. Cinque, K. Sawhney, et al., Concepts for a low emittance-high capacity storage ring for the Diamond light source, 2016.
39. C. Steier, A. Allézy, A. Anders, et al., Status of the conceptual design of ALS-U, in 9th International Particle Accelerator Conference, (2018). http://dx.doi.org/10.18429/JACOW-IPAC2017-WEPAB104
40. M. Dehler, A. Citterio, T. Garvey, et al., Conceptual Design for SLS-2, in 60th ICFA Advanced Beam Dynamics Workshop on Future Light Sources (FLS'18), Shanghai, China, (2018).
41. J. Citadini, L.N. Vilela, R. Basilio, et al., Sirius-details of the new 3.2 T permanent magnet Superbend, IEEE Trans. Appl. Supercond., (2017). https://doi.org/10.1107/S1600577518012110
42. H. Li, J. Wang, L. Tang, et al., Project of Wuhan Photon Source, in Proceedings of the 12th International Particle Accelerator Conference (IPAC 2021), Online Conference, 346-349 (2021). https://doi.org/10.18429/JACoW-IPAC2021-MOPAB092
43. A. Terebilo, Accelerator Toolbox for MATLAB, Citeseer, 2001.
44. K.J. Kim, Optical and power characteristics of synchrotron radiation sources, Opt. Eng., 34, 342-352 (1995).
45. J. Wan, P. Chu, Y. Jiao, Neural network-based multiobjective optimization algorithm for nonlinear dynamics, Phys. Rev. Accel. Beams, (2020). https://doi.org/10.1103/PhysRevAccelBeams.23.081601
46. J. Wan, P. Chu, Y. Jiao, et al., Improvement of machine learning enhanced genetic algorithm for nonlinear beam dynamics optimization, Nucl. Instrum. Methods Phys. Res., Sect. A, 946, 162683 (2019). https://doi.org/10.1016/j.nima.2019.162683
47. A.B. Ruiz, R. Saborido, M. Luque, A preference-based evolutionary algorithm for multiobjective optimization: the weighting achievement scalarizing function genetic algorithm, J. Global Optim., 62, 101-129 (2015). https://doi.org/10.1007/s10898-014-0214-y
48. J. Blank, K. Deb, P.C. Roy, Investigating the normalization procedure of NSGA-III, International Conference on Evolutionary Multi-Criterion Optimization, 229-240 (2019). https://doi.org/10.1007/978-3-030-12598-1_{19}

Note: Figure translations are in progress. See original paper for figures.