

Density-dependent relativistic mean field approach and its application to single- Λ hypernuclei in Oxygen isotopes

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Abstract

The in-medium nature of the nuclear force, which includes both nucleon-nucleon (NN) and hyperon-nucleon (ΛN) interactions, influences the description of single- Λ hypernuclei. Such effects can be revealed by analyzing the systematic evolution of bulk and single-particle properties as the mass number or isospin of hypernuclei varies. From the perspective of density-dependent meson-nucleon/hyperon couplings, a new ΛN effective interaction in covariant density functional (CDF) theory, namely DD-LZ1- $\Lambda 1$, is obtained by fitting experimental data on Λ separation energies for several single- Λ hypernuclei. This interaction is then employed to investigate the structure and transition properties of single- Λ hypernuclei in oxygen isotopes, in comparison with several selected CDF Lagrangians. Explicit discrepancies are observed in the isospin evolution of the $\Lambda 1p$ spin-orbit splitting across different effective interactions, which is attributed to the divergence of meson-hyperon coupling strengths with increasing density. In particular, density-dependent CDFs introduce an additional contribution that enhances the isospin dependence of the splitting, originating from the rearrangement terms of the Λ self-energies. Furthermore, the characteristics of hypernuclear radii are examined along the isotopic chain. Due to the impurity effect of the Λ hyperon, a size reduction is observed in the matter radii of hypernuclei relative to their normal nuclear cores, and its magnitude is further shown to correlate with the incompressibility of nuclear matter. Additionally, a significant model-dependent trend exists wherein Λ hyperon radii evolve with neutron number, which is partly determined by in-medium NN interactions as well as core polarization effects.

Full Text

Preamble

Density-dependent relativistic mean field approach and its application to single- Λ hypernuclei in Oxygen isotopes

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Abstract: The in-medium feature of nuclear force, which includes both nucleon-nucleon (NN) and hyperon-nucleon (Λ N) interactions, impacts the description of single- Λ hypernuclei. Such effects can be unveiled by analyzing the systematic evolution of bulk and single-particle properties with alternating mass number or isospin of hypernuclei. From a density-dependent meson-nucleon/hyperon coupling perspective, a new Λ N effective interaction in the covariant density functional (CDF) theory, namely DD-LZ1- Λ 1, is obtained by fitting experimental data of Λ separation energies for several single- Λ hypernuclei. It is then adopted to study the structure and transition properties of single- Λ hypernuclei in Oxygen isotopes, comparing with several selected CDF Lagrangians. Discrepancy is observed explicitly in the isospin evolution of Λ 1p spin-orbit splitting with various effective interactions, ascribed to their divergence of the meson-hyperon coupling strengths with increasing density. In particular, the density-dependent CDFs introduce an extra contribution to enhance the isospin dependence of the splitting, which originates from the rearrangement terms of Λ self-energies. Additionally, the characteristics of hypernuclear radii are studied along the isotopic chain. Owing to the impurity effect of the Λ hyperon, a size shrinkage is observed in the matter radii of hypernuclei compared to their cores of normal nuclei, while its magnitude is further elucidated to correlate with the incompressibility of nuclear matter. Besides, there exists a sizable model-dependent trend that Λ hyperon radii evolve with the neutron number, which is decided partly by the in-medium NN interactions as well as the core polarization effects.

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1 Introduction

The discovery of hyperons—particles containing strange quarks—in 1953 sparked strong interest among experimental and theoretical physicists [?]. The ability of hyperons to enter the nucleus and form hypernuclear systems makes them sensitive probes for studying nuclear structure and specific nuclear features. Studies on hyperon behavior in the nucleus help us understand the baryon-baryon interaction in nuclear medium and its effects on nuclear prop-

erties [?, ?]. In addition, hyperons are thought to be produced inside neutron stars [?, ?, ?]. The link between hypernuclear and neutron star properties benefits our comprehension of the state of matter in extreme environments, as well as strangeness-bearing nuclear force at high densities.

In recent decades, a wealth of hypernuclear data has been generated through induced reactions of meson and electron beams at various radioactive beam facilities, including the Japan Proton Accelerator Research Complex (J-PARC) [?], the Thomas Jefferson National Accelerator Facility (JLab) [?], and the Facility for Antiproton and Ion Research (FAIR) [?]. These advanced facilities have played a pivotal role in advancing our understanding of strangeness in nuclear physics. Notably, single- Λ hypernuclei have been the most extensively studied, with experimental data covering hypernuclei from ${}^3_\Lambda\text{H}$ to ${}^{208}_\Lambda\text{Pb}$ in various laboratories [?, ?, ?, ?]. When a Λ hyperon enters a nucleus, various phenomena can be observed. For instance, in ${}^7_\Lambda\text{Li}$, it has been found that the size of the ${}^6\text{Li}$ core is smaller compared to the free-space ${}^6\text{Li}$ nucleus, as suggested by the measurement of the γ -ray transition probability from $\text{E2}(5/2^+ \rightarrow 1/2^+)$ in ${}^7_\Lambda\text{C}$ [?]. In addition, in ${}^{13}_\Lambda\text{C}$, it is hinted that the Λ spin-orbit splitting is much smaller than the nucleon's [?]. Recently, the potential for producing neutron-rich hyperfragments at high-intensity heavy-ion accelerator facilities has been discussed [?, ?]. The directed flow of hypernuclei (${}^3_\Lambda\text{Li}$) was just observed at RHIC for the first time in heavy-ion collisions, providing insights into hyperon-nucleon interactions under finite pressure [?]. These advances highlight the promising prospects for investigating hypernuclear structures using the forthcoming high-intensity heavy-ion accelerator facility HIAF [?, ?]. To provide accurate predictions for these experiments, researchers have performed detailed theoretical work on observables such as hypernuclear binding energy [?, ?], spin-orbit splitting [?, ?, ?], hyperon and hypernuclear matter radius [?, ?, ?, ?, ?]. Overall, these efforts aim to provide valuable insights into the behavior of hypernuclei and to deepen our understanding of in-medium baryon interactions.

Due to their ability to provide a self-consistent and unified description of almost all nuclei on the nuclear chart, both non-relativistic and relativistic mean-field theories are widely used in calculations of finite nuclei and nuclear matter and have been extended to describe hypernuclear systems with strange degrees of freedom during the development of theoretical models [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. As a key model utilized in this work, the relativistic mean-field theory has been extensively developed to study hypernuclear properties such as hyperon separation energy [?, ?], spin-orbit splitting [?, ?, ?], hyperon halo [?], hypernuclear deformation [?, ?, ?, ?], cluster structure [?], and drip lines [?]. While most theoretical models have primarily emphasized nonlinear self-coupling interactions for studying hypernuclei, there has been a recent study that explores effective interactions for single- Λ hypernuclei within the density-dependent relativistic mean-field (DDRMF) model [?]. With three distinct fitting approaches, they propose six new sets of effective ΛN interactions and uncover a significant linear correlation between the ratios R_σ and R_ω , representing scalar and vector coupling strengths, respectively, between these

effective ΛN and NN interactions.

Recently, a new type of density-dependent relativistic mean-field Lagrangian, DD-LZ1, has been proposed, inspired by the restoration of pseudo-spin symmetry (PSS) and nuclear medium effects [?]. This new effective Lagrangian has produced satisfactory results in describing the properties of nuclear matter and finite nuclei. With its unique density-dependent form, DD-LZ1 eliminates the spurious shell closures that appeared in previous RMF calculations and reasonably restores the PSS of high orbital angular momentum near the Fermi energy [?]. Applications with this new RMF Lagrangian have been performed for several nuclear many-body characteristics, in both finite nuclei with mass ranging from light to superheavy and neutron star properties with density ranging from low to high. For instance, a comprehensive macroscopic-microscopic model was developed to evaluate the total energies for even-even nuclei with proton numbers ranging from 8 to 110 [?]. Even with the appearance of hyperons [?, ?], larger maximum masses of neutron stars could be obtained with DD-LZ1 than with several other RMF parameter sets, providing the possibility that the secondary object observed in GW190814 is a neutron star [?, ?, ?]. Utilizing the Thomas-Fermi approximation, different microscopic structures of nonuniform nuclear matter were calculated for the crust of neutron stars, and a unified equation of state was established in a vast density range [?, ?]. The different density-dependent behaviors of meson-nucleon couplings impact the microscopic structures of neutron star matter with DD-LZ1, affecting correspondingly the description of various physical processes and evolutions of neutron stars.

Apart from dealing with the different nuclear medium effects caused by the interactions themselves, the evolution of isospin also leads to significant changes in the in-medium effects of hypernuclei, thereby affecting the description of their structural properties. In recent years, a series of refined theoretical studies have been conducted on hypernuclei in different isotopic chains using various interaction models. For instance, the no-core shell model has been employed to investigate the systematic evolution of ground and excited state energies in Helium and Lithium hyperisotopes [?]. The antisymmetrized molecular dynamics method has been applied to explore the low-lying level structure of hypernuclei in Beryllium hyperisotopes [?]. The multidimensionally constrained RMF model has been used to study the shape evolution of hypernuclei in Argon hyperisotopes [?]. The beyond mean-field approach has been utilized to discuss the evolution of p-state energies and composition in Carbon hyperisotopes [?], as well as the hyperon halo structures in Boron and Carbon hyperisotopes [?, ?]. These studies exhibit the significance of the isospin role in the description of hypernuclear structure. In fact, with the development of hypernuclear spectroscopy, new experiments related to hypernuclei have been initiated, such as the planned measurements in the J-PARC project, aiming to study the Λ hyperon binding energies in neutron-rich hyperisotopes of $^{124-136}\text{Sn}$ [?, ?]. These experiments will provide crucial information about the properties of hypernuclei associated with various isospin circumstances.

In view of the essential role of nuclear in-medium effects on hypernuclear structure and their relevance to isotopic evolution, we aim to further expand the density-dependent RMF model to investigate the structure of single- Λ hypernuclei in Oxygen hyperisotopes. First, we will introduce the theoretical framework of the hypernuclear RMF approach in Sec. 2. Then, the induced Λ -nucleon (ΛN) effective interactions will be determined by fitting Λ separation energies to experimental data for the DD-LZ1 Lagrangian. To present the results and discussion, the influence of nuclear in-medium effects will be studied in Sec. 3, focusing on the isospin dependence of hypernuclear bulk properties, hyperon spin-orbit splitting, and matter/hyperon radius. Finally, a summary will be given in Sec. 4.

2 DDRMF Approach for Spherical Single- Λ Hypernuclei

To describe single- Λ hypernuclei within the meson-exchanged type of relativistic mean-field theory, the covariant Lagrangian density serves as the foundation, where the terms of free fields read as

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_\phi + \mathcal{L}_I,$$

$$\mathcal{L}_B = \sum_B \bar{\psi}_B (i\gamma^\mu \partial_\mu - M_B) \psi_B,$$

$$\mathcal{L}_\phi = +\frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu},$$

$$\mathcal{L}_\phi = +\frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$

where the index B (B') represents nucleon N or hyperon Λ , with its sum $\sum_B$ over nucleon N and hyperon Λ . The masses of the baryon and mesons are given by M_B and m_ϕ ($\phi = \sigma, \omega_\mu, \vec{\rho}_\mu$), while $\Omega_{\mu\nu}$, $\vec{R}_{\mu\nu}$ and $F^{\mu\nu}$ are the field tensors of vector mesons ω_μ , $\vec{\rho}_\mu$ and photon A_μ , respectively. The interaction between nucleon (hyperon) and mesons (photon) is involved by the Lagrangian $\mathcal{L}_I$,

$$\begin{aligned} \mathcal{L}_I = & \sum_B \bar{\psi}_B (-g_{\sigma B} \sigma - g_{\omega B} \gamma^\mu \omega_\mu) \psi_B \\ & + \bar{\psi}_N \left[-g_{\rho N} \gamma^\mu \vec{\tau} \cdot \vec{\rho}_\mu - e \gamma^\mu \frac{1 - \tau_3}{2} A_\mu \right] \psi_N. \end{aligned}$$

Here the Λ hyperon (namely ψ_B taken as ψ_Λ), which is charge neutral with isospin zero, only takes part in interactions that are mediated by isoscalar mesons. The nuclear in-medium effects are introduced phenomenologically via

the coupling strengths $g_{\phi B}$ ($g_{\phi N}$), which use baryon-density dependent functions in the density-dependent RMF (DDRMF) approach to define the strengths of different meson-baryon (meson-nucleon) couplings [?, ?].

The effective Hamiltonian operator for Λ hypernuclei can be obtained by performing the general Legendre transformation on the Lagrange density $\mathcal{L}$ in Eq. (1), and it can be written as the sum of the kinetic energy operator $\hat{T}$ and the potential energy operator $\hat{V}_\phi$,

$$\hat{H} \equiv \hat{T} + \sum_{\phi} \hat{V}_{\phi},$$

$$\hat{T} = \sum_B \int d^3x \bar{\psi}_B(x) (-i\gamma \cdot \nabla + M_B) \psi_B(x)$$

$$\hat{V}_{\phi} = \sum_B \sum_{B'} \int d^3x \int d^3x' [\bar{\psi}_B G_{\phi B} \psi_B] D_{\phi}(x, x') [\bar{\psi}_{B'} G_{\phi B'} \psi_{B'}]_{x'},$$

where x is four-vector $(t, \mathbf{x})$. Correspondingly, we define interaction vertices $G_{\phi B}(x)$ for various meson (photon)-nucleon (hyperon) coupling channels, which for isoscalar σ and ω mesons are represented as

$$G_{\sigma B}(x) = +g_{\sigma B}(x), \quad G_{\omega B}(x) = +g_{\omega B}(x)\gamma^{\mu}.$$

Notably, both nucleons and the Λ hyperon can contribute to the isoscalar meson fields. However, for the remaining isovector mesons and photon fields, it is expected that their interaction vertices solely connect to nucleons since the isoscalar and charge-zero nature of the Λ hyperon,

$$G_{\rho N}(x) = +g_{\rho N}(x)\gamma^{\mu}\vec{\tau}, \quad G_{AN}(x) = +e\gamma^{\mu}\frac{1-\tau_3}{2}.$$

As the retardation effects could be neglected in the majority of RMF models, the meson (photon) propagators D_{ϕ} (D_A) read as

$$D_{\phi}(x, x') = \frac{e^{-m_{\phi}|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|}, \quad D_A(x, x') = \frac{1}{|\mathbf{x}-\mathbf{x}'|}.$$

The baryon field operator ψ_B in the Hamiltonian (5) can be second quantized in the positive-energy space under the no-sea approximation as

$$\psi_B(x) = \sum_i f_i(x) e^{-i\epsilon_i t} c_i.$$

Here, f_i represents the Dirac spinor, while c_i denotes the annihilation operators for state i . Accordingly, the energy functional E is determined by evaluating the expectation value of the Hamiltonian with respect to a trial Hartree-Fock ground state $|\Phi_0\rangle$,

$$E = \langle \Phi_0 | \hat{H} | \Phi_0 \rangle = \langle \Phi_0 | \hat{T} | \Phi_0 \rangle + \sum_{\phi} \langle \Phi_0 | \hat{V}_{\phi} | \Phi_0 \rangle.$$

Then the binding energy of a Λ hypernucleus is written by

$$E_B = \sum_B (E_{\text{kin},B} + E_{\sigma,B} + E_{\omega,B}) + E_{\rho,N} + E_{\text{e.m.}} + E_{\text{c.m.}} + E_{\text{pair}},$$

where the kinetic energy functional of baryons is shown by $E_{\text{kin},B}$. The contributions of the potential energy functional from σ and ω are denoted by the variables $E_{\sigma,B}$ and $E_{\omega,B}$. Additionally, $E_{\rho,N}$ and $E_{\text{e.m.}}$ are used to represent the contributions from ρ and A , respectively. The center-of-mass adjustment to the mean field is represented by the term $E_{\text{c.m.}}$, while E_{pair} takes into account the contribution from nucleon pairing correlations [?].

The role of deformation in single- Λ hypernuclei has been discussed in various density functional models [?, ?, ?, ?], which may generate non-negligible effects on the single-particle energies like in Carbon hyperisotopes [?, ?, ?]. To describe single- Λ hypernuclei, particularly the Oxygen hyperisotopes discussed hereafter, we restrict the RMF approach to spherical symmetry. Correspondingly, the Dirac spinor $f_i(x)$ of the nucleon or hyperon in Eq. (9) has the following form:

$$f_{n\kappa m}(x) = \begin{pmatrix} iG_a(r)\Omega_{\kappa m}(\vartheta, \phi) \\ F_a(r)\Omega_{-\kappa m}(\vartheta, \phi) \end{pmatrix},$$

where the index a consists of the set of quantum numbers $(n\kappa) = (njl)$, and $\Omega_{\kappa m}$ is the spherical spinor. Meanwhile, the propagators can be expanded in terms of spherical Bessel and spherical harmonic functions as

$$D_{\phi}(x, x') = \sum_{L=0}^{\infty} \sum_{M=-L}^L (-1)^M R_{\phi LL}(r, r') Y_{LM}(\Omega) Y_{L-M}(\Omega'),$$

where $\Omega = (\vartheta, \phi)$, and R_{LL} contains the modified Bessel functions I and K as

$$R_{LL}(r, r') = \sqrt{\frac{1}{rr'}} \frac{2L+1}{4\pi} I_{L+\frac{1}{2}}(m_{\phi}r_{<}) K_{L+\frac{1}{2}}(m_{\phi}r_{>}),$$

with $r_{<} = \min(r, r')$ and $r_{>} = \max(r, r')$.

In the DDRMF approach, the meson-baryon coupling strengths are adopted as a function of baryon density ρ_b , which are written by

$$g_{\phi B}(\rho_b) = g_{\phi B}(0)f_{\phi B}(\xi),$$

where $\xi = \rho_b/\rho_0$ with ρ_0 the saturation density of nuclear matter, and

$$f_{\phi B}(\xi) = \frac{1 + b_{\phi B}(\xi + d_{\phi B})^2}{1 + c_{\phi B}(\xi + d_{\phi B})^2} e^{-a_{\phi B}\xi}.$$

The free coupling strength at $\rho_b = 0$ is represented by $g_{\phi B}(0)$ in the expression above. To keep the variational self-consistency between the energy density functional and single-particle properties, extra terms in baryon self-energies, namely the rearrangement terms, will occur due to the density dependence of the coupling strengths. The single-particle (nucleon or hyperon) properties can be determined by solving the Dirac equation,

$$\begin{pmatrix} \Sigma_{0,B}(r) - M_B - \Sigma_{S,B}(r) & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & \Sigma_{0,B}(r) + M_B + \Sigma_{S,B}(r) \end{pmatrix} \begin{pmatrix} G_{a,B}(r) \\ F_{a,B}(r) \end{pmatrix} = \epsilon_{a,B} \begin{pmatrix} G_{a,B}(r) \\ F_{a,B}(r) \end{pmatrix}.$$

Here the self-energies Σ_B and the time component of the vector one has $\Sigma_{0,B} \pm \Sigma_{S,B}$ composed by the vector and scalar terms. The scalar self-energy

$$\Sigma_{S,B}(r) = \Sigma_{\sigma S,B}(r) + \Sigma_R(r),$$

where $\phi = \omega, \rho$ for nucleons, and $\phi = \omega$ for Λ hyperon. The self-energies of nucleon or hyperon include scalar one $\Sigma_{S,B}$ and vector one $\Sigma_{0,B}$, in which the coupling of isoscalar mesons contributes as follows,

$$\Sigma_{\sigma S,B}(r) = -g_{\sigma B}(r) \int r'^2 dr' g_{\sigma B'}(r') \rho_{s,B'}(r') R_{\sigma 00}(r, r'),$$

$$\Sigma_{\omega 0,B}(r) = +g_{\omega B}(r) \int r'^2 dr' g_{\omega B'}(r') \rho_{b,B'}(r') R_{\omega 00}(r, r').$$

Here, $\rho_{s,B}$ and $\rho_{b,B}$ represent the scalar and baryon density, respectively [?]. Additionally, the rearrangement term Σ_R appears in the DDRMF approach, which contains the summation over all baryons for the isoscalar case of $\phi = \sigma, \omega$, but only over nucleons for the isovector one. For example, the contribution from σ -S coupling is shown as

$$\Sigma_{R,\sigma}(r) = \sum_B \rho_{s,B} \frac{\partial \Sigma_{\sigma S,B}(r)}{\partial \rho_b}.$$

3 Results and Discussion

In recent years, there has been extensive theoretical research on hypernuclei, particularly focusing on the simplest single- Λ hypernuclei, using RMF and RHF theories. In this section, we aim to extend the effective interaction DD-LZ1 [?], which has proven successful and promising in determining nuclear structure properties in both bulk and single-particle aspects, to incorporate the Λ hyperon within the framework of the RMF model. To provide a comparative study and illustrate the role of nuclear in-medium effects, the calculations with DD-LZ1 will be accompanied by several existing effective ΛN interactions within CDF models. These interactions have been significantly expanded to incorporate the degrees of freedom of the Λ hyperon and have yielded many successful findings in the study of hypernuclear structure and the properties of dense stars. In detail, density-dependent RMF effective interactions DD-LZ1 [?], PKDD [?], DD-ME2, TW99, DDV [?], density-dependent RHF (DDRHF) effective interactions PKO1, PKO2, PKO3 [?], and nonlinear RMF (NLRMF) effective interactions NL-SH [?] and PK1 [?] were selected.

In these CDF functionals, the ω -tensor coupling, which has been proved essential in reducing Λ 's spin-orbit splitting in hypernuclei [?], is ignored. The Dirac equation is solved in a radial box size of $R = 20$ fm with a step of 0.1 fm. For open-shell hypernuclei, we employ the BCS method to account for pairing correlations. As the strength of hyperon pairing correlations remains uncertain and may become essential in multi- Λ hypernuclei, our current work solely considers pairing correlations between nn and pp pairs by using the finite-range Gogny force D1S [?], see Refs. [?, ?, ?, ?] for details. In addition, the blocking effect should be taken into account for the last valence nucleon or hyperon, with a detailed description in Ref. [?].

3.1 Density dependence of ΛN effective interaction

For the theoretical study of hypernuclear structure, the ΛN interaction must be determined first. Since the Λ hyperon is an electrically neutral particle with isospin zero, our focus lies on the coupling strengths between the isoscalar-scalar σ meson and the isoscalar-vector ω meson with the Λ hyperon. For convenience, we introduce the ratio of the coupling strengths between the meson-hyperon and meson-nucleon, $g_{\phi\Lambda}/g_{\phi N}$. According to the naive quark model [?], we fix the ratio of the isoscalar-vector meson coupling strength $g_{\omega\Lambda}/g_{\omega N}$ to 0.666, while the ratio of the isoscalar-scalar one $g_{\sigma\Lambda}/g_{\sigma N}$ can be obtained by reproducing the Λ hyperon separation energy B_Λ experimental data for $^{16}_\Lambda\text{O}$, $^{40}_\Lambda\text{Ca}$, and $^{208}_\Lambda\text{Pb}$ [?, ?]. In the fitting process, the hyperon is placed in the $1s_{1/2}$ ground state, and B_Λ is defined as follows:

$$B_\Lambda(^{A+1}Z) = E(^AZ) - E(^{A+1}_\Lambda Z).$$

Based on the effective interaction DD-LZ1, we finally obtained a new set of ΛN interaction, namely DD-LZ1- $\$1$, after a fitting process of Levenberg-

Marquardt minimization. Then, we calculated the Λ separation energy B_Λ as well as the single- Λ energy, with the hyperon occupying the ground state $1s_{1/2}$ or possible excited states with higher angular momentum l_Λ . For B_Λ of DD-LZ1- $\$ \1 , a remarkable agreement with experimental data is found for most hypernuclei, except for ${}^{28}_{\Lambda}\text{Si}$ with significant deformation and Carbon hyperisotopes with light mass, as shown in Fig. 1 [Figure 1: see original paper]. Actually, a more accurate description of the light-mass Carbon hyperisotopes could be obtained by limiting the mass region of fitting and taking into account deformation effects [?]. To investigate the deviation in describing the structural properties of single- Λ hypernuclei using different CDF effective interactions, the coupling strength of DD-LZ1- $\$ \1 in comparison with other selected CDF functionals is listed in Table 1. One could check the root-mean-square deviation Δ for B_Λ between theoretical calculation and experimental value, which is defined as

$$\Delta = \sqrt{\frac{1}{N} \sum_i (B_{\Lambda,i}^{\text{exp}} - B_{\Lambda,i}^{\text{cal}})^2}.$$

To reveal the systematics, we define Δ_1 to be the deviation only for ${}^{16}_{\Lambda}\text{O}$, ${}^{40}_{\Lambda}\text{Ca}$, and ${}^{208}_{\Lambda}\text{Pb}$, as well as Δ_2 that is suitable for all hypernuclei.

From Table 1, it can be seen that different CDF theoretical models have good descriptions for ${}^{16}_{\Lambda}\text{O}$, ${}^{40}_{\Lambda}\text{Ca}$, and ${}^{208}_{\Lambda}\text{Pb}$, and most parameter sets have good consistency for hypernuclear theoretical calculations and experimental data over a large mass range from ${}^{12}_{\Lambda}\text{C}$ to ${}^{208}_{\Lambda}\text{Pb}$. In addition, by comparing three different types of CDF effective interactions, we can find that when the ratio of isospin scalar-vector meson coupling strength is fixed to the same value, the ratio of isospin scalar-scalar meson coupling strength $g_{\sigma\Lambda}/g_{\sigma N}$ may satisfy certain linear correlations with the ratio of isospin scalar-vector meson coupling strength, which has been systematically explored in some works [?, ?, ?]. It should be pointed out that the linear correlation of meson-hyperon coupling strength ratios obtained in the RMF framework is obviously not suitable for density-dependent RHF models [?].

In the DDRMF approach, the in-medium effects of nuclear force are effectively embedded in the density-dependent shape of meson-baryon coupling strength, playing a role in nuclear structure via the equilibrium of nuclear dynamics from various coupling channels. In recent years, analysis based on the equilibrium of nuclear in-medium dynamics has been applied to clarify the mechanism of pseudo-spin symmetry, shell evolution, liquid-gas phase transition, and hyperon's spin-orbit splitting in CDF models [?, ?, ?, ?, ?]. The delicate in-medium balance between nuclear attractive and repulsive interactions may be significantly altered by treating the density dependence of coupling strength differently, impacting the description of the properties of nuclear matter and finite nuclei with different CDF effective interactions.

To provide a comprehensive understanding of the in-medium equilibrium in hypernuclei, we present the density dependence of coupling strengths for selected CDF effective interactions in Fig. 2(a) and Fig. 2(b), corresponding to the isoscalar-scalar channel $g_{\sigma\Lambda}$ and isoscalar-vector one $g_{\omega\Lambda}$. There are systematic divergences of the meson-hyperon coupling strengths with increasing density among density-dependent RMF, density-dependent RHF, and nonlinear RMF effective interactions. Notably, the density dependence of $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ is significantly reduced in the DDRHF effective interaction compared to the DDRMF effective interaction. This pronounced reduction in density dependence also influences the description of single-particle properties in hypernuclei, such as Λ hyperon spin-orbit splitting [?]. Furthermore, in contrast to density-dependent interactions, the NLRMF effective interaction exhibits density-independent characteristics for $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$. Consequently, when applying these three types of CDF effective interactions to single- Λ hypernuclei, systematic deviation could take place in describing the isospin dependence of hypernuclear structure.

3.2 Bulk properties of single- Λ hypernuclei in Oxygen hyperisotopes

To focus on the isospin dependence of single-particle properties, we choose the Λ hypernuclei and their nucleonic counterparts in Oxygen (hyper)isotopes as examples, since they usually exhibit spherical symmetry. To check the accuracy of the chosen interactions in describing the properties of finite nuclei, we first calculated the binding energies E_B , charge radii R_c , and matter radii R_m for Oxygen isotopes using the DD-LZ1 effective interaction. We compared the theoretical calculations with experimental measurements, which were taken from Refs. [?, ?, ?]. From the results in Table 2, we can see that the theoretical calculations and experimental measurements are in good agreement for both the binding energies E_B and the charge radii R_c for the interaction DD-LZ1. It is worth noting that the total matter radius R_m of finite nuclei, unlike the charge radius, still has significant uncertainties based on heavy ion reaction experiments. The theoretical calculations of R_m reconcile with the experimental measurements within the existence of error bars.

Furthermore, we summarize in Table 3 the systematics of the occupied energy level of Λ hyperon, the single-particle energies of Λ hyperon, the total binding energies, the charge radii, and the matter radii of hypernuclei in Oxygen hyperisotopes. In order to give possible reference to hypernuclear experiments, we also calculated the strength of electric dipole transition $B(E1)$ between the $\Lambda 1p$ and $\Lambda 1s$ occupation states. The transition strength is expressed as

$$B(E1; J_i \rightarrow J_f) = e_\Lambda^2 |\langle f | r | i \rangle|^2 \frac{2j_f + 1}{2j_i + 1} \begin{pmatrix} j_f & 1 & j_i \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}^2,$$

where e_Λ represents the effective charge of the Λ hyperon. The integration $\langle f | r | i \rangle$ can be computed using the radial wave functions of the initial and final

single- Λ state, see Ref. [?] for details. In the framework of relativistic models, Dirac spinors with both upper and lower components could contribute to determining the value of $B(E1)$. However, it is checked that the contribution from the lower component is negligible, especially for non-charge exchange channels. Therefore, only the contribution from the upper component is preserved in current calculations as a simplification.

The inclusion of Λ hyperon causes the so-called impurity effect inside hypernuclei [?]. When the Λ hyperon is filled in the $1s_{1/2}$ state, we can see from the comparison of the total matter radii in Table 3 and Table 2 that the introduction of hyperon causes a shrinkage effect on the hypernuclei, which is approximately $0.06 - 0.13$ fm. Compared with the ground-state results, we observe a significant enhancement in Λ root-mean-square radii when the hyperon is filled in higher-lying $1p$ state. This change in the density distribution of hyperon due to different level occupations leads to an overall expansion of the hypernuclear matter radii, different from the $\Lambda 1s$ case. Additionally, with the increase of neutron filling, both the hyperon radii, matter radii, and $B(E1)$ show significant isospin dependence, which can be qualitatively explained by the density-dependence of the coupling strength. As indicated in Table 3, when the Λ hyperon occupies the $1p$ state, its density distribution spreads more outward than the nucleonic core. As isospin evolves, more neutrons are filled and their attraction to the hyperon increases, correspondingly leading to a significant reduction in the hyperon radius. For $B(E1)$, its value is determined not only by the overlap between initial and final states which are sensitive to the neutron number, but also by the effective charge. As a result, the $B(E1)$ values enlarge a little from $^{15}_{\Lambda}\text{O}$ and go down gradually as isospin evolves after $N = 8$.

3.3 Isospin dependence of Λ spin-orbit splitting

Motivated by the connection between the density-dependent effective interactions of theoretical models and the isospin-dependent properties of nuclear structure, the spin-orbit splitting of Λ hyperon in hypernuclei, as a promising observable in current hypernuclear spectroscopy, will be discussed in this subsection with the newly developed DD-LZ1- $\$1$ and other selected CDF functionals. The Λ 's spin-orbit splitting is defined by the difference of Λ single-particle energies between a couple of spin partner states, which is

$$\Delta E_{\text{SO}}^{\Lambda} \equiv \epsilon_{j_{\Lambda}=l_{\Lambda}-1/2} - \epsilon_{j_{\Lambda}=l_{\Lambda}+1/2}.$$

As shown in Fig. 3 [Figure 3: see original paper], the analysis is carried out for Λ spin partner states $1p$ in Oxygen hyperisotopes, with the Λ hyperon occupying its ground state.

In Fig. 3(a), it is seen that the isospin dependence of $\Delta E_{\text{SO}}^{\Lambda}$ is clearly distinguished with the chosen CDF functionals. The curves from NLRMF models tend to be stable with increasing neutron number, while for density-dependent

RMF or RHF functionals the splitting enlarges generally with isospin. Among them, DD-LZ1-\$ \$1 exhibits the most significant isospin dependence. Besides, it is clear that the smaller Λ spin-orbit splitting is predicted by DDRHF compared to RMF, which has been illustrated as a result in single-particle properties since the dynamical equilibrium between nuclear attraction and repulsion is dramatically changed with the appearance of Fock terms [?].

To better understand the evolution of Λ spin-orbit splitting with isospin, we could decompose $\Delta E_{\text{SO}}^{\Lambda}$ into various parts according to its source of kinetic or potential energy. The values are obtained by left-multiplying the transferred Dirac spinor to the Dirac equation Eq. (18), and separating the integrated contributions from different self-energy terms. For instance, ΔE_{rea} comes from the contribution of the rearrangement term Σ_R to Λ self-energy (19), due to the density dependence of meson-hyperon couplings. Consequently, the rest from the kinetic energy and the density-independent potential energies could be summed over, which means $\Delta E_{\text{kin}+\sigma+\omega} \equiv \Delta E_{\text{SO}}^{\Lambda} - \Delta E_{\text{rea}}$, as discussed in Fig. 3(b).

It is observed that the values of Λ spin-orbit splitting are primarily determined by $\Delta E_{\text{kin}+\sigma+\omega}$. However, the isospin dependence of the splitting is weakly controlled by $\Delta E_{\text{kin}+\sigma+\omega}$ except for ${}^{15}_{\Lambda}\text{O}$. Attributed to the occupation of $\nu 1p_{1/2}$ orbit, the Λ spin-orbit splitting predicted by various CDF functionals systematically reduces from ${}^{15}_{\Lambda}\text{O}$ to ${}^{17}_{\Lambda}\text{O}$ [?]. As has been illustrated in Ref. [?], the spin-orbit coupling potential of hyperon is determined mainly by the radial derivative of the self-energy Σ_{Λ}^{-} . In general, the more neutrons are filled into hypernuclei, the larger the density circumstance where the Λ hyperon is housed. Thus, if the model is density dependent like DDRMFs and DDRHFs given in Fig. 2, the meson-hyperon coupling strength then weakens and $\Delta E_{\text{SO}}^{\Lambda}$ should become smaller correspondingly as the neutron number increases. As seen in Fig. 3(b), such a reduction in $\Delta E_{\text{kin}+\sigma+\omega}$ is remarkable from ${}^{15}_{\Lambda}\text{O}$ to ${}^{17}_{\Lambda}\text{O}$, and relatively less significant at larger neutron numbers. Different from the NLRMF case, the density-dependent CDFs introduce extra contribution to reinforce the isospin dependence of the splitting, as demonstrated in Fig. 3(c), which cancels the reduction trend in $\Delta E_{\text{kin}+\sigma+\omega}$ overwhelmingly and finally leads to the enhancement of $\Delta E_{\text{SO}}^{\Lambda}$ with increasing neutron number in Fig. 3(a). In fact, the contribution ΔE_{rea} to Λ spin-orbit splitting originates from the rearrangement terms of Λ self-energies $\Sigma_{0,\Lambda}$ which, according to Eq. (21), depends on the density slope of the meson-hyperon coupling strength. As the neutron number increases, the density scenario where Λ lives could get more intense, consequently resulting in weaker density dependence of the meson-hyperon coupling strength, smaller density slope, and suppressed value of ΔE_{rea} . Therefore, the link between the isospin evolution of Λ spin-orbit splitting and the in-medium behavior of ΛN interaction with baryon density is elucidated from the discussion on Oxygen hyperisotopes. Consequently, possible experimental constraints on $\Delta E_{\text{SO}}^{\Lambda}$ along the hyperisotopes could assist us further in understanding the in-medium effects of nuclear force.

3.4 Isospin dependence of matter and hyperon radii

In the properties of hypernuclear structure, not only the Λ spin-orbit splitting but also the Λ impurity effect could exhibit information about in-medium nuclear interactions. In Fig. 4 Figure 4: see original paper, we selected DDRMF functionals DD-LZ1-\$1 and DD-ME2, DDRHF's PKO1-1, and NLRMF's PK1 to illustrate its influence on the dotted lines correspond to the calculated results for single- Λ hypernuclei and their nucleonic counterparts in Oxygen (hyper)isotopes, respectively. The matter radius R_m in hypernuclei goes up monotonically as the neutron number increases, regardless of the specific model used, where a steep leap from $^{17}_{\Lambda}\text{O}$ to $^{25}_{\Lambda}\text{O}$ corresponds to the effect of new occupation in $\nu 2s_{1/2}$.

Although divergent values are given for Oxygen isotopes without hyperon, all of the selected models are getting closer in size of matter radii for hypernuclei, implying R_m of hypernuclei as a possible model-independent observable. It is evident that the matter radii of Oxygen hyperisotopes contract compared to their nucleonic counterparts, namely the size shrinkage due to the impurity effect of the Λ hyperon. However, the shrinkage magnitude appears to be strongly model dependent. Among them, the DDRMF effective Lagrangian DD-LZ1-\$1 yields the largest difference between the solid and dash-dotted lines, whereas the NLRMF one PK1 shows the smallest disparity. By checking the bulk properties of nuclear matter within these CDFs, it is verified that the shrinkage magnitude correlates well with the incompressibility, which is 230.7 MeV for DD-LZ1, 250.8 MeV for DD-ME2, 250.2 MeV for PKO1, and 282.7 MeV for PK1, respectively [?, ?, ?]. In fact, the larger the incompressibility K is, the harder the nucleus is contracted by the exerted attraction from the filled hyperon inside, consequently resulting in weaker size shrinkage effect in the calculated matter radii. A similar relation could be found from Table II of a work on the isoscalar giant monopole resonance of hypernuclei, where the effective nuclear incompressibility modulus was extracted [?].

To further distinguish the effects of different interactions on the description of hypernuclear structure, we investigate the isospin evolution of the Λ hyperon radius R_{Λ} in Oxygen hyperisotopes using all selected CDF effective interactions, as shown in Fig. 5 [Figure 5: see original paper]. It is seen tangibly that R_{Λ} evolves diversely along Oxygen hyperisotopes with different CDF effective interactions. Some effective interactions, like PKO3-\$1, DD-ME2, DDV, and DD-LZ1-\$1, exhibit a reduced R_{Λ} with increasing neutron number. Especially, DD-LZ1-\$1 gives the smallest hyperon radii among all chosen CDFs and a strong declining trend. In fact, the core polarization effect due to the Λ hyperon plays a significant role in this evolution. When Λ occupies the $1s_{1/2}$ state, its density distribution is concentrated inside the hypernucleus. As a result, the Λ 's coupling or attraction with the nucleons in the core (here corresponding to ^{16}O) appears relatively stronger than that with the valence nucleons. Hence, the evolution of the hyperon radius could be comprehended more or less by the size change of the core with respect to the neutron number.

The variation of the matter radii for the ^{16}O core in Oxygen (hyper)isotopes is plotted in Fig. 4(b) with respect to the neutron number. From $N = 8$ to 14, in contrast to the situation of total matter radii R_m , there is no consistent isospin dependence for the selected CDFs in the core radius R_{core} with increasing neutron number. The nonlinear RMF functional PK1 exhibits a significant increasing trend with isospin, while the density-dependent RMF one DD-LZ1-\$\Sigma\$1 shows a noticeable decrease. Consequently, the hyperon radius R_Λ exhibits a similar isospin dependence resulting from the core polarization effect, determined mainly by the various isospin properties of CDF functionals in nucleon-nucleon channels. From such analysis, the importance of nuclear in-medium effects in affecting the hyperon radii is unveiled. Thus, the divergent isospin evolution of R_Λ given by the CDFs with different density-dependent meson-baryon couplings makes it a valuable tool to elucidate the in-medium behavior of nuclear force.

4 Summary

In summary, considering the significance of nuclear in-medium effects in nuclear many-body problems, such as eliminating spurious shell closures, we expanded the newly developed DDRMF Lagrangian DD-LZ1 to incorporate the Λ hyperon degree of freedom and determined the ΛN effective interaction by fitting experimental data of Λ separation energies for several single- Λ hypernuclei. Subsequently, with several other CDF functionals, the features including Λ separation energy and $B(E1)$ transition, and the evolution of the spin-orbit splitting as well as the characteristic radii were analyzed in detail along the Oxygen (hyper)isotopes.

By comparing results obtained from different CDF models, we further investigated the crucial impact of nuclear in-medium effects on accurately describing hyperon properties, both in terms of their bulk and single-particle characteristics. For the $1p$ spin-orbit splitting of the Λ hyperon, significant differences in isospin dependence are observed among the selected CDF effective interactions in Oxygen hyperisotopes. As the neutron number increases, the density circumstance where the hyperon is housed gradually increases, which causes the meson-hyperon coupling strengths that determine hypernuclear properties to change as well. In particular, the density-dependent CDF effective interactions introduce additional rearrangement terms that significantly enhance the isospin dependence of the Λ spin-orbit splitting, leading to more distinct variation of $\Delta E_{\text{SO}}^\Lambda$ with neutron number in DDRMF and DDRHF models.

The evolution of the hypernuclear matter radius with isospin was further investigated. Significant model dependence in the magnitude of size shrinkage due to the inclusion of Λ hyperon is observed, where the DDRMF functional DD-LZ1-\$\Sigma\$1 displays the largest shrinkage effect. The result was then explained by an anticorrelation between the incompressibility coefficients K of nuclear matter and the hyperon radii R_Λ , providing us a possible way to constrain the hyperon distribution inside a hypernucleus from better-determined bulk properties of nu-

clear matter. Additionally, it is found that the isospin evolution of the hyperon radius is primarily influenced by the density-dependent behavior of the chosen CDF functional in the NN interaction channel via the mechanism of core polarization. Thus, the sensitivity in depicting these hyperon-relevant properties in CDF models with various meson-baryon couplings holds great potential to elucidate nuclear in-medium nature in both AN and NN channels.

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