Constraints on the Average Density of Gravitationally Confined Objects with the Spin Period

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Abstract

We propose to use the rotation period to constrain the average density of an object with gravitationally confined surface. The average density is inverse proportional to the square of the rotation period, while independent of the size of the object. The lower limit of the average density can be written as $\rho_0 = 10.9\ \mathrm{g\ cm^{-3}} \left(\frac{\mathrm{hours}}{P}\right)^2$. An asteroid with rotating period shorter than 0.7 h should consist of some unknown matter, or it is a whole rock or a bulk of ice with no rubble piles on the surface.

Full Text

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Abstract

We propose using the rotation period to constrain the average density of objects with gravitationally confined surfaces. The average density is inversely proportional to the square of the rotation period, while being independent of the object's size. The lower limit of the average density can be expressed as $_0$ = 10.9 g cm^{-3} (P/h)$^{-2}$, where P is the rotation period in hours. An asteroid with a rotation period shorter than 0.7 hours must consist of some unknown matter, or alternatively, it must be a whole rock or a bulk of ice with no rubble piles on its surface.

Key words: minor planets, asteroids: general, stars: rotation

1 Introduction

There exists a lower limit on the spin period of a rotating object, determined by the equilibrium between centrifugal force and confining force. In the case of a pulsar composed of neutron matter, the lower limit of its spin period is approximately 1 ms (Shapiro & Teukolsky 1983), resulting from the equilibrium between centrifugal force and gravitational force, since a neutron star is confined by gravitational force. A pulsar with a sub-millisecond spin period would require a different composition that provides additional confinement—for example, a quark star has confinement from the strong nuclear force. To find compact stars containing quark matter, one possible approach is to search for sub-millisecond pulsars (Zheng et al. 2006).

Among main sequence stars, massive stars systematically rotate faster than low-mass stars. Several massive stars with critical rotation velocities have been found in the Milky Way (e.g., MWC 297, with a spin period of approximately 7 hours; Acke et al. 2008). For comparison, the rotation period of the Sun is approximately 25 days (Kuiper 1953).

The average density of an object (e.g., an asteroid) is an important parameter for inferring its composition. This parameter is crucial for both astrophysics and space mining. The average density can be used to constrain the nature of faint companions in pulsar binaries (e.g., in M71E; Pan et al. 2023). Asteroids rich in precious metals would have relatively higher density. Typically, density is calculated from mass and volume, but in practice, it is difficult to measure these parameters directly, making density measurements challenging. Here we demonstrate that it is possible to constrain the average density using the spin period. In this work, we describe the method for constraining average density in Section 2, present the data and results in Section 3, provide further discussion in Section 4, and give our conclusions in Section 5.

2 Methods

For simplicity, consider a spherical object with radius r. At the edge of the equatorial plane, the acceleration due to centrifugal force is

$$a_c = \omega^2 r = 0.30 \left(\frac{P}{\mathrm{hours}}\right)^{-2} \left(\frac{r}{\mathrm{km}}\right)\ \mathrm{cm\ s^{-2}},$$

where ω is the angular velocity, while the acceleration due to gravity is

$$a_g = \frac{GM}{r^2} = \frac{G\rho_0 \frac{4}{3}\pi r^3}{r^2} = \frac{4\pi G\rho_0 r}{3} = 0.028 \left(\frac{\rho_0}{\text{g cm}^{-3}}\right)\left(\frac{r}{\text{km}}\right) \text{ cm s}^{-2},$$

where G is the gravitational constant and $_0$ is the average density.

The ratio of centrifugal acceleration to gravitational acceleration, f_{cg}, can be written as

$$f_{cg} \equiv \frac{a_c}{a_g} = 10.9 \left(\frac{\rho_0}{\text{g cm}^{-3}}\right)^{-1} \left(\frac{P}{\text{hours}}\right)^{-2}.$$

If an object is held together by gravitational force, the gravitational acceleration must be larger than the centrifugal acceleration (f_{cg} < 1). When f_{cg} = 1, $_0$ serves as a lower limit of the average density:

$$\rho_0 = 10.9 \text{ g cm}^{-3} \left(\frac{P}{\text{hours}}\right)^{-2}.$$

On the other hand, the limiting period corresponding to $_0$ is

$$P = 3.3 \text{ hours} \left(\frac{\rho_0}{\text{g cm}^{-3}}\right)^{-1/2}.$$

It can be seen from Equation 2 that the average density of a gravitationally bound object is simply constrained by its spin period, independent of its radius. To find candidates with high-density matter, one possible approach is to search for fast-rotating objects.

3 Data and Results

3.1 Spin Period Distribution

We collected basic parameters for a sample of asteroids from the 3D Asteroid Catalogue[1]. Most asteroids in this sample have measured mean diameter, spin period, and albedo.

The distribution of spin periods is shown in Fig. 1 [Figure 1: see original paper]. The distribution of asteroids with spin periods longer than 6 hours follows a power law, while there are significantly fewer asteroids with spin periods shorter than 6 hours.

We calculated the lower limit of the average density for the asteroids by assuming they are gravitationally confined, as shown in Fig. 2 [Figure 2: see original paper]. The constraint on the average density is not strong for the current sample.

[1] https://3d-asteroids.space/

3.2 Asteroids with Density Measurements

There are 26 asteroids with additional parameters such as mass, density, and escape velocity (Table 1). We can compare these measurements with density estimates derived from spin periods. The estimated lower limits of density are consistent with the measurements for most asteroids. For asteroid (29075) 1950 DA, the estimated lower limit of density is larger than the value given by the catalogue. However, detailed modeling of asteroid (29075) 1950 DA suggests that its average density should be 2.5 g cm^{-3} or 3.5 g cm^{-3}, which is consistent with our estimate.

Table 1: Physical Parameters of Asteroids with Density Measurements

Asteroid	Spin Period (hours)	Albedo	Mass (kg)	Diameter (km)	Density (g/cm^3)	Escape Velocity (km/s)	Estimated Density Lower Limit (g/cm^3)
(29075) 1950 DA	0.5078	0.25	2×10^{12}	1.46	1.22	0.0008	42.3
(101155) 1992 SK	7.186	0.15	2.17×10^{12}	5.5	2.5	0.0025	0.21
(8567) 1996 HW1	8.67	0.16	2.27×10^{13}	4.57	4.8	0.0048	0.15
(52760) 1998 ML14	15.0	0.18	1.09×10^{12}	1.2	1.2	0.0012	0.05
(33342) 1998 WT24	2.697	0.55	6.251×10^{11}	0.35	2.8	0.0028	1.5
(341843) 2008 EV5	3.725	0.12	3.431×10^{10}	0.4	1.02	0.0010	0.79
(101955) Bennu	4.296	0.046	7.793×10^{10}	0.49	1.19	0.0012	0.59
(107) Camilla	4.8439	0.058	1.12×10^{19}	254	1.3	0.0013	0.46
(45) Eugenia	5.699	0.05	5.691×10^{18}	202	1.32	0.0013	0.34
(433) Eros	5.27	0.25	6.689×10^{15}	16.84	2.67	0.0027	0.39

Asteroid	Spin Period (hours)	Albedo	Mass (kg)	Diameter (km)	Density (g/cm^3)	Escape Velocity (km/s)	Estimated Density Lower Limit (g/cm^3)
(15) Eunomia	6.083	0.21	3.181×10^{19}	268	1.58	0.0016	0.29
(283) Emma	6.910	0.15	1.38×10^{18}	148	0.81	0.0008	0.23
(10) Hygiea	13.8257	0.072	1.049×10^{20}	434	2.06	0.0021	0.06
(121) Hermione	8.72	0.05	4.71×10^{18}	190	1.3	0.0013	0.14
(243) Ida	4.6336	0.24	4.121×10^{16}	31.3	2.6	0.0026	0.51
(704) Interamnia	8.726	0.074	7.493×10^{19}	316	3.6	0.0036	0.14
(25143) Itokawa	0.5054	0.53	3.147×10^{10}	0.33	1.9	0.0019	42.7
(22) Kalliope	4.148	0.14	7.358×10^{18}	166	2.8	0.0028	0.63
(216) Kleopatra	5.385	0.12	4.641×10^{18}	138	3.4	0.0034	0.38
(253) Mathilde	17.406	0.04	1.033×10^{17}	52.8	1.33	0.0013	0.04
(2) Pallas	7.8132	0.16	2.143×10^{20}	512	2.9	0.0029	0.18
(11) Parthenope	7.635	0.16	6.151×10^{18}	150	3.5	0.0035	0.19
(16) Psyche	4.196	0.15	2.293×10^{19}	226	1.9	0.0019	0.62
(87) Sylvia	5.184	0.04	1.48×10^{19}	261	1.6	0.0016	0.41
(17) Thetis	12.2704	0.09	1.43×10^{18}	90	1.2	0.0012	0.07
(4) Vesta	5.342	0.42	2.591×10^{20}	525	3.45	0.0035	0.38

4 Discussion

In our estimates above, we have assumed the asteroids are strengthless and totally confined by gravitational force. This is reasonable since space probes have imaged some asteroids, revealing surfaces covered by rubble piles.

If asteroids have confinement from other forces, their rotation periods can be much shorter for the same average density. Some asteroids may consist of whole rock—in other words, they may be monolithic. For these objects, we can calculate the corresponding limiting spin period.

The typical tensile strength of rock, P_t, is several million to several tens of million Pascal. For a cylinder of height 2h and bottom area A, the centrifugal force would equal the breakup force at the breakup limit^2, giving a corresponding centrifugal acceleration of a_c = P_t/(h), where is the density of the rock. The period at the breakup limit is then

$$P = 62.8\ \mathrm{s} \left(\frac{P_t}{10\ \mathrm{MPa}}\right)^{1/2} \left(\frac{\rho}{1\ \mathrm{g\ cm^{-3}}}\right)^{-1/2} \left(\frac{h}{1\ \mathrm{km}}\right)^{1/2}.$$

The tensile strength of ice is 0.27 MPa. If an asteroid consists of a bulk of ice, its breakup spin period is then P 382 s.

Searching for extremely fast-rotating objects will help identify objects with high density and objects consisting of a single rock, or even exotic matter (e.g., strangelets). This is important both for astrophysics and future asteroid mining, since most metallic minerals have high density and form solid rock. Based on Equation 2, an asteroid with a rotation period of 0.5 hours would have an average density of 40 g cm^{-3} if it is gravitationally confined.

Among common minerals, iridium has the highest density of 20 g cm^{-3} (Table A.1). It is believed that the iridium on Earth originated from asteroids. If we find an asteroid with a rotation period of 0.7 hours, one possibility is that it consists of some unknown matter, since the density required to provide sufficient gravitational confinement exceeds 20 g cm^{-3}. Another possibility is that it is a whole rock or a bulk of ice, so it is confined by lattice forces rather than gravitational force. We would expect no rubble piles on the surface of such an asteroid.

Among asteroids, M-type asteroids contain more metal (mainly iron and nickel; Tholen 1984) and have relatively higher average density. Observations also show that M-type asteroids rotate systematically faster than asteroids of other types (Belskaya & Lagerkvist 1996).

Among near-Earth asteroids, there exist fast-rotating objects. Asteroid 2001 OE84 has a period of 0.486 hours (Pravec et al. 2002). Only objects smaller than about 200 m rotate more rapidly (Warner et al. 2021). These asteroids must have an average density higher than 46 g cm^{-3} if they are bound by gravitational force. This density is too high for ordinary rock, suggesting they are more likely to consist of a whole rock or a bulk of ice.

5 Conclusion

After considering the constraint on the density of a self-gravitating object from its rotation period, we reach the following conclusions:

1. The rotation period provides a lower limit to the average density of an object if its surface is gravitationally confined. This constraint does not depend on the radius of the object.
2. An object with a spin period shorter than 0.7 hours could consist of high-density matter, or alternatively, it could be a single rock or a bulk of ice.
3. A fast-rotating object with rubble piles on its surface should have a high average density.

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[2] If we assume the cylinder breaks from the middle.

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Appendix A: The Density of Meteorites and Minerals with High Densities

For space mining on asteroids, we are mainly interested in precious metals such as iridium and platinum. Minerals containing these elements usually have relatively higher densities. We list some minerals in Table A.1.

Table A.1: The Density of Some Minerals

Minerals	Density (g cm^{-3})
Silver	10.5
Sperrylite	10.0
Palladium	12.0
Iridosmine	21.0
Platinum	21.5
Iridium	22.6

Data from The Engineering ToolBox (2009). Minerals - Densities. [online] Available at: https://www.engineeringtoolbox.com/mineral-density-d_{1555}.html [Accessed 9th July 2023].

Note: Figure translations are in progress. See original paper for figures.

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