Measurement of the Cavity-Loaded Quality Factor in Superconducting Radio-Frequency Systems with Mismatched Source Impedance

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Abstract

The accurate measurement of parameters such as the cavity-loaded quality factor (QL) and half bandwidth (f0.5) is essential for monitoring the performance of superconducting radio-frequency (SRF) cavities. However, the conventional "field decay method" employed to calibrate these values requires the cavity to satisfy a "zeroinput" condition. This can be challenging when the source impedance is mismatched and produce nonzero forward signals (Vf) that significantly affect the measurement accuracy. To address this limitation, we developed a modified version of the "field decay method" based on the cavity differential equation. The proposed approach enables the precise calibration of f0.5 even under mismatch conditions. We tested the proposed approach on the SRF cavities of the Chinese Accelerator Driven System Front-End Demo Superconducting Linac and compared the results with those obtained from a network analyzer. The two sets of results were consistent, indicating the usefulness of the proposed approach.

Full Text

Preamble

Measurement of the Cavity-Loaded Quality Factor in Superconducting Radio-Frequency Systems with Mismatched Source Impedance

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The accurate measurement of parameters such as the cavity-loaded quality factor (QL) and half bandwidth (f0.5) is essential for monitoring the performance of superconducting radio-frequency (SRF) cavities. However, the conventional "field decay method" employed to calibrate these values requires the cavity to satisfy a "zero-input" condition. This can be challenging when the source impedance is mismatched and produces nonzero forward signals (Vf) that significantly affect measurement accuracy. To address this limitation, we developed a modified version of the "field decay method" based on the cavity differential equation. The proposed approach enables precise calibration of f0.5 even under mismatch conditions. We tested this approach on the SRF cavities of the Chinese Accelerator Driven System Front-End Demo Superconducting Linac and compared the results with those obtained from a network analyzer. The two sets of results were consistent, indicating the usefulness of the proposed approach.

Keywords: Loaded quality factor, Field decay method, Superconducting cavity, Mismatch, Calibration, Cavity differential equation, Measurement, Accelerator driven system

Introduction

Driven by the growing demand for safe nuclear fuel post-treatment processes, the China initiative Accelerator Driven System (CiADS) is being constructed as a clean solution for nuclear fission power sources [1–3]. To showcase the potential of a high-power continuous wave (CW) proton beam for this project, the China ADS Front-End Demo Linac (CAFe) was built. This linac is a 162.5 MHz superconducting (SC) radio-frequency (RF) machine operating in CW mode and consists of both normal conducting (NC) and SC sections (Fig. 1 [Figure 1: see original paper]). The NC section includes an ion source, low-energy beam transport line, RF quadrupole accelerator, and medium-energy beam transport line. Conversely, the SC section comprises SC accelerating units, including 23 SC half-wave resonator cavities assembled into four cryomodules (CM1–CM4) [4–7]. Commissioning tests conducted on CAFe in CW mode with a current of 10 mA and energy of 20 MeV successfully demonstrated its ability to accelerate and transmit high-intensity beams.

For an SC cavity, the loaded quality factor (QL) reflects the consumption of stored electromagnetic energy inside the cavity. In an ideal situation, in the absence of a beam passing through the cavity, QL includes the power dissipation on the cavity wall caused by surface resistance (termed Q0 [8, 9]) and the power leakage from the coupler ports [10]. Thus, QL is a critical parameter that must be carefully selected to match the impedance of the RF generator with the particle beam load during operation [11]. Furthermore, dark current loading can negatively affect QL, making it an important figure of merit for identifying such effects [12, 13].

In addition, QL (or the cavity half-bandwidth (f0.5)) plays a crucial role in the design of model-based controllers [14–16]. To satisfy these application requirements, the measurement error for QL should not exceed 5%. The value of QL can be calibrated using the cavity resonant frequency (f0) and f0.5, where f0.5 = f0/2QL. Therefore, precise measurement of f0.5 is a prerequisite for calibrating QL.

The decay curves of cavity amplitude and phase (after RF power is turned off) obtained from the cavity differential equation contain information on f0.5 and the cavity detuning parameter (Δf), respectively. Many laboratories, including DESY, KEK, and CSNS, employ the "field decay method" to measure these physical quantities [17–21].

We tracked the long-term variation patterns of f0.5 based on data accumulated during CAFe facility operation when RF power was turned off. Occasionally, we found that the cavity half-bandwidth calculated from the amplitude decay curve (i.e., f0.5,decay) and the cavity detuning parameter calculated from the phase decay curve (i.e., Δfdecay) appeared to be correlated. However, in principle, they should be independent. To better understand this issue, we thoroughly examined the "field decay method." Our findings revealed that this method is based on the zero-input response of the cavity differential equation, which indicates that the RF system must satisfy the "zero-input" condition. Thus, the cavity incident power must drop to zero after RF power is turned off. However, if this condition is not met (i.e., due to impedance mismatch), the remaining incident power may influence the decay process and render the "field decay method" ineffective. We constructed an equivalent circuit that included RF power sources, transmission lines, input couplers, and SRF cavities, and derived a solution for the cavity differential equation when source impedance mismatch occurred. Finally, we modified the formula in the "field decay method" to explain the observed correlation.

The "field decay method" is routinely employed to calibrate QL. Our findings revealed that this method is based on the "zero-input" response of the cavity differential equation. If the "zero-input" condition is not satisfied due to impedance mismatch, considerable errors may occur in QL measurement. To improve measurement accuracy, this study focuses on a modified calibration algorithm based on an equivalent circuit.

II. Phenomena and Possible Interpretation

The conventional "field decay method" is briefly reviewed in this section. The naming conventions for the cavity forward voltage signal (Vf) and cavity voltage (Vc) in polar coordinates are illustrated in Fig. 2 [Figure 2: see original paper]. In polar coordinates, Vf and Vc can be expressed as Vf = (1/2) e^{i } and Vc = re^{i }, respectively, where , r, , and represent the amplitudes of 2Vf and

Vc and the phases of Vf and Vc, respectively. When RF power is turned off, immediately decreases to zero in the ideal case.

Under this condition, according to the SC cavity differential equation without beam in polar coordinates [22–24], f0.5 and Δf can be expressed as:

$$\begin{cases} f_{0.5,\mathrm{decay}} = -\frac{r'}{r} \\ \Delta f_{\mathrm{decay}} = \phi' \end{cases} \tag{1}$$

We refer to various methods used in this study to calculate f0.5 and Δf. For clarity, we first provide their detailed definitions in Table 1 .

For cavity CM3-3 (marked by a red triangle in Fig. 1) at CAFe, we measured the cavity-field decay curves for different values of Δfdecay (Fig. 3 [Figure 3: see original paper]). f0 was tuned using a frequency tuner. After RF power was turned off, the slope of the cavity phase varied with the detuning parameter (Fig. 3(c)) because the phase decay curves are directly associated with the detuning parameter according to Eq. (1). Since cavity QL is independent of the detuning parameter, the field decay curves of the cavity are expected to overlap under different detuning conditions; however, they appear to be affected by the detuning parameter (Fig. 3(a) and (b)).

We conducted several studies to address this perplexing phenomenon. First, we calibrated f0.5,decay and Δfdecay with four different values of Vc for cavity CM3-3 (Fig. 4(a)). The value of Vc is less than the onset gradient of field emission (approximately 1.2 MV). All four curves show dependencies between f0.5,decay and Δfdecay. A similar dependence appears in another cavity (CM3-4) (Fig. 4(b)).

Initially, we suspected that the frequency tuner might have disturbed the input coupler, causing variations in the coupling coefficients (β) and QL (or f0.5,decay). Consequently, we turned off the tuner and achieved cavity detuning by scanning the frequency of the signal generator on the same CM3-3. However, a similar dependence was observed in both cases (Fig. 4(c)). The deviation in Fig. 4(c) is primarily due to slight differences in cavity field levels.

Assuming that Vf* and Vr* represent the forward and reflected signals measured by the directional coupler, respectively, further calibration of Vf* and Vr* is necessary to obtain the true forward and reflected signals (Vf and Vr) using Vf = XVf* and Vr = YVr*, respectively (assuming we neglect channel crosstalk). The complex coefficients (X and Y) can be obtained by solving a linear regression equation [22]. Fig. 5 [Figure 5: see original paper] compares the Vf, Vr, and Vc signals, and a residual attenuation signal of Vf (i.e., XVf*) is observed after turning off RF power. There are two possible reasons for this (Fig. 6 [Figure 6: see original paper]):*

1. The first reason is crosstalk between measurement channels, that is, the residual signal of Vf is coupled with the signal Vr [22]. In this case, the

residual signal is a false measurement signal.
2. The second reason is source impedance mismatch, where the Vr signal is reflected from the generator side and mixed with the Vf signal. In this case, the residual signal is a true signal.

Before 2021, directional couplers with poor directivity (approximately 20 dB) were commonly used in our communication system, which we refer to as CAFe. Previous studies have suggested that the limited directivity of these couplers was primarily responsible for the residual signals [24]. In 2022, we replaced all the old directional couplers with new ones exhibiting high directivity (40 dB). This resulted in almost negligible channel crosstalk; however, we decided not to install a high-power circulator in CAFe because of cost constraints. Based on these factors, we conclude that the residual Vf signal in Fig. 5 could be attributed to impedance mismatch rather than crosstalk.

The algorithm in Eq. (1) must be modified because the "zero-input" condition is not satisfied. The specific calibration algorithms are described in Sec. III.

III. Theory and Algorithm

In this section, we establish cavity differential equations for the mismatched source impedance condition and use them to derive new formulas for calibrating f0.5 and Δf.

A. Radio-frequency and cavity circuit under mismatched source impedance condition

Fig. 7(a) [Figure 7: see original paper] presents a simplified model of an RF cavity coupled to an RF generator using a rigid coaxial line and an RF input power coupler [19]. In this model, the coaxial line is represented by a transmission line with characteristic impedance (Z0) and complex propagation constant (α + iβ). If the source impedance (Zg) on the generator side is not equal to Z0, a portion of the cavity-reflected signal is measured by the directional coupler as the cavity forward signal after turning off RF power. This process is described using an equivalent circuit (Fig. 7(b) [Figure 7: see original paper]).

Assuming that the cavity input coupler has a transformation ratio of 1:N, the voltage signals Vc, Vf, and Vr can be transformed into Ṽc, Ṽf, and Ṽr, respectively, using the following transformation equation: Ṽ = (1/N)V. In Fig. 7(b), the reflection coefficient (Γg) for the forward signal on the generator side at z = 0 can be expressed as [25]:

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{V^-}{V^+}$$

where V+ and V− represent the voltages of the incident and reflected waves at z = 0, respectively. After RF power is turned off, the reflection coefficient at z

= −L (L is the length of the transmission line) is given by:

$$\Gamma_L = \frac{\hat{V}^- e^{-(\alpha+i\beta)L}}{\hat{V}^+ e^{(\alpha+i\beta)L}} = \Gamma_g e^{-2(\alpha+i\beta)L}$$

Therefore, after turning off RF power, the transformed cavity voltage ($\hat{V}$c) can be expressed as:

$$\hat{V}_c = \hat{V}_f + \hat{V}_r$$

Inserting Eq. (3) into Eq. (4) and eliminating $\hat{V}$r, we obtain:

$$\hat{V}_f = \frac{1}{1+\Gamma_L}\hat{V}_c$$

Accordingly, signal Vf is associated with signal Vc by:

$$V_f = \frac{1}{1+\Gamma_L}V_c$$

In the absence of beam current, the cavity differential equation can be expressed as:

$$\frac{dV_c}{dt} + (\omega_{0.5} - i\Delta\omega)V_c = \omega_{0.5}u, \quad u = \frac{2V_f}{\beta_c + 1}$$

Here, the parameter βc is the coupling factor, which is generally considerably larger than 1 (i.e., βc » 1). Thus, u can be simplified as u = 2Vf. By substituting Vf with Eq. (6) into Eq. (7), we obtain:

$$\frac{dV_c}{dt} + (\omega_{0.5} - i\Delta\omega)V_c = \omega_{0.5} \cdot \alpha \cdot V_c$$

where α = 2ΓL/(1 + ΓL) denotes a complex factor. Eq. (9) does not satisfy the "zero-input" condition. However, by rearranging the terms, we obtain an equation that satisfies the following condition:

$$\frac{dV_c(t)}{dt} + [\omega_{0.5}(1-\alpha) - i\Delta\omega(t)]V_c(t) = 0$$

B. New calibration algorithm for f0.5 and Δf

It is more convenient to normalize the steady state (Vc(t)) to one, that is, Vc(0) = 1 if we assume that RF power is turned off at t = 0 and the signal Vc(t) is in steady state at t ≤ 0. Under these restrictions, the solution to Eq. (10) at t ≥ 0 is given by:

$$V_c(t) = V_c(0)e^{\int_0^t [\omega_{0.5}(\alpha-1)+i\Delta\omega(\tau)]d\tau}$$

The complex factor (α) can be decomposed into real and imaginary components (αr and αi), i.e., α = αr + iαi. Separating Eq. (11) into real and imaginary parts yields:

$$V_c(t) = e^{(\alpha_r-1)\omega_{0.5}t+i(\alpha_i\omega_{0.5}t+\int_0^\tau \Delta\omega(\tau)d\tau)}, \quad (t \geq 0)$$

Consequently, the amplitude and phase of Vc are given by:

$$\begin{cases} r = |V_c(t)| = e^{(\alpha_r-1)\omega_{0.5}t} \\ \phi = \angle V_c(t) = \alpha_i\omega_{0.5}t + \int_0^\tau \Delta\omega(\tau)d\tau \end{cases}$$

Eq. (13) provides important insights. If the source impedance (Zg) perfectly matches the transmission line impedance (Z0), then Γg = 0, ΓL = 0, and α = 0. In this scenario, according to the field decay algorithm, the cavity half bandwidth ($ 0.5)andcavitydetuningparameter(Δ $) can be easily obtained by fitting the slope of the cavity amplitude and phase, i.e., $ 0.5 = −r′/randΔ $ = ϕ'. However, if the source impedance is not correctly matched to the transmission line (Γg ≠ 0), the waves reflected from the cavity side will reflect again, resulting in nonzero forward power even after RF power is turned off. This significantly affects the shape of the field curves, rendering the original algorithm inapplicable.

Using the original field decay method, the cavity half-bandwidth ($ 0.5, decay)anddetuningparameter(Δ $decay were calibrated using:

$$\begin{cases} \omega_{0.5,\text{decay}} = -\frac{r'}{r} = (1-\alpha_r)\omega_{0.5} \\ \Delta\omega_{\text{decay}} = \phi' = \alpha_i\omega_{0.5} + \Delta\omega \end{cases}$$

To calibrate the actual cavity half-bandwidth and detuning parameters, the original algorithm must be modified as follows:

$$\begin{cases} f_{0.5,\text{cali}} = \frac{1}{2\pi} \cdot \frac{-r'/r}{1-\alpha_r} = \frac{f_{0.5,\text{decay}}}{1-\alpha_r} \\ \Delta f_{\text{cali}} = \frac{1}{2\pi}\left(\phi' - \alpha_i f_{0.5,\text{cali}}\right) = \Delta f_{\text{decay}} - \alpha_i f_{0.5,\text{cali}} \end{cases}$$

The estimation error of the original field-decay algorithm can be easily evaluated for a specified ΓL. According to Eq. (15), the accuracies of f0.5,decay and Δfdecay can be expressed as:

$$\begin{cases} \frac{f_{0.5,\mathrm{decay}}-f_{0.5,\mathrm{cali}}}{f_{0.5,\mathrm{cali}}} = -\alpha_r = -\Re\left(\frac{2\Gamma_L}{1+\Gamma_L}\right) \\ \frac{\Delta f_{\mathrm{decay}}-\Delta f_{\mathrm{cali}}}{f_{0.5,\mathrm{cali}}} = \alpha_i = \Im\left(\frac{2\Gamma_L}{1+\Gamma_L}\right) \end{cases}$$

The accuracy of the original field-decay algorithm depends on parameter α, where the real (αr) and imaginary (αi) parts of α determine the accuracies of f0.5,decay and Δfdecay, respectively. Fig. 8(a) and (b) [Figure 8: see original paper] illustrate the calibration errors of f0.5,decay and Δfdecay, respectively, as functions of ΓL. To ensure that the accuracies of f0.5,decay and Δfdecay/f0.5,cali lie within the $±$5% band, the coefficient ΓL must be located inside the red circle (i.e., |ΓL| < 0.024 -32 dB) in Fig. 8(c). Fig. 8(d) shows the specific relationship between the accuracies of f0.5,decay, Δfdecay/f0.5,cali and ΓL.

IV. Modeling and Simulation

The state-space formalism for Eq. (7) is given by [19, 26]:

$$\frac{d}{dt}\begin{bmatrix} V_{c,r} \\ V_{c,i} \end{bmatrix} = \begin{bmatrix} -\omega_{0.5} & -\Delta\omega \\ \Delta\omega & -\omega_{0.5} \end{bmatrix}\begin{bmatrix} V_{c,r} \\ V_{c,i} \end{bmatrix} + \begin{bmatrix} 0 & \omega_{0.5} \\ \omega_{0.5} & 0 \end{bmatrix}\begin{bmatrix} u_r \\ u_i \end{bmatrix}$$

Here, Vc,r and Vc,i represent the real and imaginary parts of the complex quantity (Vc), while ur and ui represent the real and imaginary parts of the complex quantity (u), respectively.

Eq. (17) can easily be transformed into its discrete-time form as follows [26, 27]:

$$\begin{bmatrix} V_{c,r}(n+1) \\ V_{c,i}(n+1) \end{bmatrix} = \begin{bmatrix} 1-T_s\omega_{0.5} & -T_s\Delta\omega(n+1) \\ T_s\Delta\omega(n+1) & 1-T_s\omega_{0.5} \end{bmatrix}\begin{bmatrix} V_{c,r}(n) \\ V_{c,i}(n) \end{bmatrix} + \begin{bmatrix} T_s\omega_{0.5} & 0 \\ 0 & T_s\omega_{0.5} \end{bmatrix}\begin{bmatrix} u_r(n) \\ u_i(n) \end{bmatrix}$$

where Ts represents the sampling period. According to Eq. (9), the drive signal u can be expressed as:

$$\begin{bmatrix} u_r(n+1) \\ u_i(n+1) \end{bmatrix} = \begin{bmatrix} \alpha_r & -\alpha_i \\ \alpha_i & \alpha_r \end{bmatrix}\begin{bmatrix} V_{c,r}(n+1) \\ V_{c,i}(n+1) \end{bmatrix}$$

Other difference equations related to the dynamic behavior of the cavity are derived in Appendix A. The simulation parameters are presented in Table 2 . After steady-state operation is achieved for 0.4 ms, RF power is turned off. The signal reflected from the cavity is also reflected as Γg is not zero. Fig. 9 [Figure

9: see original paper] compares the signals Vc, Vf, and Vr based on the cavity model (red) and real SC cavity (gray). Both results showed good consistency. In addition, the simulation results for the perfect impedance matching case (Γg = 0, indicated in green) are included for comparison.

According to Appendix A, the LFD dynamics are assumed to be determined by a first-order differential equation. Consequently, the cavity phase signal presents curved trajectories (red dashed lines) rather than linear trajectories. However, without the LFD dynamics, the cavity phase curve is linear, as indicated by the blue dotted lines in Fig. 9(a). The slope of the linear phase curve represents the cavity detuning parameter ($\Delta\omega$decay). The cavity phase curves overlap in the first 80 µs after RF power is turned off, regardless of whether LFD is included (Fig. 9(a)). For clarity, the LFD dynamics during the field decay process were examined (Fig. 10 [Figure 10: see original paper]). For the subsequent 80 µs after RF power is turned off, the LFD-induced phase error and maximum LFD value are less than 0.01 deg and approximately 0.8 Hz, respectively. Consequently, it is reasonable to fit the 80 µs cavity phase data to obtain Δf.

Table 2. RF and LFD parameters for the simulations

Parameter	Value
f0.5	149 Hz
Δfinitial	-42 deg
τ	530 µs
KLFD	0.038 m
Leff	0.15 Hz/(MV/m)^2
Ts	80 ns

V. Experimental Verification

Based on the algorithm in Eq. (15), we recalibrated the measurement results shown in Fig. 4(a). The details of the process are described below:

1. **Calibrating the actual forward and reflected signals**: Using Vf = XVf* and Vr = YVr*, respectively. The complex factors (X and Y) were determined by solving a linear regression equation [22].
2. **Determining the factor α**: According to Eq. (6), we calculated α as twice the ratio of Vf to Vc after turning off RF power. Here, we averaged α over a time interval of 50 µs (e.g., from 0.45 ms to 0.5 ms in Fig. 11 [Figure 11: see original paper]) to reduce uncertainty.
3. **Calibrating f0.5,decay and Δfdecay using the traditional "field decay method"**: To ensure a sufficient signal-to-noise ratio, we determined f0.5,decay by calculating the slope of the decay curve between 0.95 and 0.75 of the steady-state Vc (Fig. 3(b)). To avoid the LFD effect,

we determined the derivative of the cavity phase within an 80 µs interval after RF power was turned off to obtain Δfdecay.

4. **Calibrating f0.5,cali and Δfcali**: We used Eq. (15) with the known values of α, f0.5,decay, and Δfdecay to calibrate f0.5,cali and Δfcali.

We used this procedure to calibrate f0.5,cali and Δfcali (Fig. 12(a) [Figure 12: see original paper]). In contrast to the strong correlation observed between f0.5,decay and Δfdecay measured from the cavity amplitude and phase decay curves, the newly calibrated value of f0.5,cali was independent of Δfcali at different Vc levels.

We also compared Δfcali and Δfdecay with the steady-state cavity detuning parameters (Δfss), which were calculated from the steady-state values of Vf and Vc (Fig. 12(b)). The following expression was used to calibrate Δfss:

$$\Delta f_{ss} = \tan(\phi - \theta) \cdot f_{0.5,\mathrm{cali}}$$

Fig. 12(b) indicates that the discrepancy between Δfcali and Δfss is less than $±$5 Hz, whereas there is an offset of approximately 40 Hz between Δfdecay and Δfss.

To further validate the proposed algorithm, we used a network analyzer to measure CM3-3 and perform a comparison. Fig. 13(a) and (b) [Figure 13: see original paper] show the measurement setup and the frequency-response measurement with corresponding fitted curve, respectively. The measurement results were fitted using the following formula:

$$A_{\mathrm{fit}} = \sqrt{\frac{A_{\max} \cdot f_{0.5}^2}{(f - f_{\mathrm{offset}})^2 + f_{0.5}^2}}$$

where Amax is the maximum value of the amplitude-frequency-response curve. The parameters f and foffset represent the stimulus frequency and frequency offset between the cavity resonant frequency and RF, respectively. In Fig. 13(b), the estimated value of f0.5 for the fitted curve is 183.7 Hz.

Next, we utilized a network analyzer to scan the cavities in CM1–CM3 and estimated f0.5,scan by curve fitting. We then compared the deviation between f0.5,cali and f0.5,scan for 16 cavities (excluding CM2-6 and CM3-1 due to cavity faults). In addition, we plotted the deviation between f0.5,decay and f0.5,scan for comparison. The results are shown in Fig. 14 [Figure 14: see original paper]. The deviation between f0.5,scan and f0.5,cali was maintained roughly within $±$2%, indicating that the proposed calibration algorithm accurately estimated the values of f0.5.

VI. Conclusion

After RF power was turned off, the residual signal (Vf) caused by source impedance mismatch could affect the field decay process, resulting in measurement errors for f0.5 and Δf. The simulation results indicate that, to ensure measurement errors of f0.5 and Δf remain less than 5%, the reflection coefficient at the generator side must be less than -32 dB. To improve measurement accuracy, we derived a calibration algorithm based on the cavity differential equation for cases with source impedance mismatch.

Using this algorithm, we recalibrated f0.5 and Δf and compared the results with those obtained from network analyzer measurements. The maximum error between the calculated and measured values was within 2%. α is a crucial factor in the calibration algorithm that ultimately depends on the impedance Zg on the generator side. Unfortunately, we do not have a comprehensive understanding of the factors that determine Zg. One possible factor is the power output of the RF generator. For instance, different Vc and detuning parameters require different RF powers, which lead to different Zg values. Nevertheless, it is important to stress that Zg is not determined solely by the generator power, and the same power may result in different values of Zg. In the next step, we focus on identifying the mechanisms and physical quantities that affect Zg.

VII. Acknowledgment

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Appendix A: Cavity Dynamic Behavior

The Lorentz force detuning (LFD) must be considered because the cavity gradient varies after RF power is turned off. As the cavity field attenuates slowly due to the high loaded Q (narrow bandwidth) characteristic, higher-order mechanical modes are usually not excited. In this case, the LFD dynamics can be described by a first-order differential equation [19] as follows:

$$\Delta f_{\mathrm{LFD}}(t)+\tau\frac{d\Delta f_{\mathrm{LFD}}(t)}{dt}=-K_{\mathrm{LFD}}E_{\mathrm{peak}}^2$$

where KLFD and τ denote the LFD coefficient and mechanical time constant, respectively. The quantity Epeak is the cavity peak gradient, which can be defined in terms of Vc using Epeak = Vc/Leff, where Leff is the effective cavity length.

According to Ref. [27], the corresponding difference equation of Eq. (A1) is given by:

$$\Delta f_{\mathrm{LFD}}(n+1) = b\Delta f_{\mathrm{LFD}}(n) + g\left[V_{c,r}^2(n) + V_{c,i}^2(n)\right]$$

where the coefficients are b = e^{-T_s/ *{LFD}} and g = (1 - b)(-K*{LFD}). When RF power is turned off, the cavity field exponentially decreases to zero. The corresponding LFD parameter value increases from its initial value of negative dozens (hundreds) of Hz to zero. In addition, we assume that LFD is the only dominant cavity detuning that occurs after RF power is turned off (other sources may cause negligible detuning), and as Vc^2 varies from $Vc^2(0)$ to $Vc^2(t)$, the LFD dynamics can be modified as follows:

$$\Delta f_{\mathrm{LFD}}(n+1) = b\Delta f_{\mathrm{LFD}}(n) + g\left[V_{c,r}^2(n) + V_{c,i}^2(n) - 1\right]$$

The constant 1 in Eq. (A3) represents the square of the steady-state Vc amplitude, which is normalized to one for simplicity, that is, $Vc,r^2(0) + Vc,i^2(0) = 1$. Thus, the overall cavity detuning is given by:

$$\Delta\omega(n+1) = 2\pi\left[\Delta f_{\mathrm{initial}} + \Delta f_{\mathrm{LFD}}(n+1)\right]$$

Here, Δfinitial is the initial steady-state detuning parameter before the RF power source is turned off. Using the difference equations (17), (19), (A3), and (A4), a dynamic simulation of the cavity undergoing a complete field decay process can be easily performed.

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Note: Figure translations are in progress. See original paper for figures.