# Two-proton radioactivity from excited states of proton-rich nuclei within Coulomb and Proximity Potential Model

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## Abstract

We extend the Coulomb and Proximity Potential Model (CPPM) to study two-proton ($2p$) radioactivity from excited states while the proximity potential is chosen as AW95 proposed by Aage Withner in 1995. Demonstration reveals that the theoretical results acquired by CPPM exhibit a high level of consistency with prior theoretical models, such as the unified fission model (UFM), generalized liquid drop model (GLDM) and effective liquid drop model (ELDM). Furthermore, within the CPPM, we predicted the half-lives of potential $2p$ radioactive nuclei for which experimental data are currently unavailable. The predicted results were then assessed, compared with UFM, ELDM, and GLDM models, and examined in detail.

## Full Text

## Preamble

### Two-Proton Radioactivity from Excited States of Proton-Rich Nuclei within the Coulomb and Proximity Potential Model

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We extend the Coulomb and Proximity Potential Model (CPPM) to study two-proton (2p) radioactivity from excited states, employing the AW95 proximity potential proposed by Aage Winther in 1995. Our results demonstrate that the theoretical predictions from CPPM exhibit a high level of consistency with prior theoretical models, including the Unified Fission Model (UFM), Generalized Liquid Drop Model (GLDM), and Effective Liquid Drop Model (ELDM). Furthermore, within the CPPM framework, we predict the half-lives of potential 2p radioactive nuclei for which experimental data are currently unavailable. These predictions are assessed through detailed comparison with UFM, ELDM, and GLDM results.

**Keywords:** 2p radioactivity, CPPM, half-lives, excited state

## Introduction

In addition to well-known decay modes such as $\alpha$, $\beta$, $\gamma$, and cluster decays [?], exotic modes of radioactivity exist in proton-rich nuclei [?], including proton and two-proton (2p) radioactivity [?]. Two-proton radioactivity is an extremely exotic decay mode that is energetically possible in less massive nuclei near the proton drip line [?], revealing rich information about nuclear structure, including nuclear radius, wave functions of the emitted protons, spin and parity, deformation effects, and more [?]. This novel decay mode was first predicted by Zel' dovich and Goldansky in the 1960s [?], leading to extensive theoretical development over subsequent decades [?]. However, due to limitations in radioactive beam facilities and detection technology, 2p radioactivity was not experimentally confirmed until 2002, when Giovinazzo et al. at GANIL (France) and independently Pfützner et al. at GSI (Germany) observed the decay $^{45}\mathrm{Fe} \rightarrow ^{43}\mathrm{Cr} + p + p$, providing the first experimental evidence of this phenomenon [?, ?]. Since then, 2p radioactivity has been recognized as a significant decay mode for proton-rich nuclei [?], with detections in several nuclei including the resonant ground state of $^{6}$Be [?, ?] and $^{12}$O [?], as well as excited states selectively populated in $^{6}$Be [?] and $^{14}$O [?].

Numerous studies have demonstrated that 2p radioactivity occurs not only from ground states but also from short-lived excited states. Jänecke first discussed the possibility of $\beta$-delayed 2p ($ $2p) radioactivity [?], while Goldansky predicted that $ $2p radioactivity could be found in emitters with Z = 10–20. The first experimental observation of $ $2p radioactivity was reported by Cable et al. in 1983 [?]. Subsequently, an increasing number of $ $2p emitters have been detected using silicon detector telescopes, enabling precise measurement of individual proton energies. Following the discovery of $ $2p radioactivity from $^{22}$Al [?], additional $ $2p emitters were soon identified, including $^{26}$P [?] and $^{35}$Ca [?]. Other $ $2p nuclei discovered include $^{23}$Si [?], $^{27}$S [?], $^{31}$Ar [?], $^{39}$Ti [?], $^{43}$Cr [?, ?], and $^{50}$Ni [?]. Beyond $\beta$ decays, 2p radioactivity has also been observed from excited states fed by nuclear reactions such as pickup, transfer, or fragmentation, including $^{14}$O [?], $^{17,18}$Ne [?], $^{22}$Mg [?, ?], and $^{28,29}$S [?, ?]. In 2006, Mukha et al. first reported 2p radioactivity from $^{94}$Ag in an experiment at GSI [?].

From a theoretical perspective, several methods have been proposed to study 2p radioactivity mechanisms, encompassing both microscopic and phenomenological approaches. Proton-rich nuclei can emit two protons through three distinct mechanisms: (1) two-body sequential emission, (2) three-body simultaneous emission, and (3) diproton emission (also called $^{2}$He cluster emission). The $^{2}$He cluster emission represents an extreme scenario where two strongly correlated protons survive only briefly before separating after penetrating the Coulomb barrier. Recently, Yao et al. [?], Ghodsi et al. [?], and Deng et al. [?] have performed comparative studies of various proximity potential formalisms for $\alpha$ decay, proton radioactivity, and cluster radioactivity using CPPM. Since the 2p radioactivity decay process shares the same theoretical foundation as $\alpha$ decay and proton radioactivity—namely, barrier penetration [?]—we previously extended the CPPM presented in Ref. [?] to account for ground-state 2p radioactivity, finding excellent agreement between calculated results, experimental data, and other theoretical models. In addition to theoretical models, empirical formulae serve as valuable tools for investigating 2p radioactivity, including a four-parameter formula by Sreeja et al. [?] and a two-parameter formula by Liu et al. [?].

A natural question arises whether 2p radioactivity from excited states can be treated as $^{2}$He cluster emission and described by CPPM. To address this, we systematically study the half-lives of 2p radioactivity from excited states within CPPM for nuclei with $8 < Z < 47$.

This paper is organized as follows: Section II briefly describes the theoretical framework for 2p radioactivity half-lives and the CPPM formalism. Section III presents detailed calculations and discussion. Finally, Section IV provides a summary.

## Theoretical Framework

### A. The 2p Radioactivity Half-Life Formalism

The half-life of 2p radioactivity is generally determined by

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P S_{2p}}$$

where $\lambda$ is the decay constant, $\nu$ is the assault frequency related to the harmonic oscillation frequency in the Nilsson potential [?], expressed as $h\nu = \hbar\omega \approx \frac{10.1}{A^{1/3}}$ MeV. Here $h$, $\hbar$, $A$, and $\omega$ represent the Planck constant, reduced Planck constant, mass number of the parent nucleus, and angular frequency, respectively. The preformation probability of the emitted two protons in the parent nucleus, obtained by the cluster overlap approximation [?], is $S_{2p} = G_2[A/(A-2)]^{2n}\chi^2$, where $G_2 = \frac{(2n)!}{2^{2n}(n!)^2}$ [?, ?] with $n \approx (3Z)^{1/3} - 1$ being the average principal proton oscillator quantum number, and $\chi^2$ is set to 0.0143 according to Ref. [?].

The penetration probability $P$ is calculated using the WKB approximation:

$$P = \exp\left(-2\int_{r_{\text{in}}}^{r_{\text{out}}} K(r)\,dr\right)$$

where $K(r) = \sqrt{\frac{2\mu}{\hbar^2}|V(r) - Q_{2p}|}$, $\mu$ denotes the reduced mass [?], and $r$ is the center-of-mass distance between the daughter nucleus and emitted protons. The inner and outer turning points $r_{\text{in}}$ and $r_{\text{out}}$ are obtained from the conditions $V(r_{\text{in}}) = V(r_{\text{out}}) = Q_{2p}$.

The total interaction potential $V(r)$ between the emitted protons and daughter nucleus comprises the Coulomb potential $V_C(r)$, nuclear potential $V_N(r)$, and centrifugal potential $V_\ell(r)$:

$$V(r) = V_N(r) + V_C(r) + V_\ell(r)$$

In CPPM, the nuclear potential is replaced by the proximity potential (detailed in Sec. IIB). Assuming a homogeneous spherical charge distribution for the daughter nucleus, the Coulomb potential is:

$$V_C(r) = \begin{cases} \frac{Z_1^2 Z_{2e}}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right], & r < R \\ \frac{Z_1^2 Z_{2e}}{r}, & r > R \end{cases}$$

where $R = R_1 + R_2$ is the separation radius with $R_1$ and $R_2$ being the radii of the daughter nucleus and emitted protons, respectively.

For the centrifugal potential $V_\ell(r)$, we adopt the Langer-modified form $\ell(\ell+1) \to (\ell+\frac{1}{2})^2$, which provides a necessary correction for one-dimensional problems [?]:

$$V_\ell(r) = \frac{\hbar^2(\ell+\frac{1}{2})^2}{2\mu r^2}$$

where $\ell$ is the orbital angular momentum carried away by the emitted protons.

### B. The Proximity Potential Formalism

The phenomenological proximity potential was first proposed by Blocki et al. [?] in the 1970s for heavy-ion reactions. It provides a simple formula for nucleus-nucleus interaction energy as a function of the separation between approaching nuclear surfaces, with adjustable parameters based on measured nuclear surface tension and diffuseness. In CPPM, the potential barrier is modeled as the sum of Coulomb, proximity, and centrifugal potentials for both touching and separated configurations. In this work, we employ the AW95 proximity potential proposed by Aage Winther in 1995 [?] to replace the nuclear potential, expressed as:

$$V_N(r) = -\frac{V_0}{1+\exp\left(\frac{r-R_0}{a}\right)}$$

where $V_0 = 16\pi\frac{R_1 R 2}{R_1+R_2}\gamma a$, $R_i = (1.2A_i^{1/3} - 0.09)$ fm $(i = 1, 2)$, and $R_0 = R_1 + R_2$. The surface energy constant $\gamma$ takes the form:

$$\gamma = \gamma_0 \left(1 - k_s \left(\frac{N_1 - Z_1}{A_1}\right)\left(\frac{N_2 - Z_2}{A_2}\right)\right)$$

with coefficients $\gamma_0 = 0.95$ MeV/fm$^2$ and $k_s = 1.8$. Here $A_i$, $Z_i$, and $N_i$ $(i = 1, 2)$ denote the mass number, proton number, and neutron number of the daughter nucleus and emitted protons, respectively.

## Results and Discussion

Blocki first introduced the proximity potential in 1977 to characterize the interaction between two nuclei in the separation degree of freedom, based on the proximity force theorem [?]. Since then, numerous nuclear proximity potentials have been widely applied in nuclear physics research [?]. CPPM is a phenomenological model commonly used to study two-body problems in the context of 2p radioactivity, considering the interaction potential between parent and daughter nuclei as the sum of nuclear, Coulomb, and centrifugal potentials. Unlike other models, CPPM replaces the nuclear potential with a simplified formalism based on the proximity force, offering the advantage of adjustable parameters while maintaining simplicity and accuracy. In our previous work [?], we extended CPPM to study ground-state 2p radioactivity; the present work further extends CPPM to investigate 2p radioactivity from excited states.

We first performed calculations of 2p radioactivity half-lives for $^{14}$O*, $^{17}Ne$*, $^{18}$Ne*, $^{22}Mg$*, $^{29}$S*, and $^{94}Ag$* (where * denotes excited states), with all results listed in Table 1 . The first four columns of the table show the 2p decay process, experimental 2p radioactivity released energy, spin and parity of the initial and final nuclear states, and the angular momentum carried away by the emitted protons. The fifth column presents experimental half-lives for excited-state 2p radioactivity, while columns six through nine show the logarithmic half-lives obtained from CPPM, ELDM, GLDM, and UFM, respectively.

Overall, the theoretical half-lives from CPPM show high consistency with other theoretical models, except for $^{22}$Mg* (with $Q_{2p} = 6.11$ MeV) and $^{29}$S* (with $Q_{2p} = 5.12$ MeV). To understand this discrepancy, we plot the interaction potential curves for these nuclei in Fig. 1 [Figure 1: see original paper]. As described in Sec. IIA, the half-life depends on the penetration probability $P$ obtained from Eq. (3) under the conditions $V(r_{\text{in}}) = V(r_{\text{out}}) = Q_{2p}$. However, as shown in Fig. 1, the interaction potential curves for $^{22}$Mg* and $^{29}$S* cannot satisfy these conditions, preventing calculation of the corresponding $P$ and thus the half-lives.

This phenomenon likely arises from significant deformation effects. The half-lives of 2p radioactivity are highly sensitive to proton-proton interactions due to the pairing effect of valence protons. The quasi-classical $^2$He model cannot account for experimentally observed proton-proton correlations that indicate back-to-back proton emission. Moreover, for $^{22}$Mg* and $^{29}$S*, the spin-parity of the parent nuclei' s initial states are uncertain, and the $Q_{2p}$ values are not precisely determined—all factors that non-negligibly impact the $V(r)$ curve.

To illustrate the agreement between CPPM-calculated half-lives and other models for excited-state 2p radioactivity, partial theoretical results are plotted in Fig. 2 [Figure 2: see original paper]. This figure compares the logarithmic half-lives from ELDM, GLDM, and UFM with our calculations [?, ?]. For $^{14}$O*, experimental data give $\log_{10} T_{1/2} > -16.12$ s, while CPPM results are very close to those from ELDM, GLDM, and UFM, and are better than some empirical formulas such as R-matrix theory [?], Sreeja formula, and Liu formula [?], which yield half-lives of −18.12 s, −19.94 s, and −16.85 s, respectively.

To intuitively discuss the influence of angular momentum $\ell$ and release energy $Q_{2p}$ on excited-state 2p radioactivity half-lives, we examine $^{14}$O* and $^{94}$Ag* as examples. Table 1 shows that for $^{14}$O* with the same $\ell$ value, half-lives calculated by CPPM differ by nearly three orders of magnitude between different $Q_{2p}$ values (1.20 MeV vs. 3.15 MeV). Similarly, for $^{94}$Ag* with identical $Q_{2p}$ but different $\ell$ values (6 to 10), the half-lives also differ by three orders of magnitude. This conclusion aligns with our previous work [?]. Both $Q_{2p}$ and $\ell$ make non-negligible contributions to the theoretical half-lives of $^{14}$O* and $^{94}$Ag* within CPPM, ELDM, and GLDM. This strong dependency arises because these models share similar theoretical foundations, with penetration probabilities $P$ calculated using the WKB approximation. We suspect that corresponding experimental data may not be sufficiently precise.

Configuration mixing and three-body dynamics are commonly employed in nuclear and molecular physics to describe systems with many interacting particles. However, due to limitations of the phenomenological CPPM, which does not account for configuration mixing and three-body dynamics, our calculations treat the two emitted protons as strongly correlated, reducing the problem to two-body dynamics. Consequently, CPPM cannot provide comprehensive information on particle dynamics. Experimental data for excited-state 2p radioactivity remain scarce because the extremely short half-lives make observation difficult. The decay schemes of these excited nuclei reveal that one-proton (1p) radioactivity and even three-proton (3p) radioactivity coexist with 2p radioactivity. Therefore, branching ratios between 1p and 2p decay serve as crucial indicators for understanding nuclear structure and merit further investigation.

## Conclusion

In this work, we extended CPPM to study excited-state 2p radioactivity in $^{14}$O, $^{17}Ne$, $^{18}$Ne, $^{22}Mg$, $^{29}$S, *and* $^{94}Ag$. The theoretical values obtained by CPPM show high consistency with experimental data and with theoretical results from ELDM and GLDM. For $^{22}$Mg* and $^{29}$S* (with $Q_{2p} = 5.12$ MeV), CPPM is not applicable, likely because the angular momenta are unavailable due to undetermined spin-parity of the parent nuclei' s initial states. Additionally, uncertain $Q_{2p}$ values strongly influence the calculated results. We find that excited-state 2p radioactivity half-lives have a strong dependence on both $Q_{2p}$ and $\ell$. Compared with theoretical results from ELDM, GLDM, and UFM, the CPPM half-lives for excited-state 2p radioactivity are reliable and may serve as a useful guideline for future experiments.

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*Note: Figure translations are in progress. See original paper for figures.*