Overview of Research on Orbital Eccentricity in Celestial Motion: From Stellar Systems to Planetary Systems (Postprint)

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Abstract

Eccentricity is one of the key parameters describing the orbital motion of celestial bodies, providing important clues for revealing their dynamical evolution and thereby facilitating the understanding of the processes and underlying physical mechanisms of celestial formation and evolution. With the continuous advancement of astronomical observation technologies, research on celestial orbital motion has extended beyond the solar system, with the studied systems ranging from massive stellar systems to low-mass planetary systems. Focusing on the study of orbital eccentricity of celestial bodies, we review the current progress in both stellar systems (including main-sequence stars, brown dwarfs, and compact stars) and planetary systems (including extrasolar giant planets and low-mass exoplanets such as 'super-Earths' and 'sub-Neptunes'), summarize some commonalities and outstanding issues in eccentricity research across different structural scales, and, in conjunction with current and future astronomical observational facilities and projects, provide prospects for future research on the orbital eccentricity of celestial bodies.

Full Text

Preamble

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A Review of Orbital Eccentricity Studies of Celestial Bodies—from Stellar Systems to Planetary Systems

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Abstract

Orbital eccentricity is one of the key parameters describing the motion of celestial bodies and provides crucial clues for revealing their dynamical evolution, thereby helping us understand the formation and evolutionary processes and the underlying physical mechanisms. With continuous advances in observational techniques, studies of orbital motion have extended beyond the Solar System, encompassing systems ranging from massive stellar systems at the high-mass end to planetary systems at the low-mass end. Focusing on orbital eccentricity research, this paper reviews current progress in stellar systems (including main-sequence stars, brown dwarfs, and compact stars) and planetary systems (including extrasolar giant planets and low-mass exoplanets such as "super-Earths" and "sub-Neptunes"). We summarize commonalities and outstanding issues in eccentricity studies across different structural scales, and discuss future prospects for research on orbital eccentricity in conjunction with current and upcoming observational facilities and missions.

Keywords: planets and satellites: formation, planets and satellites: detection, binary star systems, planetary systems, orbital eccentricity

Kepler's first law famously states that planets in the Solar System orbit the Sun in ellipses, with the Sun located at one focus. The orbital eccentricity e—the ratio of focal length to semi-major axis—characterizes the "ellipticity" of the orbit. An eccentricity of zero corresponds to a perfect circle, while values approaching unity indicate increasingly flattened, elongated orbits. Among the eight Solar System planets, all except Mercury (the planet with the shortest orbital period, $e = 0.206$) have nearly circular orbits ($e < 0.1$, with a mean eccentricity of approximately 0.04). This prevalence of low eccentricities among Solar System planets has become a fundamental assumption in popular formation models such as long-term orbital resonances [1–4], the Nice model [5–8], and the Grand Tack model [9]. Mercury's relatively high eccentricity has spurred numerous theoretical studies of dynamical evolution within the Solar System, including "giant impact" scenarios [10–11] and theories of long-term interactions with other planets like Jupiter [12–16].

As early as 1827, Félix Savary computed the first complete orbit of a binary system—Xi Ursae Majoris—and demonstrated that it was elliptical, confirming that Newtonian mechanics and Kepler's laws apply beyond the Solar System [17]. Subsequent research on binary orbital eccentricity progressed rapidly, with astronomers conducting extensive studies of various binary systems including main-sequence stars, pulsars, and brown dwarfs, yielding numerous important results. However, due to observational limitations, the Solar System remained the only planetary system with precisely measured orbital data until 1995, when Mayor and Queloz discovered the first exoplanet orbiting a main-sequence star—51 Pegasi b [18]. Since then, observations of exoplanetary systems have proliferated, enabling systematic studies of their orbital eccentricities. In what follows, we review and summarize the progress in eccentricity research across stellar and planetary systems at different scales.

2. Orbital Eccentricity Studies of Stellar Systems

Beyond the Solar System, binary star systems are extremely common. Current star formation theories generally suggest that stellar systems tend to form as binaries or higher multiples [19][20] 118. This section primarily introduces measurement methods for binary orbital eccentricity and summarizes research on eccentricity distributions.

2.1. Measurement Methods for Stellar System Orbital Eccentricity

Binary systems can be broadly classified into four categories: visual binaries, spectroscopic binaries, eclipsing binaries, and astrometric binaries. These categories correspond to their observational techniques: visual binaries have sufficiently wide separations to be resolved by telescopes; spectroscopic binaries are identified through periodic shifts in spectral lines; eclipsing binaries have orbital inclinations near 90° (with orbital plane normals roughly perpendicular to the line of sight), causing periodic mutual eclipses and brightness variations; and astrometric binaries are recognized by detecting positional variations of the primary star on the celestial sphere (induced by gravitational perturbations from the companion). These categories can overlap, with some systems observable through multiple methods. This subsection describes how orbital eccentricity is obtained for each technique.

2.1.1. Direct Imaging Method This method is primarily used for visual binaries. Because the components are widely separated, both stars can be directly resolved by telescopes. Direct imaging of visual binaries yields the motion of the companion (the fainter star) relative to the primary (the brighter star). This observed trajectory, called the visual ellipse, represents the projection of the system's true orbital motion onto the plane of the sky. Using the visual ellipse and the observation times at each point, we can calculate the orbital elements—including eccentricity—through Kepler's laws and Newtonian mechanics [21].

2.1.2. Spectroscopic Method The motion in a binary system can be approximated as a two-body problem around the common center of mass, resulting in periodic variations in the stars' positions relative to Earth. As long as the orbital plane is not perpendicular to the line of sight, the stars' orbital velocities will have a line-of-sight component—radial velocity. This quantity characterizes how quickly the stars approach or recede from the observer. When a star moves toward us, its spectral lines are blueshifted (higher frequency); when moving away, they are redshifted (lower frequency). Binaries detected through this technique are called spectroscopic binaries. By fitting the radial velocity curve, we can determine the system's orbital eccentricity [20] 79.

2.1.3. Photometric Method When a binary system's orbital plane is nearly edge-on (inclination 90°), we can observe mutual eclipses between the components. These eclipses cause periodic brightness dips, with the deeper eclipse designated as the primary minimum and the shallower one as the secondary minimum. Continuous photometric monitoring yields the system's light curve. Binaries observed this way are called eclipsing binaries, and fitting their light curves provides the orbital eccentricity [22–25].

2.1.4. Astrometric Method Astrometry provides another way to detect stellar motion relative to the system's center of mass. Unlike spectroscopy (which measures line-of-sight motion), astrometry measures the projected motion on the celestial sphere (transverse motion), similar to direct imaging. However, it tracks the primary star's motion relative to the common center of mass, making it applicable even when the companion is too faint to be observed directly. Measuring the primary's motion yields the system's orbital eccentricity [26]. Notable instruments capable of such measurements include the Hubble Space Telescope (HST) and the Gaia space observatory.

2.1.5. Pulsar Timing Method This method targets binary systems containing rapidly rotating neutron stars. Such pulsars emit periodic electromagnetic pulses, and the pulsar's motion relative to the system's center of mass causes periodic variations in its distance from Earth. Because the speed of light is constant, these distance changes alter the arrival times of the pulses at Earth. Fitting these timing residuals yields the orbital eccentricity [27–29].

2.2. Main-Sequence Binary Systems

Early studies of binary orbital eccentricity suffered from severe selection effects [30–31], but advances in observational technology have enabled systematic investigations, producing clear and consistent results [32–40].

2.2.1. Eccentricity Distribution In 1937, Ambartsumian proposed that binary systems would reach an energy-equipartition state after sufficient dynamical evolution, leading to a Maxwellian thermal distribution of orbital eccentricities [41]. If we denote the probability density function (PDF) of binary eccentricity as P e^{ }, then for a Maxwellian distribution, $P = 2e$, implying 1 [42–44]. However, short-period binaries experience strong gravitational tidal forces, resulting in generally low eccentricities and correspondingly smaller values. For example, solar-type binaries with periods shorter than 10 days have = –0.8 [45]. Even for longer-period (>100 days) solar-type binaries where tidal effects are less pronounced, the eccentricity distribution tends toward uniformity

(0) rather than a thermal distribution [39, 45], as shown in [Figure 1: see original paper]. This figure displays the eccentricity distributions of solar-like binaries across different orbital periods ($\log P = \lg(P/\mathrm{d})$), with the horizontal axis showing relative eccentricity (e/e) and the vertical axis showing the cumulative distribution function (CDF). Here, e $= 1 - (2\ \mathrm{d}/P)$ for $P > 2$ d represents the maximum eccentricity at a given period, reflecting that the minimum orbital period for binaries is 2 days, at which point eccentricity becomes zero. The black dotted lines mark distributions with $= -1$, 0 (uniform), and 1 (thermal). The observed distributions (colored solid lines) all lie above the $= 1$ curve and are closer to $= 0$, suggesting that these stellar systems may have undergone additional dynamical processes that reduce eccentricity. The shortest-period binaries (purple line) fall between $= 0$ and $= -1$, showing the smallest value—a clear signature of tidal circularization.

2.2.2. Eccentricity-Period Relationship

As early as 1936, Finsen et al. studied the eccentricity-period distribution of binaries and found a positive correlation between mean eccentricity and period [46]. This distribution can be well explained by gravitational tidal interactions [47–48]: shorter-period binaries experience stronger tides, resulting in lower eccentricities and shorter observed circularization periods (CP) [49]. The circularization period is a monotonic function of system age, as illustrated in [Figure 2: see original paper], which shows eccentricity versus log period for solar-type binaries of different ages. Panels (a)–(h) correspond to different age subsamples, with the left side showing young systems (ages of a few to hundreds of Myr) and the right side showing old systems (Gyr ages). The horizontal solid line marks $e = 0.01$, while the curve represents the fitted function $e(P)$. The intersection between the curve and the horizontal line defines CP, where $e(CP) = 0.01$ (in practice, a small non-zero value like 0.01 is used [49]). Older binary systems exhibit larger circularization periods because they have experienced tidal circularization over longer timescales, driving the eccentricities of longer-period systems toward zero [49].

As orbital period increases, tidal effects weaken and their influence on eccentricity diminishes. [Figure 3: see original paper] shows the eccentricity distribution for intermediate-period ($10^{\{1.2\}-10}\{2.4\}$ days) solar-type binaries, with the red solid line representing old binaries (age $\tau > 3$ Gyr) and the blue line representing young binaries ($\tau < 0.7$ Gyr). The distributions are essentially identical [45], providing strong observational support for tidal theory.

The eccentricity distribution index should increase with orbital period as tidal effects weaken, approaching $= 1$ (thermal distribution). This trend is clearly observed for massive early-type stars (>5 M) in binary systems, as shown in [Figure 4: see original paper]. The figure plots versus log period for different stellar types: red points represent late-type stars (0.8–5 M), while blue points represent early-type stars (>5 M). The blue points and fitted curve (P) approach $= 1$ at longer periods. However, for late-type binaries (including solar-type stars), stops increasing beyond a certain period [40, 45]. The difference at short periods (<10^{1.2} days) can still be explained by tidal theory, as tides are more effective in cooler late-type stars [50–52]. The discrepancy at longer periods suggests different dynamical evolution histories: early-type binaries have a higher probability of containing more than three stars and may have experienced more violent dynamical evolution, while lower-mass late-type stars may have undergone more effective orbital circularization during their longer pre-main-sequence phases [45].

2.3. Brown Dwarf Systems

Brown dwarfs are generally defined as objects with masses below ~0.08 M , located to the right of M-type red dwarfs in the Hertzsprung-Russell diagram. Their low masses prevent hydrogen fusion, earning them the moniker "failed stars." While traditional theory suggests brown dwarfs form via molecular cloud collapse (top-down) [53–54], bottom-up core accretion models can also produce objects up to several tens of Jupiter masses [55–56]. Because brown dwarfs bridge the mass gap between stars and planets, studying their orbital eccentricities helps compare them with both stellar and planetary populations, providing crucial insights into formation and evolution theories.

2.3.1. Eccentricity Distribution Short-period brown dwarfs experience strong tidal forces, resulting in significantly lower eccentricities for systems with periods <12 days [57]. For intermediate periods (P 1–10^4 days, semi-major axes $a < 20$ AU), brown dwarf eccentricity distributions resemble those of short-period main-sequence binaries. At longer periods (P 10^3–10^5 days, a 5–100 AU), tidal effects become negligible, and brown dwarfs exhibit larger mean eccentricities [58–59]. [Figure 5: see original paper] illustrates this trend, showing eccentricity distributions for close (dashed line) and wide (solid line) brown dwarfs. The close population has between 0 and −1, while the wide population approaches thermal distribution (between 0 and 1), mirroring the $-P$ trend seen in main-sequence binaries. This similarity suggests that brown dwarfs and main-sequence binaries have undergone comparable dynamical evolution.

2.3.2. Eccentricity-Period Relationship Gravitational tides produce a clear positive correlation between eccentricity and period in main-sequence binaries. Despite their lower masses, brown dwarfs are similarly affected by tides at short periods. [Figure 6: see original paper] shows the eccentricity-period distribution for brown dwarfs, which generally follows the same pattern as main-sequence binaries and conforms to the same upper limit e (gray dashed line, Equation (1)). This similarity provides another important validation of tidal theory.

2.3.3. Eccentricity-Mass Relationship The eccentricity distribution of brown dwarf systems also depends on mass. [Figure 7: see original paper] shows the eccentricity-mass diagram for intermediate-period brown dwarfs and planets. Blue squares represent planets, red circles represent low-mass brown dwarfs (<42.5 M), and yellow triangles represent high-mass brown dwarfs (>42.5 M). Most mass measurements come from the radial velocity (RV) method, which yields M sin i (where i is orbital inclination). Since most inclinations are near 90° (edge-on, sin i 1), M sin i is typically close to the true mass. The distribution shows a clear break at 42.5 M , suggesting different origins for the two populations. Low-mass brown dwarfs exhibit an inverse correlation between eccentricity and mass, while high-mass brown dwarfs show no clear trend. This implies that low-mass brown dwarfs may form in protoplanetary disks and have their eccentricities excited by scattering from other massive planets or brown dwarfs. The upper envelope of the low-mass population is well explained by planet-planet scattering models [60], with the two purple curves in [Figure 7: see original paper] corresponding to scattering limits for 20 M and 25 M planets. This agreement suggests that low-mass brown dwarfs have experienced dynamical evolution similar to giant planets.

2.4. Compact Star Systems

Compact stars represent the end products of main-sequence evolution. Depending on the progenitor mass, they become white dwarfs, neutron stars, or black holes. If the supernova explosion at the end of stellar life does not disrupt the binary, the system evolves into a compact binary. Systems containing rapidly rotating neutron stars (pulsars) can be studied using the pulsar timing method described above.

2.4.1. Eccentricity-Period Relationship Compact stars have extremely strong gravitational fields, making tidal effects very efficient at circularizing orbits before the companion also becomes a compact object. Consequently, compact binary eccentricities should theoretically be very close to zero. However, [Figure 8: see original paper] reveals a bimodal distribution [61]. Square points represent double neutron star systems with massive companions, showing large eccentricities ($e > 0.01$), while circular points represent low-eccentricity systems, primarily neutron star-white dwarf binaries.

Neutron star-white dwarf systems with low-mass companions have very small eccentricities (<0.01) due to tidal circularization. However, even these values are "too large" for such strongly gravitating systems. The explanation lies in density fluctuations in the convective envelope of the giant star progenitor, which excite eccentricity through accretion flows during the neutron star's orbital motion [29, 61, 63]. This excitation mechanism correlates positively with orbital period, as shown by the black dashed line in [Figure 8: see original paper].

For double neutron star systems with more massive companions, the supernova explosion that forms the second compact star is not perfectly isotropic, imparting a kick velocity that excites orbital eccentricity [64]. The maximum eccentricity achievable also correlates with period [65]. Additionally, longer-period systems allow the first neutron star to accrete less material from the giant progenitor before the second neutron star forms (the star takes longer to fill its Roche lobe) [66]. Therefore, the supernova ejects more mass, producing larger eccentricities [65]. This leads to a positive eccentricity-period correlation for double neutron stars, as shown in [Figure 9: see original paper]. The figure displays three populations: (i) blue circles with relatively low eccentricities that will merge within a Hubble time; (ii) yellow triangles with longer periods ($P >$ 1 day) and a wide eccentricity range that will not merge; and (iii) green squares with short periods and high eccentricities near the merger threshold, aged ~100 Myr. The evolution tracks show that both eccentricity and period decrease with age, but the distribution is strongly constrained by relativistic isochrones (solid lines). Systems near the 100 Myr isochrone (population iii) will soon merge and disappear from the diagram.

2.4.2. Eccentricity-Companion Mass Relationship Low-eccentricity double neutron stars may form through electron-capture supernovae, which differ from iron-core collapse supernovae. Triggered by electron capture in oxygen-neon-magnesium cores of ~1.37–1.47 M , these explosions eject <0.2 M and produce small kick velocities [65, 72–73]. This can explain the relatively low eccentricities of some neutron star systems. Since this process favors lower-mass systems [66], a positive correlation should exist between companion mass and eccentricity [65, 74], as shown in [Figure 10: see original paper]. However, the small sample size requires confirmation from future studies.

2.4.3. Eccentricity-Spin Period Relationship Neutron stars spin up through accretion from their companions. Longer accretion times and larger accreted masses produce faster rotation (shorter spin periods). Consequently, older neutron stars that have accreted more material should have companions that eject less mass during their supernova explosions, resulting in smaller eccentricity excitation. This predicts a positive correlation between spin period and eccentricity, first discovered by McLaughlin et al. [75] and Faulkner et al. [76] in 2004. However, short-period, high-eccentricity double neutron stars experience strong gravitational wave radiation and have short lifetimes, yet they also undergo longer accretion phases and thus faster rotation. This selection effect might bias the observed correlation [77]. Dewi et al. confirmed the correlation using simulations that excluded short-period systems and showed it can constrain kick velocities [78], as demonstrated in [Figure 11: see original paper]. The figure reveals that excessive kick velocities (left panel) destroy the correlation, while moderate kicks (right panel) reproduce it.

3. Orbital Eccentricity Studies of Planetary Systems

Due to observational limitations, exoplanetary systems remained undetected until the late 19th century. The first exoplanet system was discovered in 1992 by Wolszczan and Frail using pulsar timing around PSR 1257+12 [79]. The first exoplanet orbiting a main-sequence star—51 Pegasi b—was found in 1995 by Mayor and Queloz using the radial velocity method [18], earning them the 2019 Nobel Prize. Since then, RV and transit methods have discovered thousands of exoplanets, enabling comprehensive studies of their orbital eccentricities.

3.1. Measurement Methods for Planetary System Orbital Eccentricity

Exoplanet detection parallels binary star studies, but with planetary-mass companions requiring higher observational precision. This section mirrors Section 2.1, describing analogous methods for planetary systems.

3.1.1. Direct Imaging Method Similar to visual binaries, direct imaging could theoretically measure exoplanet orbits. However, the extreme brightness contrast poses enormous challenges: for a Sun-like star at 10 pc with a Jupiter-like companion at 0.5 arcsec separation, the contrast is ~10^{-9} in visible light, and ~10^{-10} for Earth-like planets [80–81]. Current direct imaging is limited to young, hot, self-luminous giant planets at large separations, typically observed in infrared bands where contrast improves by 2–3 orders of magnitude [82–86]. Next-generation instruments will enable more sensitive observations, including JWST [87–88], CSST's Cool-Planet Imaging Coronagraph (CPIC) [89], Subaru's SCExAO [90–91], Magellan's MagAO-X [92], Roman Space Telescope's CGI [93–94], and future 30-meter-class telescopes (TMT [95], ELT [96], GMT [97]) as well as space missions like HabEx [98] and LUVOIR [99] [80, 100–101].

3.1.2. Radial Velocity (RV) Method The RV method detects the star's motion around the common center of mass caused by planetary perturbations, analogous to spectroscopic binaries. By measuring spectral line shifts, we obtain the stellar radial velocity curve, which can be fitted to determine planetary eccentricity. However, exoplanet detection demands extreme precision (~10 m/s for Jupiter analogs [102]), which is why the first success only came in 1995. Since then, RV has become a major discovery method, with instruments like HARPS finding numerous planets [103]. Limitations include difficulty fitting low signal-to-noise or high-eccentricity systems [100] and strong observational biases favoring short-period, massive planets [100, 104–106].

3.1.3. Transit Method Transit photometry, exemplified by NASA's Kepler mission [107], has become the most prolific exoplanet discovery method. When planetary orbits are nearly edge-on (inclination 90°), planets passing in front of their host stars cause periodic brightness dips (primary transits); secondary eclipses occur when planets pass behind the star. Space missions like Kepler and TESS [108–109] and ground-based surveys like HATNet, WASP, and NGTS [110–113] have discovered thousands of transiting exoplanets. Fitting transit light curves can yield orbital inclination and eccentricity, but for Jupiter-sized planets around Sun-like stars, eccentricity-induced variations are only ~10^{-5} in normalized flux [114], making eccentricity determination difficult. Consequently, only a small fraction of systems have measured eccentricities from transits alone [115–116].

For multi-planet transit samples, however, the transit duration ratio (TDR) can constrain eccentricity distributions [117], though this requires accurate stellar density measurements [118–120]. Additionally, gravitational perturbations from other planets in the system cause transit timing variations (TTV), which can be fitted to determine masses and eccentricities of both transiting and perturbing planets [121–125].

3.1.4. Astrometric Method Astrometry measures the star's projected motion on the celestial sphere due to planetary perturbations, providing full three-dimensional orbital characterization and decoupled mass measurements. It favors nearby stars with widely separated planets. Detecting Jupiter-like planets at 10 pc requires milliarcsecond (mas) precision, while Earth-like planets need microarcsecond (µas) precision—currently unattainable. Gaia achieves tens of µas precision [126–128], while ground-based instruments reach ~100 µas [129–134]. Consequently, only a few giant planets have been confirmed astrometrically, such as in Gaia DR3 [128]. Future astrometry missions like Theia [135], GaiaNIR [136], and China's CHES [137–138] will enable detection and characterization of habitable-zone Earth-like planets with sub-µas precision.

3.1.5. Pulsar Timing Method The first confirmed exoplanet system, PSR 1257+12, orbits a pulsar rather than a main-sequence star [79]. The planets were discovered through timing residuals caused by planetary perturbations, similar to binary pulsar timing. This method can also be applied to other systems with stable periodic signals, including some binaries [100, 139], white dwarf systems [100, 139–140], and extreme horizontal branch (sdB) stars [100, 139, 141–146]. However, suitable systems are rare, yielding few detections.

3.2. Eccentricity Distribution

In the early 21st century, RV surveys discovered numerous giant planets and some Neptune-like worlds (Doppler planets), which have a moderate mean eccentricity ($\bar{e} = 0.29$) [147]. Unlike binary stars, exoplanets are influenced not only by tides but also by planet-planet interactions, planet-disk interactions, and stellar encounters [60, 149–161]. [Figure 12: see original paper] shows exoplanet eccentricity distributions across different periods. Short-period planets ($P <$ 10 days, dashed line) have the smallest (approaching –1), while intermediate-period (1–10^4 days, dot-dashed line) and long-period (10^3–10^5 days, solid line) planets have similar distributions with between 0 and –1.

Most known exoplanets are intermediate-period (1–10^4 days, $a < 20$ AU). Their similarity to intermediate-period brown dwarfs and short-period binaries (between 0 and –1) indicates tidal influence, particularly for the shortest-period planets ($P <$ 10 days) where approaches –1. However, unlike massive binaries and brown dwarfs, long-period planets (10^3–10^5 days, a 5–100 AU) do not approach the thermal distribution (= 1) despite negligible tidal effects. Instead, their distribution resembles that of intermediate-period planets, suggesting that other dynamical processes dominate over tides for both intermediate- and long-period planets [58–59]. This indicates that planets have experienced different dynamical evolution than brown dwarfs and binaries. However, the long-period sample consists mainly of directly imaged giant planets and is subject to selection effects [58], requiring future confirmation.

Current exoplanet eccentricities are primarily measured by RV, biasing the sample toward giant planets and Neptunes. This does not reflect the distribution of lower-mass "super-Earths" and "sub-Neptunes," which constitute 30–50% of planets around Sun-like stars and are even more common around M dwarfs [162–167]. Observing these planets is limited by instrumental precision and selection effects, particularly at long periods, necessitating future technological advances.

3.3. Eccentricity and Planetary Multiplicity

Over half of discovered exoplanets are in single-planet systems. Comparative studies reveal that single-planet systems have significantly higher eccentricities than multi-planet systems [168–173]. Using precise stellar parameters from LAMOST, Xie et al. applied the TDR method to Kepler planets, finding mean eccentricities of ~0.3 for single-planet systems versus ~0.04 for multi-planet systems [172]. [Figure 13: see original paper] shows TDR distributions for Kepler systems with different numbers of transiting planets. As the number of transiting planets (a proxy for system multiplicity) increases, the TDR distribution becomes less dispersed (smaller σ), indicating lower eccentricities. This phenomenon is termed the "eccentricity dichotomy" [172].

This anti-correlation between eccentricity and multiplicity aligns with the Solar System's characteristics: multi-planet systems tend toward circular, coplanar orbits. Proposed explanations include: (1) "natural selection," where low eccentricities ensure long-term stability in multi-planet systems [168, 174–175]; (2) age effects, where older systems have fewer transiting planets due to dynamical excitation of eccentricities and inclinations [176]; (3) external giant planets exciting inner planet eccentricities and reducing observed multiplicity [177]; (4) stellar flybys exciting eccentricities and depleting planet numbers [178]; and (5) protoplanetary disk parameters reproducing the dichotomy [179].

3.4. Eccentricity-Period Relationship

Doppler planets show a positive eccentricity-period correlation similar to binaries, but with both high- and low-eccentricity populations at all periods, indicating that tides are not the only shaping mechanism [147–148]. Winn et al. derived a period-dependent upper limit from tidal theory that matches Equation (1) and correlates positively with period [148], as shown in [Figure 14: see original paper]. The similarity between exoplanet, brown dwarf, and binary eccentricity-period distributions provides further validation of tidal theory.

However, Correia et al. found that short-period ($P < 5$ days) Neptune-like planets (3–9 R) have significantly non-zero eccentricities that show no period dependence, contradicting tidal predictions [180]. Recently, Shin et al. discovered an inverse eccentricity-period correlation for short-period ($P < 150$ days), low-mass ($M < 30$ M) planets [181], as shown in [Figure 15: see original paper]. The best-fit relation is:

$$\mathrm{Ecc} = -0.09 \pm 0.01\ \mathrm{lg(Per/d)} + 0.2 \pm 0.01$$

with a p-value of 10^{-4}, strongly favoring this inverse correlation over a constant model. This opposite trend compared to binaries and giant planets arises from angular momentum deficit (AMD) transfer, suggesting distinct dynamical histories for low-mass planets [181].

3.5. Eccentricity-Mass Relationship

Doppler planets show a positive eccentricity-mass correlation [147–148], as illustrated in [Figure 16: see original paper]. The figure includes both planets and brown dwarfs, with masses expressed as $M \sin i$ for RV detections. The overall positive correlation may reflect that low-mass planets are more commonly found in multi-planet systems with lower eccentricities [182], consistent with the eccentricity dichotomy.

Two vertical lines at $M = 30$ M and $M = 42.5$ M divide the sample into three regions. For giant planets (>30 M) and low-mass brown dwarfs (<42.5 M), an inverse eccentricity-mass correlation is observed (right dotted line), suggesting that planet-planet scattering [60, 157] influences both populations and that low-mass brown dwarfs and giant planets may share origins and dynamical histories [57]. For the lowest-mass planets (<30 M), an inverse correlation may also exist, likely caused by AMD transfer effects [13, 183], which more strongly affect lower-mass planets.

3.6. Eccentricity-Stellar Metallicity Relationship

Doppler planets show a positive correlation between eccentricity and host star metallicity [148], as shown in [Figure 17: see original paper]. Metal-rich host stars (squares) produce planets with higher average eccentricities than metal-poor hosts (circles), particularly for giant planets ($M > 80$ M) where the correlation is significant [184–185]. Metal-rich disks have more solid material, forming more and larger planets that drive stronger planet-planet interactions and eccentricity excitation [184].

For low-mass planets, the correlation appears weaker. Van Eylen et al. found no clear relation for $R < 6$ R planets using asteroseismology [171], while Mills et al. found that high-eccentricity small planets prefer metal-rich hosts [173]. Recent work by An et al. confirmed a positive correlation for $R < 4$ R planets [186].

3.7. Eccentricity-Mutual Inclination Relationship

Like eccentricity, mutual orbital inclinations provide crucial clues about dynamical history. In the Solar System, satellites, asteroids, and trans-Neptunian objects show a linear correlation between eccentricity and inclination. Exoplanet multi-planet systems follow the same trend, as shown in [Figure 18: see original paper] [172]. The figure plots mean eccentricity versus mean mutual inclination for various populations, with the line $\bar{e} = 2\bar{\imath}$ dividing the parameter space. All data lie in the region $\bar{e} < 2\bar{\imath}$ and roughly follow $\bar{e} = \bar{\imath}$ (dashed line), consistent with formation models [187–188]. Exoplanet multi-planet systems have small eccentricities ($\bar{e} < 0.1$) and are nearly coplanar ($\bar{\imath} < 6°$), resembling the Solar System's architecture.

4. Summary and Outlook

This review has summarized progress on orbital eccentricity studies across stellar systems (main-sequence stars, brown dwarfs, compact stars) and planetary systems (giant planets, super-Earths, sub-Neptunes). These investigations have yielded rich results on eccentricity distributions and correlations with orbital period, mass, spin period, metallicity, and other properties. Several common themes emerge:

1. **Comparison with thermal distribution**: Deviations from the Maxwellian thermal distribution ($= 1$) reveal the intensity of dynamical evolution. Short-period systems have < 0 due to tides, while massive early-type binaries and brown dwarfs approach thermal distribution at long periods. Late-type binaries and exoplanets deviate even at long periods, suggesting quieter dynamical histories with eccentricity-damping mechanisms.
2. **Eccentricity-period correlation**: Almost all systems (except short-period low-mass exoplanets) show positive eccentricity-period correlations, as summarized in [Figure 19: see original paper]. This universal trend, following the envelope given by Equation (1), demonstrates that massive bodies have experienced similar dynamical evolution with tidal circularization playing a key role. The contrasting inverse correlation for short-period low-mass planets highlights different evolutionary pathways at low masses.
3. **Eccentricity-mass relationship**: Lower-mass stars and planets generally have smaller mean eccentricities, reflecting gentler dynamical evolution. However, low-mass brown dwarfs, giant planets, and small planets all show inverse eccentricity-mass correlations caused by planet-planet interactions, indicating similar underlying mechanisms across different mass regimes.

Current limitations include: incomplete mass and age measurements for binary samples; limited sample sizes for brown dwarfs and compact stars; and observational incompleteness and selection effects that hinder studies of sub-Earth, Earth-like, super-Earth, and sub-Neptune planets. Many host stars also lack precise characterization.

Future facilities will address these challenges. For binary systems: TESS [109], PLATO [190], and ET2.0 [191] will discover eclipsing binaries; SDSS-V [192], LAMOST [193], and 30-meter telescopes will provide spectroscopic data; Gaia [194], Theia [135], GaiaNIR [136], and CHES [137–138] will deliver astrometric data. These will dramatically expand samples and enable precise measurements of masses, ages, and orbits. The multi-messenger era, with LISA [196] and Chinese space-based detectors [197–198], will revolutionize compact object studies [199–200].

For planetary systems: SDSS-V and LAMOST will characterize host stars; TESS continues Kepler's transit legacy; JWST [201] will study planets through imaging and transits; Gaia DR4 will provide astrometric orbits for giant planets [202]. Next-generation missions—PLATO, ET2.0, Theia, GaiaNIR, CHES, and 30-meter telescopes with advanced coronagraphs—will complete the census of small planets and characterize their orbits, clarifying formation and evolution processes.

Finally, eccentricity studies will extend to exomoon systems. While current knowledge is limited to Solar System satellites, several exomoon candidates have been identified [203–205], heralding a new frontier for orbital eccentricity research.

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