

Exploring the Equation of State of the Early Universe: Insights from BBN, CMB, and PTA Observations

Authors: Zhi-Chao Zhao, Qing-Hua Zhu, Sai Wang, Xin Zhang, Sai Wang, Xin Zhang

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Abstract

Strong evidence for a gravitational-wave background (GWB) has been reported in the nano-Hertz band. Interpreting the origin of this background to be scalar-induced gravitational waves (SIGWs), we explore the equation of state (EoS) of the early universe by performing Bayes parameter inferences across the big-bang nucleosynthesis (BBN), cosmic microwave background (CMB), and pulsar timing array (PTA) joint observations for the first time. Assuming a monochromatic power spectrum for primordial curvature perturbations, we obtain the spectral amplitude $A \sim 10^{-3} - 10^{-1}$ and spectral peak frequency $f_* \sim 10^{-7} - 10^{-6}$ Hz. We find that the radiation domination with EoS $w = 1/3$ is compatible with the current observational data, the kination domination with EoS $w = 1$ is not forbidden, while the early matter domination with EoS $w = 0$ is excluded at more than 2σ confidence level. These results can be tested with future observations.

Full Text

Preamble

Exploring the Equation of State of the Early Universe: Insights from BBN, CMB, and PTA Observations

Zhi-Chao Zhao,¹ Qing-Hua Zhu,² Sai Wang,^{3,4,*} and Xin Zhang^{5,6,7,†}

¹Department of Applied Physics, College of Science, China Agricultural University, Qinghua East Road, Beijing 100083, People's Republic of China

²Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

³Theoretical Physics Division, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China

⁴School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China

⁵Key Laboratory of Cosmology and Astrophysics (Liaoning) & College of Sciences, Northeastern University, Shenyang 110819, People's Republic of China

⁶National Frontiers Science Center for Industrial Intelligence and Systems Optimization, Northeastern University, Shenyang 110819, People's Republic of China

⁷Key Laboratory of Data Analytics and Optimization for Smart Industry (Ministry of Education), Northeastern University, Shenyang 110819, People's Republic of China

Strong evidence for a gravitational-wave background (GWB) has been reported in the nano-Hertz band. Interpreting this background as scalar-induced gravitational waves (SIGWs), we explore the equation of state (EoS) of the early universe by performing Bayesian parameter inference across joint observations of big-bang nucleosynthesis (BBN), cosmic microwave background (CMB), and pulsar timing array (PTA) data for the first time. Assuming a monochromatic power spectrum for primordial curvature perturbations, we obtain a spectral amplitude $A \sim 10^{-3} - 10^{-1}$ and spectral peak frequency $f_* \sim 10^{-7} - 10^{-6}$ Hz. We find that radiation domination with EoS $w = 1/3$ is compatible with current observational data, kination domination with EoS $w = 1$ is not forbidden, while early matter domination with EoS $w = 0$ is excluded at more than 2σ confidence level. These results can be tested with future observations.

* Corresponding author: wangsai@ihep.ac.cn

† Corresponding author: zhangxin@mail.neu.edu.cn

Introduction

The equation of state (EoS) is a critical parameter characterizing the thermal evolution of the universe [1]. However, little is known about the EoS of the early universe during the era between inflation and radiation domination. For an adiabatic fluid, the density ρ and pressure P can be obtained via the Boltzmann approach [2]. In this context, one finds that in the non-relativistic approximation ($p^2 \ll E^2$), the EoS parameter is $w = P/\rho = 0$, while in the relativistic limit ($E^2 \simeq p^2$), we have $w = 1/3$. Therefore, in realistic scenarios, ordinary matter should yield an EoS parameter w ranging from 0 to $1/3$. For example, for thermal plasma exhibiting strong QCD interactions, the evolution of relativistic degrees of freedom causes w to vary within this range [3–5]. If the early universe is filled with a scalar matter field ϕ , the EoS parameter is primarily determined by the effective potential $V(\phi)$ of the scalar field [6–8]. Recently, oscillating scalar field models have been employed to resolve the Hubble tension [9,10] and address the small-scale crisis of large-scale structure [11,12]. Additionally, an epoch between the end of inflation and the onset of radiation domination may be matter-dominated due to primordial black holes (PBHs), oscillons, and other mechanisms [13–17]. For the early universe, it is reasonable to anticipate

that the EoS may exhibit mixed behavior, as described in these scenarios. It is therefore well-motivated to explore the value of the EoS parameter w through phenomenological studies [18–21].

Since the universe is transparent to gravitational waves [22], the EoS of the early universe can be directly probed via gravitational waves produced in that epoch. Scalar-induced gravitational waves (SIGWs) can be nonlinearly produced by linear cosmological curvature perturbations when the latter re-enter the Hubble horizon after inflation [23–28]. In mid-2023, pulsar timing array (PTA) data releases showed significant evidence for a gravitational-wave background (GWB) in the nano-Hertz band [29–32]. This signal has been interpreted as SIGWs by several authors [33–50]. Since the energy-density fraction spectrum of SIGWs is sensitive to the EoS parameter of the early universe [18–20], if such an interpretation is confirmed in the future, SIGWs would become a powerful probe of the early universe, particularly for the EoS (see review in Ref. [51]).

In this work, for the first time, we infer the EoS parameter of the early universe during the era between inflation and radiation domination by conducting Bayesian analysis of the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) 15-year dataset [31]. Other model parameters are inferred simultaneously. Comparing our results with existing literature [52,53], we demonstrate the significant impact of the EoS parameter on the inference of these other parameters. Once produced in the early universe, SIGWs behave like additional relativistic components, thereby increasing the expansion rate of the early universe. Therefore, we incorporate upper bounds on the effective number of relativistic species from big-bang nucleosynthesis (BBN) [54] and cosmic microwave background (CMB) observations [55]. Analyzing this joint dataset further refines our results by more tightly constraining the parameter space.

The remainder of this paper is organized as follows. In Section II, we develop a theory of SIGWs considering different EoS parameters of the early universe. In Section III, we describe our data analysis methodology and present parameter inference results along with their physical implications. Section IV provides concluding remarks.

II. SIGWs Produced in the Early Universe with Different EoS

Limited by our current observational capabilities on such small cosmological scales, we have little knowledge of the matter state in the early universe. In this paper, we phenomenologically describe the early-time universe preceding the radiation-dominated epoch by assuming an EoS $P = w\rho$ with varying values of $w \in [0, 1]$. We study SIGWs produced both in this early-time epoch and in the radiation-dominated epoch, as well as the transition between them.

The perturbed spatially-flat Friedmann-Robertson-Walker (FRW) metric in conformal coordinates is given by

$$ds^2 = a^2(\eta) [-(1 + 2\phi)d\eta^2 + ((1 - 2\psi)\delta_{ij} + h_{ij}) dx^i dx^j]$$

where η is conformal time, $a(\eta)$ is the scale factor, ϕ and ψ are the Newtonian potential and curvature perturbations (linear scalar metric perturbations), and h_{ij} describes second-order tensor metric perturbations (i.e., SIGWs). Due to the absence of anisotropic stress, we have $\psi \simeq \phi$. In a universe filled with perfect fluids with EoS parameter w , the equation of motion for SIGWs is

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij} = -4\Lambda_{ij}^{ab}S_{ab},$$

where Λ_{ij}^{ab} is the transverse-traceless operator, \mathcal{H} is the conformal Hubble parameter, and the source term S_{ab} consisting of linear scalar perturbations ϕ is

$$S_{ab} = \frac{2(5 + 3w)}{3(1 + w)} \partial_a \phi \partial_b \phi + 3(1 + w)\mathcal{H}(\partial_a \phi \partial_b \phi' + \partial_a \phi' \partial_b \phi) + 3(1 + w)\mathcal{H}^2 \partial_a \phi' \partial_b \phi'.$$

The evolution of ϕ is determined by the master equation

$$\phi'' + 3(1 + w)\mathcal{H}\phi' - w\Delta\phi = 0,$$

where we have used the adiabatic sound speed $c_s^2 = w$ and focus on $0 \leq w \leq 1$ for phenomenological studies.

We follow the scenario in Ref. [28] for an extensive study of SIGWs during the transition from the early-time epoch to radiation domination. The scale factor evolves differently in different epochs:

$$a(\eta) \simeq \begin{cases} \left(\frac{\eta}{\eta_R}\right)^{\frac{2}{1+3w}} & \eta < \eta_R \\ \frac{\eta}{\eta_R} & \eta > \eta_R \end{cases}$$

where η_R is the transition time. This leads to a transition of the Hubble parameter from $\mathcal{H} = 2[(1 + 3w)\eta]^{-1}$ in the early-time epoch to $\mathcal{H} = \eta^{-1}$ in the radiation-dominated epoch. Based on this setup, we obtain the solutions for h_{ij} in momentum space from Eq. (2):

$$h_{ij,k}^{(ET)} = (k\bar{\eta})^\beta Y_\beta(k\eta) - (k\eta)^\beta J_\beta(k\eta)$$

$$h_{ij,k}^{(RD)} = \sin(k\eta)h_{ij,k}^{(RD,0)} + \left[\frac{\pi}{2} \cos(k\eta) \int_{\eta_R}^{\eta} \frac{d\bar{\eta}}{\bar{\eta}} \frac{x^{1-\beta} J_\beta(k\bar{\eta}) \Lambda_{ij}^{ab}(\hat{k}) S_{ab,k}^{(ET)}}{(k\bar{\eta})^{1-\beta} Y_\beta(k\bar{\eta}) \Lambda_{ij}^{ab}(\hat{k}) S_{ab,k}^{(ET)}} \right]$$

$$h_{ij,k}^{(RD,1)} + \sin(k\eta) \int_{\eta_R}^{\eta} \frac{d\bar{\eta}}{\bar{\eta}} \cos(k\bar{\eta}) \Lambda_{ij}^{ab}(\hat{k}) S_{ab,k}^{(RD)} + \cos(k\eta) \int_{\eta_R}^{\eta} \frac{d\bar{\eta}}{\bar{\eta}} \sin(k\bar{\eta}) \Lambda_{ij}^{ab}(\hat{k}) S_{ab,k}^{(RD)},$$

where $Y_n(x)$ and $J_n(x)$ are Bessel functions of the second and first kind, respectively, $\beta \equiv -3(1-w)/[2(1+3w)]$, and the superscripts (ET) and (RD) represent “early-time” and “radiation domination,” respectively. We require continuity and differentiability of $h_{ij,k}$, i.e., $h_{ij,k}^{(ET)} = h_{ij,k}^{(RD)}$ and $h_{ij,k}^{(ET)'} = h_{ij,k}^{(RD)'}$ at η_R , which determines the time-independent quantities $h_{ij,k}^{(RD,0)}$ and $h_{ij,k}^{(RD,1)}$ as

$$h_{ij,k}^{(RD,0)} = k\eta_R \cos(k\eta_R) h_{ij,k}^{(ET)}(\eta_R) + [\cos(k\eta_R) + k\eta_R \sin(k\eta_R)] h_{ij,k}^{(ET)' }(\eta_R)$$

$$h_{ij,k}^{(RD,1)} = k\eta_R \cos(k\eta_R) h_{ij,k}^{(ET)' }(\eta_R) + k\eta_R \sin(k\eta_R) h_{ij,k}^{(ET)}(\eta_R).$$

The source terms in Eqs. (6) and (7), denoted as $S_{ab,k}^{(ET)}$ and $S_{ab,k}^{(RD)}$, are provided by Eq. (3) with $\phi^{(ET)}$ and $\phi^{(RD)}$ defined for the early-time and radiation-dominated epochs, respectively. Solving Eq. (4) yields their expressions:

$$\phi^{(ET)} = {}_0F_1 \left(\frac{7+9w}{2(1+3w)}; -\frac{wk^2\eta^2}{4} \right) \Phi_k$$

$$\phi^{(RD)} = \left[\frac{3k\eta \cos(k\eta) + (k^2\eta^2 - 3) \sin(k\eta)}{(k\eta)^3} \right] \phi^{(RD,0)} + \left[\frac{-3 \cos(k\eta) + 3k\eta \sin(k\eta)}{(k\eta)^3} \right] \phi^{(RD,1)}$$

where Φ_k is a random variable related to primordial curvature perturbations and ${}_0F_1$ is the confluent hypergeometric function. The continuity and differentiability of ϕ_k determine $\phi^{(RD,0)}$ and $\phi^{(RD,1)}$ as

$$\begin{aligned} \phi^{(RD,0)} &= \frac{[(k\eta_R)^2 - 9] \left(\frac{k\eta_R \sqrt{w}}{2} \right)^{\frac{1-3w}{2(1+3w)}} J_{\frac{1-3w}{2(1+3w)}}(k\eta_R \sqrt{w}) + 3k\eta_R \sin(k\eta_R)}{6 \cos(k\eta_R) - 6(k\eta_R)^2 \cos(k\eta_R) + 3k\eta_R [(k\eta_R)^2 - 3] \sin(k\eta_R)} \phi^{(ET)}(\eta_R) \\ &+ \frac{3(4(k\eta_R)^2 - 9) \sin(k\eta_R) - 3k\eta_R [(k\eta_R)^2 - 18] \cos(k\eta_R)}{6 \cos(k\eta_R) - 6(k\eta_R)^2 \cos(k\eta_R) + 3k\eta_R [(k\eta_R)^2 - 3] \sin(k\eta_R)} \phi^{(ET)' }(\eta_R) \\ \phi^{(RD,1)} &= \frac{-(k\eta_R)^3 + k\eta_R [(k\eta_R)^2 - 18] \cos(k\eta_R) + 3[4(k\eta_R)^2 - 9] \sin(k\eta_R)}{6 \cos(k\eta_R) - 6(k\eta_R)^2 \cos(k\eta_R) + 3k\eta_R [(k\eta_R)^2 - 3] \sin(k\eta_R)} \phi^{(ET)}(\eta_R) \end{aligned}$$

$$+ \frac{6(k\eta_R)^2 \cos(k\eta_R) - 6 \cos(k\eta_R) - 3k\eta_R[(k\eta_R)^2 - 3] \sin(k\eta_R)}{6 \cos(k\eta_R) - 6(k\eta_R)^2 \cos(k\eta_R) + 3k\eta_R[(k\eta_R)^2 - 3] \sin(k\eta_R)} \phi^{(ET)'}(\eta_R).$$

We find that the early-time epoch leaves an imprint on the evolution of linear scalar perturbations during the radiation-dominated epoch, which subsequently affects gravitational wave generation. This suggests that SIGWs in the context of an early-time epoch cannot be simplistically understood as a superposition of waves across all epochs. The additional influence of scalar perturbations on gravitational waves underscores the nonlinear characteristics of Einstein's theory of gravity.

The energy-density fraction spectrum of SIGWs at $k\eta \gg 1$ during the radiation-dominated epoch is defined as [23–28]

$$\Omega_{GW}(k, \eta) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \ln k} = \frac{k^3}{12a^2 H^2} \int \frac{d^3 k'}{(2\pi)^3} \left\langle h_{ij,k}^{(RD)'} h_{ij,k'}^{(RD)'} + h_{ij,k}^{(RD)} h_{ij,k'}^{(RD)} \right\rangle,$$

where ρ_c is the critical density. In the small-scale limit ($k\eta \gg 1$), we can use $\bar{P}_h(k) = P_h(k)$. The two-point correlations of h_{ij} and h'_{ij} are defined as

$$\langle h_{ij,k}^{(RD)} h_{ij,\bar{k}}^{(RD)} \rangle = 2(2\pi)^3 \delta^{(3)}(k + \bar{k}) \frac{2\pi^2}{k^3} P_h(k, \eta)$$

$$\langle h_{ij,k}^{(RD)'} h_{ij,\bar{k}}^{(RD)'} \rangle = 2(2\pi)^3 \delta^{(3)}(k + \bar{k}) \frac{2\pi^2}{k^3} \bar{P}_h(k, \eta).$$

These quantities are determined by Eq. (9). The stochastic nature of $h_{ij,k}^{(RD)}$ and $h_{ij,k}^{(RD)'}$ originates from that of Φ_k . The two-point correlations of Φ_k can be formally expressed as

$$\langle \Phi_k \Phi_{\bar{k}} \rangle = (2\pi)^3 \delta^{(3)}(k + \bar{k}) \left(\frac{3(1+w)}{5+3w} \right)^2 \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k),$$

where $\mathcal{P}_\zeta(k)$ is the power spectrum of primordial curvature perturbations ζ . For simplicity, we adopt a monochromatic spectrum, $\mathcal{P}_\zeta = Ak_* \delta(k - k_*)$. Such a spectrum might be related to PBH formation [56] and has been extensively studied in the literature (see review in Refs. [57,58] and references therein).

[Figure 1: see original paper]

Figure 1: Energy-density fraction spectra for selected EoS parameters w and transition time η_R (multiplied with k_* to make the combination $k_* \eta_R$ dimensionless).

Figure 1 depicts the energy-density fraction spectrum for various parameters. For $w = 1/3$, the result corresponds to SIGWs in a purely radiation-dominated epoch [24,59]. For $w = 0$, the energy-density fraction spectrum experiences enhancement due to the transition to radiation domination. This enhancement mechanism for the early-time matter domination model has also been discussed in Ref. [60].

For parameter inference in the following section, we consider the energy-density fraction spectrum of SIGWs in the present universe:

$$h^2\Omega_{GW,0}(k) \simeq h^2\Omega_{r,0} \times \Omega_{GW}(k, \eta),$$

where $\Omega_{GW}(k, \eta)$ is given by Eq. (14) and the physical energy-density fraction of radiation in the present universe is $h^2\Omega_{r,0} \simeq 4.2 \times 10^{-5}$ with the dimensionless Hubble constant $h = 0.6766$ measured by the Planck satellite [61].

III. Parameter Inferences from the PTA, BBN, and CMB Datasets

In this study, we consider two distinct dataset combinations. The first consists solely of NANOGrav 15-year PTA data [31], while the second additionally incorporates BBN [54] and CMB [55] constraints on the effective number of relativistic species. In terms of the integrated energy-density fraction, defined as $\int_{k_{\min}}^{\infty} dk h^2\Omega_{GW,0}(k)$, the upper limits are 1.3×10^{-6} for BBN [54] and 2.9×10^{-7} for CMB [55]. The lower bound of the integral is $k_{\min} = 2\pi f_{\min}$, where f_{\min} is 1.5×10^{-11} Hz for BBN and 3×10^{-17} Hz for CMB [2]. BBN and CMB data have also been considered in studies of SIGWs [44] and phase-transition gravitational waves [62].

For the energy-density fraction spectrum of SIGWs in Eq. (18), we examine three scenarios with $k_*\eta_R$ set to 10, 10^2 , and 10^3 , respectively.

[Figure 2: see original paper]

Figure 2: Posterior distributions of three independent parameters inferred from the NANOGrav 15-year PTA dataset, for scenarios with $k_*\eta_R = 10, 10^2, 10^3$.

[Figure 3: see original paper]

Figure 3: Posterior distributions of three independent parameters inferred from the joint BBN, CMB, and NANOGrav 15-year PTA dataset, for scenarios with $k_*\eta_R = 10, 10^2, 10^3$.

The parameter space under investigation is defined by $\log_{10} A$, $\log_{10}(f_*/\text{Hz})$, and w . Throughout this work, we take $k_* = 2\pi f_*$. We apply uniform priors: $\log_{10} A \in [-3.5, 0]$, $\log_{10}(f_*/\text{Hz}) \in [-9, -4]$, and $w \in [0, 1]$. Following Ref. [52], we perform Bayesian parameter inference across this parameter space for both dataset combinations. The BBN and CMB constraints are incorporated by

assigning negative infinity to the log-likelihood if the integrated energy-density fraction exceeds the specified upper limits.

The Bayesian analysis results for the posterior distributions are shown in Figs. 2 and 3. Figure 2 considers only NANOGrav 15-year PTA data, while Fig. 3 additionally incorporates BBN and CMB bounds. Other settings are identical across all scenarios.

When considering only PTA data (Fig. 2), we find that the parameter A is bounded from below but not from above, i.e., $A \gtrsim 10^{-3}$. Meanwhile, the peak frequency f_* is constrained to $\sim 10^{-7} - 10^{-5}$ Hz, indicating a micro-Hertz frequency band. Since PTA is sensitive to the nano-Hertz band ($\sim 10^{-9} - 10^{-7}$ Hz) and considering the spectral profiles in Fig. 1, only the infrared tails of the energy-density fraction spectra can fit the NANOGrav 15-year dataset well. Depending on the scenario, loose constraints are obtained for the EoS parameter w .

In contrast, incorporating BBN and CMB bounds refines these results by more tightly constraining the parameter space (Fig. 3). Parameter regions that make the integrated energy-density fraction exceed the specified upper limits are excluded from our analysis. For all scenarios, we find that the spectral amplitude A is bounded both from below and above, i.e., $A \sim 10^{-3} - 10^{-1}$. The peak frequency f_* is also more tightly constrained to $f_* \sim 10^{-7} - 10^{-6}$ Hz. Additionally, we still obtain loose constraints on the EoS parameter w . However, the most important result concerns the exclusion of early matter domination with $w = 0$ at more than 2σ confidence level.

As shown in Figs. 2 and 3, radiation domination with $w = 1/3$ is compatible with our dataset combinations for all scenarios considered, and an epoch of kination domination with $w = 1$ [63] is not forbidden.

IV. Conclusions and Discussions

In this work, we studied the equation of state of the early universe by assuming the SIGW interpretation of recent PTA data releases. We computed the energy-density fraction spectrum of SIGWs during the transition from an arbitrary $w \in [0, 1]$ to $w = 1/3$ (see Fig. 1 for illustrative examples). To perform Bayesian parameter inference, we analyzed both the NANOGrav 15-year PTA dataset alone and in combination with BBN and CMB bounds. To our knowledge, this is the first Bayesian data analysis on this topic. The posterior distributions are shown in Figs. 2 and 3. For the energy-density fraction spectrum of SIGWs, we found the allowed parameter region to be $A \sim 10^{-3} - 10^{-1}$ and $f_* \sim 10^{-7} - 10^{-6}$ Hz from the joint dataset. In particular, we found that early matter domination with $w = 0$ is excluded at $> 2\sigma$ confidence level. However, radiation domination with $w = 1/3$ remains compatible with our joint analysis. Additionally, we found that kination domination with $w = 1$ for the early universe is not forbidden by current dataset combinations. These results can be further tested with future observations.

We generalized the theory of SIGWs to characterize a transition from an early-time epoch with arbitrary w to the radiation-dominated epoch. While SIGW production in the early universe with arbitrary w has been studied in Refs. [18–20], in those models gravitational waves propagate freely during the subsequent radiation-dominated epoch but are no longer produced. However, for SIGWs produced during an early matter-dominated epoch and subsequent radiation-dominated epoch, enhanced production can occur due to a sudden transition from early-matter domination to radiation domination [60]. It has been shown that the energy-density fraction spectrum of SIGWs produced via this enhancement mechanism can dominate the total spectrum. Therefore, to study SIGWs comprehensively, we considered production in both the early-time epoch with arbitrary $w \in [0, 1]$ and the subsequent radiation-dominated epoch with $w = 1/3$, following the approach of Refs. [28,60].

Our findings reveal that the EoS parameter $w = 0$ is inconsistent with the joint dataset examined. This result also demonstrates the importance of BBN and CMB bounds for studies of SIGWs produced in the early universe with varying EoS. Given that f_* is constrained to roughly $10^{-7} - 10^{-6}$ Hz, our results suggest that an extended period of early-matter domination lasting more than 10^7 seconds (yielding $k_* \eta_R \gg 1$) is not supported by the data. When the universe was approximately 10^7 seconds old, its temperature would have been around 10^{-3} MeV, lower than that during BBN [1]. Since early-matter domination pertains to the cosmic evolution from the end of inflation to the onset of radiation domination, our research offers fresh perspectives for investigations into early universe physics [13–17].

Our results rely on the assumption that the GWB evidence in the PTA band can be interpreted using SIGW theory. However, this assumption may not hold, as alternative interpretations exist [52,53]. More generally, we should consider these possible GWB origins to draw definitive conclusions about the early universe EoS. In particular, the astrophysical origin of GWB—namely, inspiraling supermassive black hole (SMBH) binaries—should be included in realistic data analyses [53,64]. Given these considerations and the large uncertainties in current PTA observations, providing confident conclusions about the early universe EoS remains challenging at this stage. Nevertheless, our study demonstrates that SIGWs can be a powerful probe of early-universe physics.

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References

- [1] S. Dodelson, *Modern Cosmology* (Elsevier Science, 2003).
- [2] M. Maggiore, *Gravitational Waves. Vol. 2: Astrophysics and Cosmology* (Oxford University Press, 2018).
- [3] M. Drees, F. Hajkarim, and E. R. Schmitz, JCAP 06, 025 (2015), arXiv:1503.03513 [hep-ph].
- [4] K. Saikawa and S. Shirai, JCAP 1805, 035 (2018), arXiv:1803.01038 [hep-ph].
- [5] B. Carr, S. Clesse, J. García-Bellido, and F. Kühnel, Phys. Dark Univ. 31, 100755 (2021), arXiv:1906.08217 [astro-ph.CO].
- [6] M. S. Turner, Phys. Rev. D 28, 1243 (1983).
- [7] V. Poulin, T. L. Smith, D. Grin, T. Karwal, and M. Kamionkowski, Phys. Rev. D 98, 083525 (2018), arXiv:1806.10608 [astro-ph.CO].
- [8] A. Vikman, Phys. Rev. D 71, 023515 (2005), arXiv:astro-ph/0407107.
- [9] V. Poulin, T. L. Smith, T. Karwal, and M. Kamionkowski, Phys. Rev. Lett. 122, 221301 (2019), arXiv:1811.04083 [astro-ph.CO].
- [10] J. C. Hill, E. McDonough, M. W. Toomey, and S. Alexander, Phys. Rev. D 102, 043507 (2020), arXiv:2003.07355 [astro-ph.CO].
- [11] R. Hlozek, D. Grin, D. J. E. Marsh, and P. G. Ferreira, Phys. Rev. D 91, 103512 (2015), arXiv:1410.2896 [astro-ph.CO].
- [12] D. J. E. Marsh, Phys. Rept. 643, 1 (2016), arXiv:1510.07633 [astro-ph.CO].
- [13] I. Dalianis and C. Kouvaris, JCAP 07, 046 (2021), arXiv:2012.09255 [astro-ph.CO].
- [14] G. Domènech, C. Lin, and M. Sasaki, JCAP 04, 062 (2021), [Erratum: JCAP 11, E01 (2021)], arXiv:2012.08151 [gr-qc].
- [15] K. D. Lozanov and V. Takhistov, Phys. Rev. Lett. 130, 181002 (2023), arXiv:2204.07152 [astro-ph.CO].
- [16] N. Bhaumik and R. K. Jain, Phys. Rev. D 104, 023531 (2021), arXiv:2009.10424 [astro-ph.CO].
- [17] M. R. Haque, D. Maity, T. Paul, and L. Sriramkumar, Phys. Rev. D 104, 063513 (2021), arXiv:2105.09242 [astro-ph.CO].
- [18] F. Hajkarim and J. Schaffner-Bielich, Phys. Rev. D 101, 043522 (2020), arXiv:1910.12357 [hep-ph].
- [19] G. Domènech, Int. J. Mod. Phys. D 29, 2050028 (2020), arXiv:1912.05583 [gr-qc].
- [20] G. Domènech, S. Pi, and M. Sasaki, JCAP 08, 017 (2020), arXiv:2005.12314 [gr-qc].
- [21] G. Domènech and S. Pi, Sci. China Phys. Mech. Astron. 65, 230411 (2022), arXiv:2010.03976 [astro-ph.CO].
- [22] R. Flauger and S. Weinberg, Phys. Rev. D 99, 123030 (2019), arXiv:1906.04853 [hep-th].
- [23] K. N. Ananda, C. Clarkson, and D. Wands, Phys. Rev. D 75, 123518 (2007), arXiv:gr-qc/0612013 [gr-qc].
- [24] D. Baumann, P. J. Steinhardt, K. Takahashi, and K. Ichiki, Phys.

- Rev. D76, 084019 (2007), arXiv:hep-th/0703290 [hep-th].
- [25] S. Mollerach, D. Harari, and S. Matarrese, Phys. Rev. D69, 063002 (2004), arXiv:astro-ph/0310711 [astro-ph].
- [26] H. Assadollahi and D. Wands, Phys. Rev. D81, 023527 (2010), arXiv:0907.4073 [astro-ph.CO].
- [27] J. R. Espinosa, D. Racco, and A. Riotto, JCAP 09, 012 (2018), arXiv:1804.07732 [hep-ph].
- [28] K. Kohri and T. Terada, Phys. Rev. D97, 123532 (2018), arXiv:1804.08577 [gr-qc].
- [29] H. Xu et al., Res. Astron. Astrophys. 23, 075024 (2023), arXiv:2306.16216 [astro-ph.HE].
- [30] J. Antoniadis et al., (2023), arXiv:2306.16214 [astro-ph.HE].
- [31] G. Agazie et al. (NANOGrav), Astrophys. J. Lett. 951 (2023), 10.3847/2041-8213/acdac6, arXiv:2306.16213 [astro-ph.HE].
- [32] D. J. Reardon et al., Astrophys. J. Lett. 951 (2023), 10.3847/2041-8213/acdd02, arXiv:2306.16215 [astro-ph.HE].
- [33] G. Franciolini, A. Iovino, Junior., V. Vaskonen, and H. Veermäe, (2023), arXiv:2306.17149 [astro-ph.CO].
- [34] K. Inomata, K. Kohri, and T. Terada, (2023), arXiv:2306.17834 [astro-ph.CO].
- [35] Y.-F. Cai, X.-C. He, X. Ma, S.-F. Yan, and G.-W. Yuan, (2023), arXiv:2306.17822 [gr-qc].
- [36] S. Wang, Z.-C. Zhao, J.-P. Li, and Q.-H. Zhu, (2023), arXiv:2307.00572 [astro-ph.CO].
- [37] L. Liu, Z.-C. Chen, and Q.-G. Huang, (2023), arXiv:2307.01102 [astro-ph.CO].
- [38] K. T. Abe and Y. Tada, (2023), arXiv:2307.01653 [astro-ph.CO].
- [39] R. Ebadi, S. Kumar, A. McCune, H. Tai, and L.-T. Wang, (2023), arXiv:2307.01248 [astro-ph.CO].
- [40] D. G. Figueroa, M. Pieroni, A. Ricciardone, and P. Simakachorn, (2023), arXiv:2307.02399 [astro-ph.CO].
- [41] Z. Yi, Q. Gao, Y. Gong, Y. Wang, and F. Zhang, (2023), arXiv:2307.02467 [gr-qc].
- [42] E. Madge, E. Morgante, C. P. Ibáñez, N. Ramberg, and S. Schenk, (2023), arXiv:2306.14856 [hep-ph].
- [43] H. Firouzjahi and A. Talebian, (2023), arXiv:2307.03164 [gr-qc].
- [44] Q.-H. Zhu, Z.-C. Zhao, and S. Wang, (2023), arXiv:2307.03095 [astro-ph.CO].
- [45] Z.-Q. You, Z. Yi, and Y. Wu, (2023), arXiv:2307.04419 [gr-qc].
- [46] G. Ye and A. Silvestri, (2023), arXiv:2307.05455 [astro-ph.CO].
- [47] S. A. Hosseini Mansoori, F. Felegray, A. Talebian, and M. Sami, (2023), arXiv:2307.06757 [astro-ph.CO].
- [48] S. Balaji, G. Domènech, and G. Franciolini, (2023), arXiv:2307.08552 [gr-qc].
- [49] J.-H. Jin, Z.-C. Chen, Z. Yi, Z.-Q. You, L. Liu, and Y. Wu, (2023), arXiv:2307.08687 [astro-ph.CO].

- [50] B. Das, N. Jaman, and M. Sami, (2023), arXiv:2307.12913 [gr-qc].
- [51] G. Domènech, Universe 7, 398 (2021), arXiv:2109.01398 [gr-qc].
- [52] A. Afzal et al. (NANOGrav), Astrophys. J. Lett. 951 (2023), 10.3847/2041-8213/acdc91, arXiv:2306.16219 [astro-ph.HE].
- [53] J. Antoniadis et al., (2023), arXiv:2306.16227 [astro-ph.CO].
- [54] R. Cooke, M. Pettini, R. A. Jorgenson, M. T. Murphy, and C. C. Steidel, Astrophys. J. 781, 31 (2014), arXiv:1308.3240 [astro-ph.CO].
- [55] T. J. Clarke, E. J. Copeland, and A. Moss, JCAP 10, 002 (2020), arXiv:2004.11396 [astro-ph.CO].
- [56] S. Hawking, Mon. Not. Roy. Astron. Soc. 152, 75 (1971).
- [57] B. Carr and F. Kuhnel, Ann. Rev. Nucl. Part. Sci. 70, 355 (2020), arXiv:2006.02838 [astro-ph.CO].
- [58] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Class. Quant. Grav. 35, 063001 (2018), arXiv:1801.05235 [astro-ph.CO].
- [59] K. Kohri and T. Terada, Class. Quant. Grav. 35, 235017 (2018), arXiv:1802.06785 [astro-ph.CO].
- [60] K. Inomata, K. Kohri, T. Nakama, and T. Terada, Phys. Rev. D 100, 043532 (2019), arXiv:1904.12879 [astro-ph.CO].
- [61] N. Aghanim et al. (Planck), Astron. Astrophys. 641, A6 (2020), [Erratum: Astron. Astrophys. 652, C4 (2021)], arXiv:1807.06209 [astro-ph.CO].
- [62] T. Bringmann, P. F. Depta, T. Konstandin, K. Schmidt-Hoberg, and C. Tasillo, (2023), arXiv:2306.09411 [astro-ph.CO].
- [63] Y. Gouttenoire, G. Servant, and P. Simakachorn, (2021), arXiv:2111.01150 [hep-ph].
- [64] G. Agazie et al. (NANOGrav), (2023), arXiv:2306.16220 [astro-ph.HE].

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