

## Postprint of Theoretical Research on the Quantification of Wave-Particle Interactions in Collisionless Plasmas

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### Abstract

In collisionless plasmas, wave-particle interactions induce energy transfer between electromagnetic fields and particles, one consequence of which is the reshaping of particle velocity distribution functions. Therefore, the quantification of wave-particle interactions constitutes a fundamental problem in heliospheric and astrophysical plasma research. In recent years, significant advances have been achieved in studies on quantifying wave-particle interactions. This review will primarily introduce progress in related theoretical research, with particular emphasis on newly proposed theoretical methods for quantifying resonant and non-resonant wave-particle interactions. Additionally, applications of these methods to quantify wave-particle interactions in inner-heliospheric Alfvén-mode waves, proton beam instability, and electron heat flux instability will be introduced.

### Full Text

## On the Theory of Quantifying Wave-particle Interactions in Collisionless Plasmas

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### Abstract

In collisionless plasmas, wave-particle interactions can cause energy transfer between electromagnetic fields and particles, one consequence of which is reshaping the particle velocity distribution function. Therefore, how to quantify

wave-particle interactions is a fundamental problem in heliospheric and astrophysical plasma research. In recent years, many important achievements have been made in the study of quantifying wave-particle interactions. This paper will mainly introduce the progress in related theoretical research, particularly focusing on a newly proposed theoretical method for measuring both resonant and non-resonant wave-particle interactions. It will also introduce the application of this method in quantifying wave-particle interactions for Alfvén-mode waves, proton beam instabilities, and electron heat flux instabilities in the inner heliosphere.

**Key words:** Sun; heliosphere; waves; plasmas; wave-particle interaction

**Classification code:** P182; **Document code:** A

## 1. Introduction

In heliospheric and astrophysical plasma environments, particles are mostly in weakly collisional or collisionless states, and wave-particle interactions significantly influence particle dynamics. On one hand, an unstable particle velocity distribution function can release free energy through wave-particle interactions to reach a relatively stable state [?]. On the other hand, plasma waves can energize particles, triggering plasma heating and particle acceleration [?]. Furthermore, wave-particle interactions are widely regarded as an effective energy dissipation mechanism for space and astrophysical turbulence [?, ?, ?]. Therefore, how to quantify wave-particle interactions has always been a fundamental problem in heliospheric and astrophysical plasma research.

Plasmas contain multiple intrinsic wave modes with very broad temporal and spatial scales, and the corresponding wave-particle interactions affect particle dynamics across different time and space scales [?]. Based on different resonance conditions and mechanisms, wave-particle interactions are generally categorized as Landau interactions, transit-time interactions, and normal/anomalous cyclotron interactions [?, ?, ?]. As early as the 1960s, the plasma theory community recognized that these wave-particle interactions originate from the dot product of electric field  $\mathbf{E}$  and current density  $\mathbf{J}$ , i.e.,  $\mathbf{E} \cdot \mathbf{J}$  [?]. When plasma waves appear,  $\mathbf{E}$  and  $\mathbf{J}$  become coherent, making  $\mathbf{E} \cdot \mathbf{J}$  non-zero and leading to energy conversion between electromagnetic energy and particle kinetic energy.

Theoretical research on wave-particle interactions in the 1960s mainly focused on interactions between Langmuir waves and electrons [?, ?, ?]. With the development of space detection technology, researchers began to explore how to directly identify and quantify various types of wave-particle interactions in observations, particularly those related to electromagnetic wave modes. Over the past two decades, significant progress has been made in this direction.

Theoretically, Quataert [?] derived an expression for the wave-particle energy transfer rate based on the weak damping approximation in 1998, comparing the contributions of Landau resonance and transit-time resonance mechanisms to kinetic Alfvén wave damping. Klein et al. [?] proposed a new method in 2016

to identify and quantify wave-particle interactions in velocity space, which they termed the field-particle correlation method. Subsequently, a series of studies [?, ?, ?] used numerical simulation data and the field-particle correlation method to predict signals of various wave-particle resonance mechanisms (electron Landau resonance, ion Landau resonance, and ion cyclotron resonance) in velocity space. Meanwhile, based on analysis of space satellite data with high-precision particle measurements, researchers identified observational signatures of wave-particle interactions associated with kinetic Alfvén waves (electron Landau resonance) and ion cyclotron waves (ion cyclotron resonance) [?, ?, ?, ?].

Recently, Zhao et al. [?] developed a theory for quantifying wave-particle interactions. This theory incorporates an interaction factor term and can analyze both resonant ( $v_{\parallel}k_{\parallel} - \omega + n\Omega_{cs} = 0$ ) and non-resonant ( $v_{\parallel}k_{\parallel} - \omega + n\Omega_{cs} \neq 0$ ) wave-particle interactions of different types (depending on the  $n$  value), where  $v_{\parallel}$  represents the particle velocity parallel to the magnetic field,  $k_{\parallel}$  denotes the wave number parallel to the magnetic field,  $\omega$  is the wave frequency,  $\Omega_{cs}$  is the cyclotron frequency of particle species “s”, and  $n$  is an integer. Zhao et al. [?] also defined multiple energy transfer rate expressions to measure different types of wave-particle interactions. This paper will focus on introducing this theory and its applications: Section 2 presents the basic theory; Sections 3 through 5 introduce three applications [?, ?, ?], namely quantifying wave-particle interactions for Alfvén-mode waves, proton beam instabilities, and electron heat flux instabilities; and Section 6 provides a brief summary.

## 2. Basic Theory

In weakly collisional or collisionless plasmas, particle dynamics can be described by the Vlasov equation:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0,$$

where  $f_s$ ,  $q_s$ , and  $m_s$  represent the velocity distribution function, charge, and mass of particle species “s”, respectively;  $\mathbf{E}$  and  $\mathbf{B}$  denote the electric and magnetic fields;  $t$  represents time;  $\mathbf{v}$  represents velocity; and  $\mathbf{r}$  represents position.

Below we derive the control equations for energy conversion between electromagnetic energy and particle kinetic energy. First, we linearize equation (1). Assume any physical variable  $x$  can be written as  $x = x_0 + \epsilon x_1 + \epsilon^2 x_2$ , where  $x_0$  represents the zeroth-order variable,  $x_1$  the first-order variable,  $x_2$  the second-order variable, and  $\epsilon$  is a small quantity with  $\epsilon \ll 1$ . This paper considers the existence of a background velocity distribution function ( $f_{s0} \neq 0$ ) and background magnetic field ( $\mathbf{B}_0 \neq 0$ ), but no background electric field ( $\mathbf{E}_0 = 0$ ). Based on equation (1) and the linearized expressions for physical variables, we can derive the first- and second-order Vlasov equations:

$$\frac{\partial f_{s1}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{s1}}{\partial \mathbf{r}} + \frac{q_s}{m_s} (\mathbf{v} \times \mathbf{B}_0) \cdot \frac{\partial f_{s1}}{\partial \mathbf{v}} = -\frac{q_s}{m_s} (\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1) \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}},$$

$$\frac{\partial f_{s2}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{s2}}{\partial \mathbf{r}} + \frac{q_s}{m_s} (\mathbf{v} \times \mathbf{B}_0) \cdot \frac{\partial f_{s2}}{\partial \mathbf{v}} = -\frac{q_s}{m_s} (\mathbf{E}_2 + \mathbf{v} \times \mathbf{B}_2) \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}} - \frac{q_s}{m_s} (\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1) \cdot \frac{\partial f_{s1}}{\partial \mathbf{v}},$$

where  $f_{s1}$  and  $f_{s2}$  represent the first- and second-order particle velocity distribution functions,  $\mathbf{E}_1$  and  $\mathbf{E}_2$  the first- and second-order electric field perturbations, and  $\mathbf{B}_1$  and  $\mathbf{B}_2$  the first- and second-order magnetic field perturbations.

### 2.1.1 First-order Particle Kinetic Energy

Multiplying equation (2) by  $m_s v^2/2$  and integrating over position and velocity space yields the control equation for first-order particle kinetic energy:

$$\frac{\partial E_{s1}}{\partial t} = - \int d\mathbf{r} d\mathbf{v} \left[ \frac{m_s v^2}{2} \mathbf{v} \cdot \frac{\partial f_{s1}}{\partial \mathbf{r}} + \frac{q_s v^2}{2} (\mathbf{v} \times \mathbf{B}_0) \cdot \frac{\partial f_{s1}}{\partial \mathbf{v}} + \frac{q_s v^2}{2} (\mathbf{v} \times \mathbf{B}_1) \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}} + \frac{q_s v^2}{2} \mathbf{E}_1 \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}} \right],$$

where  $E_{s1} = \int d\mathbf{r} d\mathbf{v} (m_s v^2 f_{s1}/2)$  represents the first-order kinetic energy of particle species “s”.

The first term on the right-hand side of equation (4) is related to particle bounce motion [?, ?]. This term represents the integral in position space of the gradient of first-order particle kinetic energy density,  $\partial_{\mathbf{r}}(m_s v^2 f_{s1}/2)$ . For periodically varying  $f_{s1}$ , its integral in position space is zero [?, ?]. The second and third terms on the right-hand side of equation (4) are related to particle Lorentz motion. Since the Lorentz force only changes the direction of particle motion without altering the magnitude of particle kinetic energy, these two terms contribute nothing to particle kinetic energy changes.

The fourth term on the right-hand side of equation (4) can be written as  $\int d\mathbf{r} (\mathbf{E}_1 \cdot \mathbf{J}_{s0})$ , where  $\mathbf{J}_{s0} = \int d\mathbf{v} (q_s \mathbf{v} f_{s0})$  represents the current density carried by drifting particles. When considering the plasma zero-current condition, i.e.,  $\sum_s \mathbf{J}_{s0} = 0$ , we have  $\sum_s (\mathbf{E}_1 \cdot \mathbf{J}_{s0}) = 0$ , indicating that the fourth term does not cause a net change in the total first-order kinetic energy of all particles.

### 2.1.2 Second-order Particle Kinetic Energy

Using a derivation method similar to Section 2.1.1 and based on equation (3), we obtain the control equation for second-order particle kinetic energy:

$$\frac{\partial E_{s2}}{\partial t} = - \int d\mathbf{r} d\mathbf{v} \left[ \frac{m_s v^2}{2} \mathbf{v} \cdot \frac{\partial f_{s2}}{\partial \mathbf{r}} + \frac{q_s v^2}{2} (\mathbf{v} \times \mathbf{B}_0) \cdot \frac{\partial f_{s2}}{\partial \mathbf{v}} + \frac{q_s v^2}{2} (\mathbf{v} \times \mathbf{B}_2) \cdot \frac{\partial f_{s0}}{\partial \mathbf{v}} + \frac{q_s v^2}{2} (\mathbf{v} \times \mathbf{B}_1) \cdot \frac{\partial f_{s1}}{\partial \mathbf{v}} + \frac{q_s v^2}{2} \right]$$

where the definition of particle second-order kinetic energy is

$$E_{s2} = \int d\mathbf{r}d\mathbf{v} \frac{m_s v^2}{2} f_{s2}.$$

The first term on the right-hand side of equation (5) represents particle bounce motion related to  $f_{s2}$ , while the third to fourth terms are related to particle Lorentz motion (arising from different Lorentz forces). Based on the physical discussion in Section 2.1.1, particle bounce motion and Lorentz motion do not change the magnitude of particle kinetic energy, so the first through fourth terms on the right-hand side contribute nothing to changes in  $E_{s2}$ . The fifth term can be written as  $\int d\mathbf{r}(\mathbf{E}_2 \cdot \mathbf{J}_{s0})$ , which under zero-current conditions does not cause a net change in the total second-order kinetic energy of all particles ( $\sum_s E_{s2}$ ). Thus, only the sixth term on the right-hand side of equation (5) can cause a net change in particle second-order kinetic energy. Therefore, equation (5) simplifies to:

$$\frac{\partial E_{s2}}{\partial t} = - \int d\mathbf{r}d\mathbf{v} \frac{q_s v^2}{2} \mathbf{E}_1 \cdot \frac{\partial f_{s1}}{\partial \mathbf{v}}.$$

This equation can also be written as:

$$\frac{\partial E_{s2}}{\partial t} = \int d\mathbf{r}(\mathbf{E}_1 \cdot \mathbf{J}_{s1}),$$

where  $\mathbf{J}_{s1} = \int d\mathbf{v}(q_s \mathbf{v} f_{s1})$ . Equation (8) indicates that energy transfer between electromagnetic perturbations and particles is governed by the  $(\mathbf{E}_1 \cdot \mathbf{J}_{s1})$  term.

Considering only this effect, the control equation for particle kinetic energy in phase space,  $e_s$ , can be written as:

$$\frac{\partial e_s}{\partial t} + \mathbf{v} \cdot \frac{\partial e_s}{\partial \mathbf{r}} + \frac{q_s}{m_s} (\mathbf{v} \times \mathbf{B}_0) \cdot \frac{\partial e_s}{\partial \mathbf{v}} = q_s \mathbf{E}_1 \cdot \mathbf{v} f_{s1},$$

where the phase-space particle kinetic energy is defined as  $e_s \equiv m_s v^2 f'_{s2}/2$ . It should be noted that  $f'_{s2}$  represents an equivalent second-order perturbation of the particle velocity distribution function, different from  $f_{s2}$  (see equation (3)). The former is only affected by electric field forces, while the latter is subject to the combined effects of pressure gradient forces, Lorentz forces, and electric field forces.

Integrating over velocity space yields:

$$\frac{d}{dt} \int d\mathbf{v} e_s = \int d\mathbf{v} (\mathbf{E}_1 \cdot \mathbf{v} f_{s1}).$$

When  $|v| \rightarrow \infty$ ,  $(m_s v^2 f_{s1}/2) \rightarrow 0$ , making the first term on the right-hand side of equation (7) zero. Equation (7) can be further written as:

$$\frac{d}{dt} \int d\mathbf{v} e_s = \mathbf{E}_1 \cdot \mathbf{J}_{s1}.$$

## 2.2 Energy Transfer and Energy Transfer Rate from Monochromatic Waves

Various wave modes exist in heliospheric and astrophysical plasma environments, generally carrying electromagnetic energy that can be converted with particle kinetic energy. This paper discusses ideal plasma waves, i.e., plane waves, whose first-order electric field, current density, and particle velocity distribution function perturbations can be written as:

$$\mathbf{E}_1(\mathbf{r}, t) = \tilde{\mathbf{E}}_1 e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} + \tilde{\mathbf{E}}_1^* e^{i(\omega^* t - \mathbf{k} \cdot \mathbf{r})},$$

$$\mathbf{J}_{s1}(\mathbf{r}, t) = \tilde{\mathbf{J}}_{s1} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} + \tilde{\mathbf{J}}_{s1}^* e^{i(\omega^* t - \mathbf{k} \cdot \mathbf{r})},$$

$$f_{s1}(\mathbf{r}, \mathbf{v}, t) = \tilde{f}_{s1} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} + \tilde{f}_{s1}^* e^{i(\omega^* t - \mathbf{k} \cdot \mathbf{r})},$$

where  $\tilde{\mathbf{E}}_1$ ,  $\tilde{\mathbf{J}}_{s1}$ , and  $\tilde{f}_{s1}$  represent the Fourier-space expressions for first-order electric field, current density, and particle velocity distribution function perturbations;  $\omega$  denotes the wave frequency;  $\mathbf{k}$  represents the wave vector; and the superscript "\*" denotes complex conjugate.

Using equation (11), equations (9) and (10) can be written as:

$$\frac{d}{dt} \int d\mathbf{v} e_s = \int d\mathbf{r} \int d\mathbf{v} (\mathbf{E}_1 \cdot \mathbf{v} f_{s1}) = \int d\mathbf{r} (\mathbf{E}_1 \cdot \mathbf{J}_{s1}),$$

where

$$\mathbf{E}_1 \cdot \mathbf{v} f_{s1} = \tilde{\mathbf{E}}_1 \cdot \mathbf{v} \tilde{f}_{s1} e^{-i2(\omega t - \mathbf{k} \cdot \mathbf{r})} + \tilde{\mathbf{E}}_1^* \cdot \mathbf{v} \tilde{f}_{s1}^* e^{i2(\omega^* t - \mathbf{k} \cdot \mathbf{r})} + \tilde{\mathbf{E}}_1 \cdot \mathbf{v} \tilde{f}_{s1}^* e^{i(\omega^* - \omega)t} + \tilde{\mathbf{E}}_1^* \cdot \mathbf{v} \tilde{f}_{s1} e^{i(\omega - \omega^*)t},$$

and

$$\mathbf{E}_1 \cdot \mathbf{J}_{s1} = \tilde{\mathbf{E}}_1 \cdot \tilde{\mathbf{J}}_{s1} e^{-i2(\omega t - \mathbf{k} \cdot \mathbf{r})} + \tilde{\mathbf{E}}_1^* \cdot \tilde{\mathbf{J}}_{s1}^* e^{i2(\omega^* t - \mathbf{k} \cdot \mathbf{r})} + \tilde{\mathbf{E}}_1 \cdot \tilde{\mathbf{J}}_{s1}^* e^{i(\omega^* - \omega)t} + \tilde{\mathbf{E}}_1^* \cdot \tilde{\mathbf{J}}_{s1} e^{i(\omega - \omega^*)t}.$$

Since the wave frequency can be decomposed into real and imaginary parts, i.e.,  $\omega = \omega_r + i\gamma$  ( $\omega_r$  represents wave oscillation,  $\gamma$  represents wave damping or

growth rate),  $e_s$  and  $d \int d\mathbf{v} e_s / dt$  in equations (12) and (13) have two timescales:  $\propto \exp(\pm i2\omega_r t)$  and  $\propto \exp(2\gamma t)$ . The former represents periodic oscillations of energy with period  $1/(2\omega_r)$ , while the latter represents continuous growth or decay of energy. When considering time-averaging effects, the periodic oscillation terms do not lead to particle energy changes; the non-oscillatory terms cause wave-particle energy transfer.

Based on equations (12) and (13), the control equations for wave-particle energy transfer can be written as:

$$\frac{d}{dt} \int d\mathbf{v} e_s = 4\Re \left( \tilde{\mathbf{E}}_1 \cdot \tilde{\mathbf{J}}_{s1}^* \right) e^{2\gamma t},$$

$$\frac{d}{dt} \int d\mathbf{v} e_s = 4\Re \left( \tilde{\mathbf{E}}_1 \cdot \mathbf{v} \tilde{f}_{s1}^* \right) e^{2\gamma t}.$$

Using the non-oscillatory part of electromagnetic energy, i.e.,  $W_{EB} \exp(2\gamma t)$ , as a normalization factor, the above two equations can be rewritten as:

$$\frac{d}{dt} \int d\mathbf{v} e_s = P_s(\mathbf{v}) W_{EB} e^{2\gamma t},$$

$$\frac{d}{dt} \int d\mathbf{v} e_s = P_{st} W_{EB} e^{2\gamma t},$$

where

$$P_s(\mathbf{v}) = \frac{4\Re \left( \tilde{\mathbf{E}}_1 \cdot \mathbf{v} \tilde{f}_{s1}^* \right)}{W_{EB}},$$

$$P_{st} = \int d\mathbf{v} P_s(\mathbf{v}) = \frac{4\Re \left( \tilde{\mathbf{E}}_1 \cdot \tilde{\mathbf{J}}_{s1}^* \right)}{W_{EB}},$$

and  $W_{EB} = \epsilon_0 |\tilde{\mathbf{E}}|^2 / 4 + |\tilde{\mathbf{B}}|^2 / 4\mu_0$ .

$P_s(\mathbf{v})$  and  $P_{st}$  can directly measure wave-particle interactions. The specific physical meaning of  $P_s(\mathbf{v})$  is the rate of energy emission or absorption per unit phase space, per unit electromagnetic energy, and per unit time. The specific physical meaning of  $P_{st}$  is the rate of energy emission or absorption per unit position space, per unit electromagnetic energy, and per unit time.

Since  $\sum_s P_{st} = -2\gamma$  [?, ?], using expressions (18) and (19), we can quantitatively calculate the contributions of different particle species and wave-particle interaction mechanisms to plasma wave damping ( $\gamma < 0$ ) or growth ( $\gamma > 0$ ).

To effectively measure various types of wave-particle interactions, reference [?] defined multiple energy transfer rate expressions, some of which were first introduced in references [?, ?]. These are summarized as follows:

- (1) One- and two-dimensional energy transfer rates in Cartesian coordinates:

$$P_s(v_i) = \int dv_j dv_k P_s(\mathbf{v}), \quad P_s(v_i, v_j) = \int dv_k P_s(\mathbf{v}),$$

where  $i$ ,  $j$ , and  $k$  represent the three coordinate axes ( $x$ ,  $y$ ,  $z$ ) in the Cartesian coordinate system.

- (2) One- and two-dimensional energy transfer rates in cylindrical coordinates ( $v_{\parallel}$ ,  $v_{\perp}$ ,  $\phi$ ):

$$P_s(v_{\parallel}) = \int dv_{\perp} d\phi v_{\perp} P_s(v_{\parallel}, v_{\perp}, \phi),$$

$$P_s(v_{\perp}) = \int dv_{\parallel} d\phi v_{\perp} P_s(v_{\parallel}, v_{\perp}, \phi),$$

$$P_s(v_{\parallel}, v_{\perp}) = \int d\phi v_{\perp} P_s(v_{\parallel}, v_{\perp}, \phi).$$

The transformation from Cartesian coordinates ( $v_x$ ,  $v_y$ ,  $v_z$ ) to cylindrical coordinates ( $v_{\parallel}$ ,  $v_{\perp}$ ,  $\phi$ ) is given by:  $v_{\parallel} = v_z$ ,  $v_{\perp} = (v_x^2 + v_y^2)^{1/2}$ , and  $\phi = \arctan(v_y/v_x)$ .

- (3) Energy transfer rates for different components and different  $n$  values:

$$P_{sl}(\mathbf{v}) = \frac{4\Re(\tilde{E}_{1l} \tilde{v}_l \tilde{f}_{s1}^*(\mathbf{v}, n))}{W_{EB}},$$

$$P_s(\mathbf{v}, n) = \frac{4\Re(\tilde{\mathbf{E}}_1 \cdot \mathbf{v} \tilde{f}_{s1}^*(\mathbf{v}, n))}{W_{EB}},$$

$$P_s(n) = \int d\mathbf{v} P_s(\mathbf{v}, n),$$

where “ $l$ ” represents  $x$ ,  $y$ , or  $z$ .

It should be noted that because the particle velocity distribution function perturbation contains the interaction factor  $v_{\parallel} k_{\parallel} - \omega + n\Omega_{cs}$ , the velocity-space energy transfer rate expressions can simultaneously measure both resonant ( $v_{\parallel} k_{\parallel} - \omega + n\Omega_{cs} = 0$ ) and non-resonant ( $v_{\parallel} k_{\parallel} - \omega + n\Omega_{cs} \neq 0$ ) wave-particle interactions. Detailed explanations can be found in reference [?].

Furthermore, in plasma instability research, an important issue is the source of free energy and the energy transfer process. This problem can be addressed using energy transfer rate expressions for different particles, as detailed in references [?, ?].

### 3. Application 1: Wave-particle Interactions in Alfvén-mode Waves

Based on the energy transfer rate expressions listed in Section 2, Zhao et al. [?] explored the feasibility of quantifying wave-particle interactions in Alfvén-mode waves. Using observational data from the Parker Solar Probe, reference [?] discussed Alfvén-mode waves in a typical inner heliospheric environment (solar wind at a heliocentric distance of about 36 solar radii). The plasma and magnetic field parameters used were: proton number density  $n_p \simeq 286 \text{ cm}^{-3}$  and proton temperature  $T_p \simeq 27 \text{ eV}$ ; for electron number density and temperature, they were assumed equal to ion parameters,  $n_e = n_p$  and  $T_e = T_p$ ; magnetic field strength  $B_0 \simeq 91 \text{ nT}$ . Thus, reference [?] focused on a moderate- $\beta$  plasma environment with plasma beta values  $\beta_p = \beta_e \simeq 0.37$ .

Below we introduce some important results from reference [?] on quantifying wave-particle interactions in Alfvén-mode waves using total energy transfer rate and velocity-space energy transfer rate expressions.

Using the total energy transfer rate expression  $P_{pt}(n)$ , reference [?] presented the distribution of energy transfer rates on the Alfvén-mode wave dispersion surface, revealing the contributions of different types of wave-particle interactions to wave damping.

Figure 1 [Figure 1: see original paper] shows the dispersion surface and damping rate distribution of Alfvén-mode waves, while Figure 2 [Figure 2: see original paper] displays the energy transfer rate distribution between electromagnetic fields and particles (protons and electrons). It should be noted that the energy transfer rate distribution shown in Figure 2 includes both resonant and non-resonant wave-particle interaction sources [?], where resonant wave-particle interactions lead to positive energy transfer rates, while non-resonant wave-particle interactions lead to negative energy transfer rates.

Based on Figure 2, reference [?] divided Alfvén-mode waves into four main categories, see regions I, II, III, and IV labeled in Figure 2:

- (1) Region I: Quasi-parallel and medium-angle oblique MHD Alfvén waves. The energy transfer rate distribution characteristics in this region are:  $P_{pt}(n=0)/(-2\gamma) \gtrsim 1$ ,  $P_{pt}(n=1)/(-2\gamma) \lesssim -0.5$ , and  $P_{et}(n=0)/(-2\gamma) \gtrsim 0.1$ . Therefore, the damping mechanism of this wave type mainly comes from proton and electron  $n=0$  resonant wave-particle interactions.
- (2) Region II: Quasi-parallel ion cyclotron waves. In this region,  $P_{pt}(n=1)/(-2\gamma) \gtrsim 0.5$ , indicating that wave damping mainly comes from the proton cyclotron resonance mechanism. However, when Region II waves have a finite  $\lambda_p k_\perp$  ( $\lambda_p$  and  $k_\perp$  represent the proton inertial length and perpendicular wavenumber, respectively),  $P_{pt}(n=0)$  is a small positive value, indicating that proton  $n=0$  resonant wave-particle interactions also contribute to wave damping.

Figure 1(a) shows the dispersion surface of the Alfvén-mode wave,  $\omega_r/\Omega_{cp}$ , where the normalized parameter is the proton cyclotron frequency  $\Omega_{cp}$ . (b) shows the damping rate of the Alfvén-mode wave,  $-\gamma/\omega_r$ , where the normalized parameter is the wave frequency  $\omega_r$ . The dotted curve represents the contour at  $\gamma = -0.01\omega_r$ . From reference [?].

Figure 2 shows the total energy transfer rates  $P_{st}$  of protons ( $s = p$ , top panels) and electrons ( $s = e$ , bottom panels). The panels from left to right illustrate different  $n$ : (a)  $n = 1$ ; (b)  $n = -1$ ; and (c)  $n = 0$ . The data are normalized by  $(-2\gamma)$ . The four regions labeled by I, II, III, and IV correspond to: the quasi-parallel and medium oblique MHD Alfvén wave regime (surrounded by the black thin dotted curve) where  $P_{pt}(n = 0)/(-2\gamma) \gtrsim 1$  and  $P_{pt}(n = 1)/(-2\gamma) \lesssim -0.5$ ; the quasi-parallel ion cyclotron wave regime (surrounded by the black thick dotted curve) where  $P_{pt}(n = 1)/(-2\gamma) \gtrsim 0.5$  and  $\theta < 45^\circ$ ; the quasi-perpendicular MHD and ion-scale Alfvén wave regime (surrounded by the blue thin dotted curve) where  $P_{et}(n = 0)/(-2\gamma) \gtrsim 1$ ,  $P_{pt}(n = 0)/(-2\gamma) \gtrsim 0.5$  and  $\theta > 45^\circ$ ; and the quasi-perpendicular sub-ion scale Alfvén wave regime (surrounded by the blue thick dotted curve) where  $P_{et}(n = 0)/(-2\gamma) \gtrsim 1$  and  $\theta > 45^\circ$ , respectively. From reference [?].

- (3) Region III: Quasi-perpendicular MHD and ion-scale Alfvén waves. Since  $P_{et}(n = 0)/(-2\gamma) \gtrsim 1$  and  $P_{pt}(n = 0)/(-2\gamma) \gtrsim 0.5$ , wave damping mainly comes from electron and proton  $n = 0$  resonant wave-particle interactions. In this region,  $P_{pt}(n = 1) < 0$  and  $P_{pt}(n = -1) < 0$ , which shows that non-resonant protons release energy.
- (4) Region IV: Quasi-perpendicular sub-ion-scale Alfvén waves. In this region,  $P_{et}(n = 0)/(-2\gamma) \gtrsim 1$ . Wave damping mainly comes from electron  $n = 0$  resonant wave-particle interactions, including Landau resonance and transit-time resonance wave-particle interactions. Additionally,  $P_{pt}(n = 1) < 0$  and  $P_{pt}(n = -1) < 0$  indicate that non-resonant protons release energy.

In addition to these four main types of Alfvén-mode waves, Figure 2 also shows that in regions with finite  $\lambda_p k_\perp$  and  $\lambda_p k_\parallel$  (e.g.,  $\lambda_p k_\perp \sim 1$  and  $\lambda_p k_\parallel \sim 0.5$ , where waves are obliquely propagating ion-scale waves), since  $P_{pt}(n = 1)/(-2\gamma) \gtrsim 0.5$  and  $P_{pt}(n = 0)/(-2\gamma) \gtrsim 0.5$ , wave damping in these regions mainly comes from ion cyclotron and Landau resonance mechanisms.

Using velocity-space energy transfer rate expressions, reference [?] displayed for the first time the signatures of resonant and non-resonant wave-particle interactions. This paper briefly introduces the ion cyclotron waves in Region II and sub-ion-scale Alfvén waves in Region IV (Figures 3 [Figure 3: see original paper] and 4 [Figure 4: see original paper]), with complete discussions available in reference [?].

Figure 3 shows the proton energy transfer rate distribution for a typical ion cyclotron wave ( $\lambda_p k_\perp = 0.01$  and  $\lambda_p k_\parallel = 0.7$ ). According to the resonance condition, the proton resonant velocities corresponding to  $n = 1$  and  $n = 0$

wave-particle resonant interactions are  $v_{\parallel} \simeq -1.5V_{tp}$  and  $0.9V_{tp}$ , respectively, where  $V_{tp}$  is the proton thermal speed. The wave-particle energy transfer rates at these two resonant velocity positions mainly come from resonant wave-particle interactions, while those at other velocity positions come from non-resonant wave-particle interactions. Figure 3 clearly shows the occurrence signature of resonant wave-particle interactions—red lines at the resonant velocities. Compared with the cyclotron resonance interaction at  $v_{\parallel} \simeq -1.5V_{tp}$ , the transit-time and Landau resonance interactions at  $v_{\parallel} \simeq 0.9V_{tp}$  are both weak. Figure 3 also clearly shows the occurrence signature of non-resonant wave-particle interactions, namely that the energy transfer rates they cause are generally negative, with this effect being strongest at  $v_{\parallel} \simeq -1.5V_{tp}$ .

Figure 3. The proton energy transfer rate distributions of a typical quasi-parallel ion-cyclotron wave which has  $\lambda_p k_{\perp} = 0.01$  and  $\lambda_p k_{\parallel} = 0.7$ . In this figure,  $V_{tp\parallel}$  and  $V_{tp\perp}$  denote the parallel and perpendicular proton thermal speed, respectively. (a) The normalized total energy transfer rate,  $\bar{P}_{pt}$ ; (b) the normalized  $x$  component of the energy transfer rate,  $\bar{P}_{px}$ ; (c) the normalized  $y$  component of the energy transfer rate,  $\bar{P}_{py}$ ; and (d) the normalized  $z$  component (or parallel) energy transfer rate,  $\bar{P}_{pz}$ . The normalized parameter in the 2D distributions is the maximum  $P_{pt}$  in the 2D velocity space. The normalized parameter in the 1D distributions corresponds to the maximum  $P_{pt}(v_{\parallel})$  or the maximum  $P_{pt}(v_{\perp})$ . The 1D energy transfer rate coordinates are logarithmic coordinates, and the colorbars are also presented in logarithmic form. From reference [?].

Figure 4 shows the energy transfer rate distribution for a typical sub-ion-scale Alfvén wave ( $\lambda_p k_{\parallel} = 0.01$  and  $\lambda_p k_{\perp} = 10$ ). This scale of Alfvén wave is mainly affected by  $n = 0$  wave-particle interactions (see Figure 2), so Figure 4 only shows the energy transfer rate distribution caused by this type of interaction. Theoretically, the predicted resonant velocity is  $v_{\parallel} \simeq 0.2V_{te}$ , and Figure 4 clearly displays the resonant wave-particle interaction signature at this location. Comparing the transit-time resonance ( $\bar{P}_{ey}$ ) with the Landau resonance ( $\bar{P}_{ez}$ ) signatures, the former's intensity is about half of the latter's, indicating that the Landau resonance mechanism is the main dissipation mechanism for sub-ion-scale Alfvén waves. Figure 4 also shows that the non-resonant energy transfer rate distribution strongly depends on velocity and wave-particle interaction type.

Figure 4. The distributions of electron energy transfer rates at  $n = 0$  of a typical sub-ion Alfvén-mode wave having  $\lambda_p k_{\parallel} = 0.01$  and  $\lambda_p k_{\perp} = 10$ . In this figure,  $V_{te\parallel}$  and  $V_{te\perp}$  denote the parallel and perpendicular electron thermal speed, respectively. (a) The normalized total energy transfer rate,  $\bar{P}_{et}$ ; (b) the normalized  $x$  component of the energy transfer rate,  $\bar{P}_{ex}$ ; (c) the normalized  $y$  component of the energy transfer rate,  $\bar{P}_{ey}$ ; and (d) the normalized  $z$  component (or parallel) energy transfer rate,  $\bar{P}_{ez}$ . The normalized parameter in the 2D distributions is the maximum  $P_{et}$  in the 2D velocity space. The normalized parameter in the 1D distributions corresponds to the maximum  $P_{et}(v_{\parallel})$  or the maximum  $P_{et}(v_{\perp})$ . The 1D energy transfer rate coordinates are logarithmic

coordinates, and the colorbars are also presented in logarithmic form. From reference [?].

It should be particularly mentioned that reference [?] physically explained non-resonant wave-particle interactions. This paper briefly quotes that explanation: For plasma waves, in addition to electromagnetic perturbations, the wave also has macroscopic velocity perturbations of the plasma, which are determined by the velocity distribution function perturbations. When wave damping occurs, in addition to the decrease in the wave's electromagnetic energy, its kinetic energy also decreases, with the corresponding microscopic manifestation being that non-resonant energy transfer rates are predominantly negative.

#### 4. Application 2: Wave-particle Interactions in Proton Beam Instabilities

Liu et al. [?] used multiple energy transfer rate expressions to quantify wave-particle interactions in proton beam instabilities in the inner heliosphere, considering a four-component plasma (core protons, beam protons, alpha particles, and electrons).

Based on the plasma and magnetic field model summarized by Bale et al. [?], reference [?] presented the distributions of basic parameters for proton beam instabilities in the inner heliosphere, as shown in Figure 5 [Figure 5: see original paper]. According to the distribution characteristics of electric field, wave number, and angle of instability-excited waves, reference [?] identified five types of ion beam instabilities, labeled in Figure 5 as I (Oblique Alfvén/Ion Cyclotron instability, i.e., OA/IC instability), II (Oblique Fast Magnetosonic/Whistler instability, i.e., OFM/W instability), III (Oblique Alfvén/Ion Beam instability, i.e., OA/IB instability), IV (Parallel Fast Magnetosonic/Whistler instability, i.e., PFM/W instability), and V (Parallel Alfvén/Ion Cyclotron instability, i.e., PA/IC instability).

Figure 5 shows the radial distribution of proton beam instabilities in the inner heliosphere. In this figure,  $r/R_S$  denotes the normalized radial distance ( $R_S$  is the solar radius), and  $V_{pb}$  denotes the drift speed of beam protons. (a) The maximum growth rate  $\gamma_{max}$ ; (b) the wave frequency  $\omega_r$ ; (c) the argument of  $E_y/E_x$ ,  $\arg(E_y/E_x)$ ; (d) the absolute value of  $E_y/E_x$ ,  $|E_y/E_x|$ ; (e) the wavenumber  $\lambda_p k$ , where  $\lambda_p$  denotes the proton inertial length; and (f) the wave normal angle  $\theta$ . The OA/IC, OFM/W, OA/IB, PFM/W, and PA/IC instabilities are labeled by I, II, III, IV, and V, respectively. The dotted curves represent the boundaries between two types of instabilities, and two solid curves represent  $V_{pb} = V_A$  and  $2V_A$ , respectively, where  $V_A$  denotes the Alfvén speed.

Reference [?] presented the radial distributions of energy transfer rates for the five proton beam instabilities mentioned above, as shown in Figure 6 [Figure 6: see original paper], and this paper focuses on introducing these research results. It should be noted that reference [?] also analyzed the energy transfer rate distributions for different  $n$  values for solar wind at a specific location (10 solar

radii), clarifying the excitation mechanisms of various proton beam instabilities. Detailed analysis and conclusions can be found in reference [?].

Figure 6 shows the radial distributions of parallel, perpendicular, and total energy transfer rates in proton beam instabilities in the inner heliosphere. The energy conversion processes in various instabilities are summarized as follows:

- (1) In the oblique Alfvén/ion cyclotron instability, proton beams provide the free energy source. This energy mainly flows out of the beam proton component in the parallel direction. In addition to oblique Alfvén/ion cyclotron waves gaining partial energy, some energy flows into the electron component in the parallel direction and into core protons and alpha particles in the perpendicular direction.
- (2) In the oblique fast magnetosonic/whistler instability, the free energy carried by proton beams flows out in both parallel and perpendicular directions, flowing into oblique fast magnetosonic/whistler waves in the perpendicular direction and into electron components in both parallel and perpendicular directions.
- (3) In the oblique Alfvén/ion beam instability, the free energy carried by proton beams flows out in both parallel and perpendicular directions, with partial energy flowing into oblique Alfvén/ion cyclotron waves in the parallel direction, and the remaining energy mainly flowing into core proton components in the perpendicular direction.
- (4) In the parallel fast magnetosonic/whistler instability, the free energy carried by proton beams flows out in the perpendicular direction, flowing into parallel fast magnetosonic/whistler waves and all particle components in the perpendicular direction.
- (5) In the parallel Alfvén/ion cyclotron instability, electron components provide the free energy source, which mainly flows into parallel Alfvén/ion cyclotron waves and core proton components.

The effect of particle components gaining energy in parallel and perpendicular directions can be simply corresponded to increases in parallel and perpendicular temperatures.

Finally, it should be noted that proton beam components in the inner heliosphere are generally distributed in the region  $V_{pb} \sim 0.5 - 2V_A$  [?]. Therefore, it is difficult to trigger the parallel Alfvén/ion cyclotron instability with a high excitation threshold of  $V_{pb} \gtrsim 4.5V_A$  in the inner heliosphere environment [?], where  $V_{pb}$  and  $V_A$  represent the drift speed of the proton beam component and the Alfvén speed, respectively.

## 5. Application 3: Wave-particle Interactions in Electron Heat Flux Instabilities

Using a methodology similar to reference [?], Sun et al. [?] studied wave-particle interactions in electron heat flux instabilities in the inner heliosphere. Unlike the plasma model in reference [?], reference [?] used a three-component plasma including core electrons, beam electrons, and protons. The existence of core and beam electron components is the cause of electron heat flux formation.

This paper focuses on introducing the results of electron heat flux instabilities and their energy transfer rate radial distributions from reference [?], as shown in Figure 7 [Figure 7: see original paper]. Based on the wave number, wave propagation angle, and electric field distribution characteristics in Figure 7, reference [?] found that there are mainly four types of electron beam instabilities in the inner heliosphere: (I) Electron Acoustic Heat Flux Instability (EA-HFI); (II) Lower Hybrid Heat Flux Instability (LH-HFI); (III) Oblique Alfvén Heat Flux Instability (OA-HFI); and (IV) Parallel Whistler Heat Flux Instability (PW-HFI).

These four instabilities dominate in different electron beam and radial regions. Figure 7 measures the wave-particle interactions in electron heat flux instabilities. By analyzing the parallel, perpendicular, and total energy transfer rate distributions, the energy conversion processes in various instabilities are summarized as follows:

- (1) In the electron acoustic heat flux instability, the beam electron component provides the free energy, which mainly flows out of the beam electron component in the parallel direction and flows into electron acoustic waves and core electron components in the parallel direction.
- (2) In the lower hybrid heat flux instability, the free energy carried by beam electrons mainly flows out in the perpendicular direction, and also flows into lower hybrid waves and core electron components in the perpendicular direction.
- (3) In the oblique Alfvén heat flux instability, the core electron component carries the free energy, which mainly flows out of the core electron component in the parallel direction, flows into oblique Alfvén waves mainly in the parallel direction, and flows into proton components in the perpendicular direction.
- (4) In the parallel whistler heat flux instability, the beam electron component provides the free energy, which flows out of the beam electron component in the perpendicular direction and also flows into whistler waves and electron components in the perpendicular direction.

Figure 7 shows the radial distributions of basic parameters and energy transfer rates of the electron heat flux instability in the inner heliosphere. In this figure,  $r/R_S$  denotes the normalized radial distance ( $R_S$  is the solar radius), and  $V_{eb}$

denotes the drift speed of beam electrons. (a) The maximum growth rate  $\gamma$ ; (b) the absolute value of the wave frequency  $|\omega_r|$ ; (c) the wavenumber  $\lambda_e k$ , where  $\lambda_e$  denotes the electron inertial length; (d) the wave normal angle  $\theta$ ; (e) the argument of  $E_y/E_x$ ,  $\arg(E_y/E_x)$ ; (f) the absolute value of  $E_y/E_x$ ,  $|E_y/E_x|$ . (g)–(j) present the energy transfer rate of the waves, beam electrons, core electrons, and protons, respectively, where the panels from top to bottom denote the parallel  $P_{\parallel} = \max(|P_s|)$ , perpendicular  $P_{\perp} = \max(|P_s|)$ , and total  $P_t = \max(|P_s|)$  energy transfer rate, respectively.  $\max(|P_s|)$  is the energy transfer rate of the particles with major free energy. The EA-HFI, LH-HFI, OA-HFI, and PW-HFI are labeled as I, II, III, and IV, respectively, where the boundaries between two types of instabilities are denoted by the dotted curves. The data with  $\gamma < 0.1 \text{ rad} \cdot \text{s}^{-1}$  are removed. From reference [?].

## 6. Summary

This paper has introduced the latest progress in theoretical research on quantifying wave-particle interactions. The newly developed theory proposes to directly measure wave-particle interactions using velocity distribution function perturbations. Since this velocity perturbation distribution function includes an interaction factor term, the new theory can simultaneously measure both resonant and non-resonant wave-particle interactions in velocity space. The new theory also proposes energy transfer rate expressions to measure different types of wave-particle interactions.

This paper has also introduced the results of quantifying wave-particle interactions in Alfvén-mode waves, proton beam instabilities, and electron heat flux instabilities in the inner heliosphere using the energy transfer rate method. The important results include: (1) proposing that the Alfvén-mode wave dispersion surface can be divided into four regions, each controlled by different wave-particle interactions; (2) revealing the occurrence signatures of resonant and non-resonant wave-particle interactions for ion cyclotron waves and sub-ion-scale Alfvén waves; (3) revealing the free energy sources and energy transfer processes in various proton beam instabilities; (4) revealing the free energy sources and energy conversion processes in various electron heat flux instabilities.

These research results demonstrate that the energy transfer rate method can be used to quantify wave-particle interactions in any wave or instability.

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