

The Zhang lattice: a simple lattice naturally has type-II Dirac points

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Abstract

I review the discovery as well as the band structure of the Zhang lattice.

Full Text

The Zhang Lattice: A Simple Lattice with Natural Type-II Dirac Points

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I review the discovery and band structure of the Zhang lattice.

Dirac materials [1] are special because their band structures contain Dirac points, meaning that the excited quasi-particles are fermionic and obey the massless Dirac equation [2]. The most frequently studied Dirac materials are graphene (or the honeycomb lattice), the Lieb lattice, and the kagome lattice [3]. However, the Dirac points supported by these lattices are type-I [4]. Considering half-filling, the Fermi surface is a point for a type-I Dirac point, a line for a type-III Dirac point, and a pair of crossing lines for a type-II Dirac point. Compared to materials supporting type-I Dirac points, materials that support type-II and type-III Dirac points are much rarer, especially those supporting type-II Dirac points. To obtain type-II Dirac points, various methods have been developed [5–10].

I would like to point out that all these efforts are indirect. One cannot help but wonder: is there a material or artificial material that naturally possesses type-II Dirac points, just as graphene naturally possesses type-I Dirac points?

To address this question, I began pondering it after noticing Ref. [6] and finally succeeded in 2020 [11] after many attempts and efforts [12–14].

The designed lattice, i.e., the Zhang lattice, is shown in Fig. 1(a), which has three sites in each unit cell. In each unit cell (indicated by a hexagon) there are three sites labeled as A, B, and C. The basis vectors of the Bravais lattice are $\mathbf{v}_1 = [-a/2, \sqrt{3}a/2]$ and $\mathbf{v}_2 = [+a/2, \sqrt{3}a/2]$, with a being the lattice constant. The corresponding far-field diffraction pattern in Fig. 1(b) shows the first Brillouin zone (the innermost hexagon) clearly.

I believe the complexity of the Zhang lattice is at the same level as that of the Lieb lattice and the kagome lattice, and therefore the Zhang lattice is a simple lattice.

[Figure 1: see original paper]

The band structure of the Zhang lattice was calculated using both the discrete model (i.e., the tight-binding method) and the continuous model (i.e., the Schrödinger-like paraxial wave equation), and both results demonstrate the existence of type-II Dirac points. According to the tight-binding method with only nearest-neighbor hopping considered, the Hamiltonian can be written as

$$H = t \begin{pmatrix} 0 & 2 \cos[\mathbf{k} \cdot (\mathbf{v}_2 - \mathbf{v}_1)] & \exp(+i\mathbf{k} \cdot \mathbf{v}_1) + \exp(+i\mathbf{k} \cdot \mathbf{v}_2) \\ 2 \cos[\mathbf{k} \cdot (\mathbf{v}_2 - \mathbf{v}_1)] & 0 & \exp(-i\mathbf{k} \cdot \mathbf{v}_1) + \exp(-i\mathbf{k} \cdot \mathbf{v}_2) \\ \exp(-i\mathbf{k} \cdot \mathbf{v}_1) + \exp(-i\mathbf{k} \cdot \mathbf{v}_2) & 2 \cos[\mathbf{k} \cdot (\mathbf{v}_2 - \mathbf{v}_1)] & 0 \end{pmatrix}$$

with $\mathbf{k} = [k_x, k_y]$ being the Bloch momentum and t the hopping strength. The band structure, i.e., the eigenvalues of the Hamiltonian versus k_x and k_y , is shown in Fig. 2(a). There are three bands in the band structure, and the intersections between each pair of bands are type-II Dirac points. The appearance of type-II Dirac points is natural and no additional operation is required for the Zhang lattice.

[Figure 2: see original paper]

The continuous model includes all hoppings as well as the concrete profile of the sites, and therefore is more accurate than the discrete model. Assuming the Zhang lattice is inscribed in a transparent optical medium (e.g., fused silica) by femtosecond laser direct writing, the dimensionless Schrödinger-like paraxial wave equation should be written as

$$i \frac{\partial \psi}{\partial z} + \frac{1}{2k_0} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi = -R(x, y)\psi,$$

where R represents the Zhang lattice that can be described by Gaussian functions

$$R(x, y) = p \sum_{m,n} \exp \left[-\frac{(x - x_{m,n})^2}{2\sigma_x^2} - \frac{(y - y_{m,n})^2}{2\sigma_y^2} \right],$$

with p being the lattice depth, (σ_x, σ_y) accounting for the beam width, and $(x_{m,n}, y_{m,n})$ being the coordinates of the lattice grids. Considering a set of realistic experimental parameters [15, 16], e.g., $\sigma_x = 0.25$ (2.5 μm), $\sigma_y = 0.75$ (7.5 μm), $a = 3$ (30 μm), $\lambda = 600$ nm, and $p = 5$ (refractive index change $\sim 5.5 \times 10^{-4}$), the corresponding band structure of the Zhang lattice is shown in Fig. 2(b), obtained by introducing the ansatz $\psi(x, y, z) = u(x, y) \exp(ibz)$ into Eq. (3) with b being the propagation constant and $u(x, y)$ the Bloch state. The result based on the continuous model is quite similar to that based on the discrete model, and the type-II Dirac points are definitely supported by the Zhang lattice.

Note that the Zhang lattice has been induced in photorefractive SBN crystals [17] and also in atomic vapors [18] with the aid of a spatial modulator. In the Zhang lattice, conical diffraction [11], Klein tunneling [11], and scalar as well as vector valley Hall edge solitons have been reported [17, 19, 20]. In the future, I believe more and more interesting physical principles and phenomena will be reported and verified based on the Zhang lattice.

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