Quantum Enhancement Effects of Macroscopic Quantum Entangled States

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Abstract

Atomic systems possessing reflection symmetry and parity can generate macroscopic photonic entangled states via stimulated emission from atoms and amplification in a parallel-plane resonator [4].

It is a quantum entangled state of 2N photons with definite parity, zero total momentum, definite energy, and definite angular momentum. Observations of its temporal and spatial positions exhibit uncertainty and randomness. According to Heisenberg's uncertainty principle, its energy (frequency) and momentum are completely determined. The measurement precision can reach the Heisenberg quantum limit, exhibiting a quantum 2N enhancement effect (where 2N is the number of entangled photons).

Through experimental measurement of its probability distribution P2N(t) and its Fourier transform P2N(w-w0), the experimental results agree with theoretical predictions. The lifetime of the macroscopic photonic entangled state was also observed to exhibit a 2N enhancement factor, and the experimental results agree with theoretical predictions.

Full Text

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Abstract

Atomic systems possess reflection symmetry and parity. Atomic stimulated radiation, amplified by a parallel-plane resonant cavity, can generate macroscopic photon entangled states:

$$>= 1\{|\ 2N_{RK}, 0 > + \mid 0, 2N_{L-K} >\} \quad (a)$$

$$>= 1\{|\ 2N_{LK}, 0 > + \mid 0, 2N_{R-K} >\} \quad (b)$$

States (a) and (b) are quantum entangled states of 2N photons with definite parity, zero total momentum, definite energy, and definite angular momentum. Observations of (a) and (b) exhibit uncertainty and randomness in time and spatial position. According to the Heisenberg uncertainty principle, their energy (frequency) and momentum are completely determined. The measurement precision can reach the Heisenberg quantum limit, exhibiting a quantum 2N enhancement effect (where 2N is the number of entangled photons).

Through experiments, we measured the probability distribution $P_{2N}(t)$ of $|\ >$ and its Fourier transform $P_{2N}(\omega - \omega_0)$. The experimental results are consistent with theoretical predictions. We also observed the lifetime of the macroscopic photon entangled state, which similarly exhibits a 2N enhancement factor, with experimental results matching theoretical expectations.

Keywords: multiphoton entanglement; quantum enhancement; Heisenberg quantum limit

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H. F. proposed a method for generating arbitrary N-photon maximally entangled states (polarization entangled states) [1] with the following expression:

$$(N00N)\{|\ N_R, 0 > + \mid 0, N_L >\} \quad (1.1)$$

R—right circular polarization; L—left circular polarization; (1.1) is an N-photon co-mode polarization entangled state.

Z. Y. Ou proposed a method for preparing arbitrary N-photon translational entangled states [2], which has the following form:

$$(N00N)K\{|\ N_K, 0 > + \mid 0, N(-K) >\} \quad (1.2)$$

Z. Y. Ou pointed out that if the photon entangled state represented by (1.2) is passed through a beam splitter for interference, as shown in the figure below, with controllable phase modulation at one input port and observation of N-photon coincidence counts at the output port, the probability of the (N00N) composite particle overall state appearing, i.e., the N-photon coincidence counting rate, is:

$$P_N = 1(1 + cosN\phi_c) \quad (1.3)$$

[Figure 1: see original paper] N-photon interference of photon-number maximally entangled states

From (1.3), it can be seen that the phase measurement precision of N-photon interference can reach the Heisenberg limit:

$$\phi_c = \quad (1.4)$$

The lowest limit of the Heisenberg uncertainty relation:

$$\Delta\phi \cdot \Delta N = 1 \quad (1.5)$$

The effective wavelength of the N-photon entangled state overall state:

$$\lambda_D = \lambda_0, \text{ where } \lambda_0 \text{ is the single-photon wavelength.}$$

Although (N00N) N-photon entangled states can be prepared theoretically, experimental verification has been limited to only a few photons. M. W. Mitchell et al. used a three-photon (3003) entangled state to perform super-high-resolution phase measurements experimentally, achieving the Heisenberg limit [3]. Experiments demonstrate that the quantum enhancement effect of entangled light is proportional to the number of entangled photons.

For N-photon entangled state interference experiments, as N increases to the macroscopic quantum entangled state regime, N-photon coincidence counting becomes infeasible.

Controllable phase modulation is introduced in N-photon entangled interference experiments, as shown in Figure 1. According to the lowest limit of the uncertainty principle (1.5), ΔN is completely uncertain, and the phase measurement precision can reach the Heisenberg limit $\Delta\phi = 1$. Since $\Delta\phi = K_0 \cdot \Delta S$, the optical path difference is also completely determined. According to the uncertainty relation, the overall state wavenumber is completely uncertain. With ΔS completely determined, the time delay Δt is completely determined, and according to the uncertainty principle, energy and frequency are completely uncertain.

II. Macroscopic Multi-Photon Entangled States

The N-photon entangled state we employ was proposed in reference [4] and has the following expressions:

$$(N00N) >= 1\{\mid 2N_{RK}, 0 > + \mid 0, 2N_{L-K} >\} \quad (2.1)$$

$$>= 1\{\mid 2N_{LK}, 0 > + \mid 0, 2N_{R-K} >\} \quad (2.2)$$

The number of entangled photons 2N is even.

$$>= 1\{\mid N_{RK}, N_{R-K} > + \mid N_{LK}, N_{L-K} >\} \quad (2.3)$$

| > has even parity. The total spin angular momentum is 2Nħ for one state and -2Nħ for another. | > has even parity and zero total spin angular momentum.

R_K—right circular polarization, spin of 1, wave vector K along +Z direction; L_K—left circular polarization, spin of -1, wave vector K along +Z direction; $\mid R_K >=\mid L_{-K} >$, $\mid L_{-K} >=\mid R_K >$, $\mid L_K >=\mid R_{-K} >$, $\mid R_{-K} >=\mid L_K >$, where is the parity operator with two eigenstates and eigenvalues $±$1. States with parity must be quantum superposition states of two reflection-symmetric states.

$\mid \quad >$ and $\mid \quad >$ are generated by a single-mode He-Ne laser with symmetric structure outputting along opposite directions of the optical axis. For atomic systems, quantum mechanics has proven that the energy operator commutes with the parity operator, and the angular momentum operator commutes with the parity operator. States with parity have definite energy and angular momentum. Energy, angular momentum, and parity can completely describe the state of photons.

Energy, angular momentum, and parity are conserved quantities. Ne atom $3S_2 \rightarrow 2P_4$ spontaneous radiation: $3S_2$ has odd parity, $2P_4$ has even parity, and parity is conserved in spontaneous radiation. The single-photon angular momentum of spontaneous radiation is -1, which determines that the single-photon state has the following expression:

$$\mid \Psi_1 >= 1\{\mid L_K > - \mid R_{-K} >\} \quad (2.4)$$

$\mid \Psi_1 >$ undergoes stimulated radiation feedback through the laser resonator cavity. Atomic stimulated radiation conserves parity and angular momentum. The two-photon state from stimulated radiation has odd parity and definite spin angular momentum, uniquely determining that the two-photon state has the following form:

$$>= 1\{\mid 2L_K, 0 > + \mid 0, 2R_{-K} >\} \quad (2.5)$$

After one reflection by the resonator cavity mirror, (2.5) transforms into (2.6):

$$>= 1\{\mid 2R_K, 0 > + \mid 0, 2L_{-K} >\} \quad (2.6)$$

Stimulated radiation involves photon-atom interaction, leading to interaction between the two photons. It cannot be decomposed into a direct product state of two independent photons, so (2.5) and (2.6) are two-photon entangled states. The two-photon entangled state (2.6) induces stimulated absorption in two Ne atoms, simultaneously entangling the two Ne atoms. The two-photon entangled state (2.5) propagates back and forth once in the resonator cavity, inducing stimulated radiation from two entangled Ne atoms and generating a four-photon entangled state $1\{\mid 4L_K, 0 > + \mid 0, 4R_{-K} >\}$. Continuing this process, entangled photons and entangled atoms interact in the resonator cavity, continuously amplifying until stable macroscopic entangled photons and macroscopic entangled Ne atoms are produced. Macroscopic entangled light is coupled out through both ends of the resonator cavity mirrors, and the entangled photons have interactions.

Entangled atoms are difficult to observe directly in experiments, but the spectrum of entangled light from entangled atom stimulated radiation can be directly observed experimentally. The spectrum of entangled light carries information about the entangled Ne atoms, allowing the quantum properties of the entangled atomic electronic states to be retrieved from the entangled spectrum.

III. Properties of 2N Photon Entangled States

(1) Due to the interaction between entangled photons and entangled atoms in the resonator cavity, entangled photons have interactions.

(2) $|\ >$ has reflection symmetry and parity; energy, angular momentum, and parity are conserved.

(3) $|\ >$ has rotational symmetry, angular momentum conservation, and phase invariance (rotation only causes a phase shift of the overall state without changing its quantum properties).

(4) The quantum superposition entangled state $|\ >$ has a 1/2 probability of randomly jumping to $|\ 2N_{RK}, 0>$ or $|\ 0, 2N_{L-K}>$, continuously switching between the two configurations. The time and position when simultaneously in the quantum superposition state are completely uncertain. This is precisely the manifestation of wave-particle duality and can be observed experimentally.

$|\ >$ has parity, zero total momentum, momentum correlation, and completely determined momentum. According to the Heisenberg uncertainty principle, position is completely uncertain, but the position difference is completely determined, positions are correlated, and energy and frequency are completely determined. Time is completely uncertain, but the time difference is completely determined, and time is correlated.

If particle properties are observed, energy, angular momentum, and parity are conserved quantities. If wave properties are observed by sending the $|\ >$ overall state into an interferometer, the phase of the interference pattern is completely determined, and time and position are also completely determined. However, energy, angular momentum, and photon number become completely uncertain.

Before measurement, it is impossible to predict which state $|\ >$ is in. $|\ >$ is the spin wavefunction of entangled photons. Since the orbital angular momentum of the photon entangled state is zero, the general angular momentum wavefunction simplifies to a spin wavefunction. Furthermore, because the photon entangled state has parity, the spin wavefunction further simplifies to a simple form of $|\ >$.

IV. Experimental Observation of Macroscopic Photon Entangled State Overall State Probability

The macroscopic photon entangled state:

$$>= 1\{|\ 2N_{RK}, 0 > + \mid 0, 2N_{L-K} >\}$$

with $2N = 4.4474 \times 10^{11}$.

The experimental setup for observing the probability of the macroscopic quantum entangled state overall state is shown below:

[Figure 2: see original paper] Experimental setup for observing macroscopic entangled state overall state probability

The two detectors are arranged to satisfy reflection symmetry conditions. That is, detector DI is placed along the laser axis (at Z0 in the positive Z direction), and detector DII is placed along the negative Z direction at -Z0. The momentum direction of entangled photons is parallel to the Z axis.

The experimental observation time was 4500 seconds. The probability pattern of the $|\ >$ overall state is shown in Figure 3 [Figure 3: see original paper].

[Figure 3: see original paper] (a) Probability pattern of the $|\ >$ overall state [Figure 3: see original paper] (b) Signal detection from DI (Right Camera) and DII (left camera) in Figure 2. The coincidence measurement of the two signals (multiplied and normalized) yields Figure 3(a).

The probability of the $|\ >$ overall state:

$$P_{2N} =|\ \ |^2 \quad (4.1)$$

The probability pattern shows: (1) When the experimental observation time exceeds 2410.9 seconds, the overall state probability pattern disappears. We observe the probability of both configurations $|\ 2N_{RK}, 0 >$ and $|\ 0, 2N_{L-K} >$ simultaneously being in quantum superposition—the probability of the overall state appearing. This can be used to determine that the macroscopic entangled photon state has decohered into classical light. Therefore, this time can be determined as the lifetime of the $|\ >$ entangled light overall state:

$$\tau_{lifetime} = 2.4109 \times 10^3 \text{ seconds} \quad (4.2)$$

The lifetime of entangled light originates from the lifetime of entangled atoms, meaning the experiment simultaneously determines the lifetime of entangled atoms. This is also the probability of entangled Ne atoms undergoing stimulated radiation.

According to the lowest limit of the quantum mechanical uncertainty relation:

$$\tau_{lifetime} \cdot \Delta\omega = 1 \quad (4.3)$$

$$\Delta\omega = \frac{1}{2.4109 \times 10^3} = 4.1478 \times 10^{-4} \text{ Hz} \quad (4.4)$$

where $\Delta\omega$ is the frequency width of the macroscopic photon entangled state overall state. (High-end lasers have frequency widths of 10^3 to 10^4 Hz).

Performing a time-domain Fourier transform on the probability pattern in Figure 3 yields the frequency-domain probability $P(\omega - \omega_0)$, as shown in Figure 4 [Figure 4: see original paper]. This is the spectrum of entangled light from entangled atom collective stimulated radiation. The frequency width determined from this spectral diagram is basically consistent with that determined from the uncertainty relation.

[Figure 4: see original paper] (a) Spectrum of entangled light from entangled atom collective stimulated radiation [Figure 4: see original paper] (b) Local magnification of Figure 4(a)

The spatial lifetime of macroscopic entangled light:

$$L_{lifetime} = C \cdot \tau_{lifetime} = 7.2276 \times 10^{13} \text{ cm} \quad (C = 2.9979 \times 10^{10} \text{ cm} \cdot \text{s}^{-1})$$

When the spatial distance between $\mid 2N_{RK}, 0 >$ and $\mid 0, 2N_{L-K} >$ exceeds $L_{lifetime}$, they decohere into classical states.

According to the lowest limit of the quantum mechanical uncertainty relation:

$$\Delta K \cdot L_{lifetime} = 1, \text{ then } \Delta K = 1/L_{lifetime} = 1.3836 \times 10^{-14} \text{ cm}^{-1}$$

where ΔK is the wavenumber width of the macroscopic photon entangled state overall state.

The macroscopic photon entangled state $\mid \ >$ has zero total momentum, which is completely determined. According to the uncertainty relation, position is completely uncertain. $\mid \ >$ has reflection symmetry; although position is completely uncertain, the position difference is determined, positions are correlated, and energy and frequency are completely determined. Time is completely uncertain, but the time difference is completely determined, and time is correlated.

V. Experimental Results and Discussion

(1) Macroscopic entangled photon number: $2N = 4.4474 \times 10^{11}$

(2) Macroscopic photon entangled state average lifetime: $\tau_{lifetime} = 2.4109 \times 10^3$ seconds.

Single-photon superposition state average lifetime: $\tau_{single-photon} = 5.4209 \times 10^{-9}$ seconds. The He-Ne laser natural lifetime (i.e., the lifetime of spontaneous radiation single-photon superposition state) is $\tau_{natural} = 7.9592 \times 10^{-9}$ seconds. The average lifetime of macroscopic entangled light is approximately 2N times the He-Ne laser natural lifetime.

(3) Macroscopic entangled photon frequency width: $\Delta\omega = 1/(2.4109 \times 10^3) = 4.1478 \times 10^{-4}$ Hz, which is approximately 1/(2N) of the natural linewidth.

(4) Macroscopic entangled photon spatial average lifetime and wavenumber width: $L_{lifetime} = 7.2276 \times 10^{13}$ cm, $\Delta K = 1/L_{lifetime} = 1.3836 \times 10^{-14}$ cm^{-1}, which is approximately 1/(2N) of the single-photon wavenumber width.

The ratios of macroscopic entangled light's average lifetime, frequency width, and wavenumber width to the corresponding single-photon values all exhibit a quantum enhancement factor of 2N. The measurement precision of entangled light frequency and wavenumber reaches the Heisenberg limit. These quantum enhancement effects have no classical analog.

(5) The ultra-long lifetime of macroscopic entangled light, which is proportional to the number of entangled photons, provides strong resistance to decoherence and can be used as physical qubits (existing superconducting interferometer flux qubits have lifetimes of about 20 microseconds), offering large operation time and space. It can be used to test interactions and correlations of entangled light over long distances.

The ultra-long spatial lifetime of macroscopic entangled light enables secure and reliable transmission of information over long distances.

(6) Macroscopic entangled light carries information about macroscopic entangled Ne atoms. The lifetime and frequency width of entangled Ne atoms can be obtained. The frequency measurement precision also exhibits a quantum enhancement effect related to the number of entangled atoms.

(7) The interaction between entangled photons and entangled atoms in the resonator cavity causes entangled photons to have interactions. Photons rarely interact, and other schemes for making photons interact are very complex. Photon interactions are important for quantum information processing and quantum computing (the prerequisite for implementing controlled-NOT gates and controlled phase-shift gates in quantum computing is that photons must have interactions).

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Note: Figure translations are in progress. See original paper for figures.