Exploring the Implications of 2023 Pular Timing Array Datasets for Scalar-Induced Gravitational Waves and Primordial Black Holes

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Abstract

Significant evidence for a gravitational-wave background was reported by several pulsar-timing-array collaborations. By assuming that this signal is interpreted by the scalar-induced gravitational waves, we study physical implications of the observed signal for the nature of primordial curvature perturbations and primordial black holes. In particular, we explore the effects of primordial non-Gaussianity on the inferences of model parameters, and obtain the parameter region allowed by the observed signal, i.e., the primordial scalar spectral amplitude $A_S \sim 10^{-2} - 1$, the primordial non-Gaussian parameter $-10 \lesssim f_{\rm NL} \lesssim 10$, and the mass of primordial black holes $m_{\rm pbh} \sim 10^{-3} - 0.1 M_\odot$. We find that the non-Gaussianity suppressing the abundance of primordial black holes is preferred by the observed signal. We show that the anisotropies of scalar-induced gravitational waves are a powerful probe for measurements of the non-Gaussian parameter $f_{\rm NL}$, and conduct a complete analysis of the angular power spectrum in the nano-Hertz band. We expect that the Square Kilometre Array project has potentials to measure such anisotropies.

Full Text

Preamble

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Several pulsar-timing-array collaborations have reported significant evidence for a gravitational-wave background. By interpreting this signal as scalar-induced gravitational waves, we investigate the physical implications for the nature of primordial curvature perturbations and primordial black holes. In particular, we explore the effects of primordial non-Gaussianity on parameter inference and obtain the parameter region allowed by the observed signal: primordial scalar spectral amplitude $A_S \sim 10^{-2} - 1$, primordial non-Gaussian parameter $-10 \lesssim f_{\rm NL} \lesssim 10$, and primordial black hole mass $m_{\rm PBH} \sim 10^{-3} - 0.1 M_\odot$. We find that non-Gaussianity suppressing the abundance of primordial black holes is preferred by the observed signal. We demonstrate that the anisotropies of scalar-induced gravitational waves provide a powerful probe for measuring the non-Gaussian parameter $f_{\rm NL}$ and conduct a complete analysis of the angular power spectrum in the nano-Hertz band. We expect that the Square Kilometre Array project has the potential to measure such anisotropies.

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Introduction

Recently, several pulsar timing array (PTA) collaborations have reported significant evidence for an excess signal with Hellings-Downs (HD) correlations [1–4], indicating the gravitational-wave origin of this signal. The strain amplitude of such a gravitational-wave background (GWB) was found to be of order 10^{-15} at the pivot frequency of 1 $\rm yr^{-1}$. The inferred GWB spectrum was found to be consistent with an astrophysical origin from inspiraling supermassive black hole (SMBH) binaries [5]. However, the current datasets cannot exclude possibilities of cosmological origins (and other exotic astrophysical sources), which were studied by the collaborations in several accompanying papers [6, 7]. In particular, many cosmological origin models have been shown to provide even better fits to the observed signal than the SMBH-binary interpretation. If confirmed in the future, they may point to evidence for new physics.

In this work, we focus on the cosmological interpretation of the observed signal, specifically scalar-induced gravitational waves (SIGWs) [8–13]. This possibility was previously considered to account for the NANOGrav 12.5-year dataset [14] by the authors of Refs. [15–24]. Recently, it was revisited by the collaborations in Refs. [6, 7], but only Gaussian primordial scalar (or equivalently, comoving curvature) perturbations were considered in those studies. However, it has been shown that primordial non-Gaussianity contributes significantly to the energy density of SIGWs [25–33], indicating substantial modifications to the energy-density fraction spectrum that is essential for PTA data analysis. By interpreting the observed signal as having a SIGW origin, we therefore study the implications of PTA datasets for the nature of primordial scalar perturbations, including both the power spectrum and local-type primordial scalar non-Gaussianity.

We also investigate the physical implications for primordial black hole (PBH) scenarios. The formation process of PBHs was inevitably accompanied by the production of SIGWs. In fact, enhanced curvature perturbations not only produced PBHs through gravitational collapse in the early universe [34], but also induced a GWB via nonlinear mode couplings. Therefore, we can explore PBH scenarios using SIGWs [35–38]. Related works analyzing realistic datasets can be found in Refs. [6, 7, 20–22, 38, 39]. Strong influences of primordial non-Gaussianity on the PBH mass function were also studied in the literature [24, 40–52].

In this work, by taking into account the effects of primordial non-Gaussianity, we recast the constraints on primordial perturbations into constraints on the PBH mass function. To further explore primordial non-Gaussianity, we study the anisotropies in SIGWs in the PTA band and provide a complete analysis of the angular power spectrum in this band. The energy-density spectrum of isotropic SIGWs has significant degeneracies in model parameters [33]. By conducting a complete analysis of the angular power spectrum, we show that it is useful for determining f_{NL} because these degeneracies would be broken if anisotropic SIGWs are considered [33]. Earlier related works can be found in Refs. [54–62]. In this work, we further study the angular power spectrum for anisotropies in SIGWs in the PTA band, which may be useful not only for determining f_{NL} but also for discriminating between different GWB sources in realistic data analysis.

The remainder of this paper is arranged as follows. In Section II, we provide a brief summary of isotropic SIGWs. In Section III, we show the implications of current datasets for the power spectrum of primordial curvature perturbations and then for the PBH mass function. In Section IV, we study the anisotropies in SIGWs and show the angular power spectrum in the PTA band. In Section V, we present concluding remarks.

II. Energy-Density Fraction Spectrum of Scalar-Induced Gravitational Waves

In this section, we provide a brief but self-consistent summary of the main results of SIGW theory. The energy-density fraction spectrum of the isotropic GWB is $\bar{\Omega}_{\mathrm{gw}}(\eta, q) = \bar{\rho}_{\mathrm{gw}}(\eta, q)/\rho_{\mathrm{crit}}(\eta)$ [63], where q is the wavenumber of gravitational waves (GWs), ρ_{crit} is the critical energy density of the universe at conformal time η, and the overbar denotes background-level physical quantities. This definition indicates that $\int \bar{\rho}_{\mathrm{gw}}(\eta, q) d \ln q$ is the total energy-density fraction of the GWB [63]. The spectrum is formally expressed as $\bar{\rho}_{\mathrm{gw}}(\eta, q) \sim \langle h_{ij,l} h_{ij,l} \rangle$, where $h_{ij}(\eta, q)$ denotes the strain with wavevector q in Fourier space and the angle brackets define an ensemble average.

For SIGWs on subhorizon scales, we have $h_{ij} \sim \zeta^2$ and then $\bar{\Omega}_{\mathrm{gw}}(\eta, q) \sim \langle \zeta^4 \rangle$

[8, 9], where $\zeta(q)$ denotes the primordial curvature perturbations in the early universe. In the case of primordial Gaussianity, the semi-analytic formula for $\bar{\Omega}_{\rm gw}(\eta, q)$ was shown in Refs. [12, 13], with earlier related works in Refs. [8, 9]. However, in the case of primordial non-Gaussianity, such a semi-analytic formula does not exist. Related studies can be found in recent literature [25–33, 64]. In this work, we follow the conventions of our previous work [33].

To derive the contributions of primordial non-Gaussianity to the energy density, we express the primordial curvature perturbations ζ in terms of their Gaussian components ζ_g [65]:

$$\zeta(q) = \zeta_g(q) + \frac{(2\pi)^{3/2}}{2} \int \frac{d^3k}{(2\pi)^3} \frac{\zeta_g(k)\zeta_g(q-k)}{\sqrt{k^3|q-k|^3}}$$

where $f_{\rm NL}$ is the non-linear parameter characterizing local-type primordial non-Gaussianity. We introduce a new quantity $F_{\rm NL} = 3f_{\rm NL}/5$ that simplifies the analytic formulae in the following. Note that the validation of perturbation theory requires $A_S F_{\rm NL}^2 < 1$, where A_S will be defined below.

We define the dimensionless power spectrum of ζ_g as follows:

$$\langle \zeta_g(q)\zeta_g(q')\rangle = \delta^{(3)}(q+q')\frac{2\pi^2}{q^3}\Delta_g^2(q)$$

where $\Delta_g^2(q)$ is assumed to be a normal function with respect to $\ln q$ in this work [21, 31, 66, 67]:

$$\Delta_g^2(q) = \frac{A_S}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\ln^2(q/q_*)}{2\sigma^2}\right]$$

Here, A_S stands for the spectral amplitude at the spectral peak wavenumber q_*, and σ denotes the standard deviation characterizing the width of the spectrum. The wavenumber q can be straightforwardly recast into frequency ν, namely $q = 2\pi\nu$ and $q_* = 2\pi\nu_*$.

By conducting a tedious but straightforward derivation, we can decompose $\bar{\Omega}_{\rm gw} \sim \langle\zeta^4\rangle$ into three components depending on the power of $f_{\rm NL}$, based on Wick's theorem. However, the complete derivations have been simplified by following an approach using Feynman-like diagrams [25, 28, 30–33]. We summarize only the final results as follows:

$$\bar{\Omega}_{\rm gw}(\eta, q) = \bar{\Omega}_{\rm gw}^{(0)}(\eta, q) + \bar{\Omega}_{\rm gw}^{(1)}(\eta, q) + \bar{\Omega}_{\rm gw}^{(2)}(\eta, q)$$

where we show the analytic expressions of $\bar{\Omega}_{\mathrm{gw}}^{(n)} \propto A_S^{2+n} F_{\mathrm{NL}}^n$ explicitly in Appendix A. They have been computed with the vegas package [68], and the numerical results are reproduced in Fig. 1. In particular, $\bar{\Omega}_{\mathrm{gw}}^{(0)}$ corresponds to the case of primordial Gaussianity, while $\bar{\Omega}_{\mathrm{gw}}^{(1)}$ and $\bar{\Omega}_{\mathrm{gw}}^{(2)}$ completely describe the contributions of local-type primordial non-Gaussianity.

The energy-density fraction spectrum of SIGWs at the current conformal time η_0 is given by:

$$\bar{\Omega}_{\mathrm{gw},0}(\nu) = \Omega_{\mathrm{rad},0} \frac{g_{*,\rho}(T)}{g_{*,\rho}(T_{\mathrm{eq}})} \frac{g_{*,s}(T_{\mathrm{eq}})}{g_{*,s}(T)} \bar{\Omega}_{\mathrm{gw}}(\eta, q)$$

where $\Omega_{\mathrm{rad},0} h^2 = 4.2 \times 10^{-5}$ is the physical energy-density fraction of radiation in the present universe [69], T (and T_{eq}) labels the cosmic temperatures at the emission time (and the epoch of matter-radiation equality), and ν denotes the gravitational-wave frequency [21]:

$$\nu = 26.5\,\mathrm{nHz} \left(\frac{T}{10^4\,\mathrm{GeV}}\right) \left(\frac{g_{*,\rho}(T)}{106.75}\right)^{1/2} \left(\frac{g_{*,s}(T)}{106.75}\right)^{-1/3}$$

Here, the effective relativistic degrees of freedom of the universe, $g_{*,\rho}$ and $g_{*,s}$, are tabulated functions of T, as shown in Ref. [70]. To illustrate the SIGW interpretation of the current datasets, we depict $\bar{\Omega}_{\mathrm{gw},0}(\nu)$ as a function of ν in Fig. 2, by choosing a particular set of model parameters.

III. Implications of PTA Datasets for Primordial Curvature Perturbations and Primordial Black Holes

In this section, we study possible constraints on the parameter space of the primordial power spectrum and PBHs from the NG15 data. Constraints from other PTA datasets can also be obtained following the same approach, but are disregarded in this work.

A. Primordial Curvature Perturbations

Based on the principle of PTA observations, the energy density of a given GWB, denoted $\Omega_{\mathrm{gw}}(\nu)$, is related to the timing residual power spectral density $S(\nu)$ [7]:

$$\Omega_{\mathrm{gw}}(\nu) = \frac{8\pi^4 \nu^5}{3 H_0^2 \nu_{\mathrm{yr}}^3} S(\nu)$$

where ν_{yr} is a pivot frequency related to a duration time of one year. For realistic data analysis, $S(\nu)$ could be assumed to be a power law [7]:

$$S(\nu) = A\left(\frac{\nu}{\nu_{\rm yr}}\right)^{-\gamma}$$

where the amplitude A and index γ have been constrained by the NG15 data. For example, based on Fig. 1 of Ref. [7], the green contours in the $\log_{10} A - \gamma$ plane represent 68% (inner) and 95% (outer) probability regions.

Although a full Bayesian analysis will be necessary in the future, we can immediately gain useful insights by recasting the contours in the $\log_{10} A - \gamma$ plane into contours in the $A_S - \nu_*$ plane, once σ and $|F_{\rm NL}|$ take definitive values. Fig. 3 displays such contours for a series of $|F_{\rm NL}|$ values, denoted by colored dots. Hereafter, we fix the value of σ, namely $\sigma = 1$, though a generalization is straightforward. In fact, this choice is consistent with the best-fit σ from the NG15 data, which resulted in a 68% credible interval $\sigma \in [0.51, 2.07]$ [7]. To conduct such recasting, for a given set of A and γ, we vary A_S and ν_* to make $\bar{\Omega}_{\rm gw,0}(\nu)$ follow the same power law as $\Omega_{\rm gw}(\nu)$. Here, we have fixed the pivot scale to be $\nu_{\rm yr}$ for the model parameters A_S and ν_*. This approach was already adopted in the literature [17].

Based on Fig. 3, we find significant effects of the non-Gaussian parameter $|F_{\rm NL}|$ on the inferences of other parameters A_S and ν_*, and vice versa. In particular, the contours are shifted to lower-A_S regimes with increasing $|F_{\rm NL}|$, since the energy density of SIGWs would be overproduced in the presence of primordial non-Gaussianity. This is one of the important results of this work. Though our research method can be straightforwardly generalized to other values of σ, it is essential to conduct a full Bayesian analysis, which we leave to future work. Note that the above results are independent of the non-Gaussian sign. However, the non-Gaussian sign would significantly affect the mass function of PBHs, which we study below.

We further pin down the parameter region of A_S and $F_{\rm NL}$ by considering the validation of perturbation theory that requires $A_S F_{\rm NL}^2 \leq 1$. In Fig. 4, the blue shaded area corresponds to this requirement, while the orange shaded area corresponds to the inferred parameter region (68% confidence level) from the NG15 data. Only the overlap area is simultaneously allowed by theory and observations. In this sense, PTA observations have already become a powerful probe of the early universe.

B. Primordial Black Holes

We further recast the constraints on primordial curvature perturbations into constraints on PBHs. Due to large uncertainties in PBH formation scenarios (e.g., see reviews in Ref. [54]), we consider a simplified scenario used by Ref. [48] to demonstrate the importance of primordial non-Gaussianity. The initial mass function of PBHs is given by:

$$\beta \equiv \frac{\rho_{\rm PBH}}{\rho_{\rm tot}} = \int_{\zeta>\zeta_c} P(\zeta)d\zeta = \int_{\zeta(\zeta_g)>\zeta_c} P(\zeta_g)d\zeta_g$$

where $P(\zeta)$ is a probability distribution function (PDF) of primordial curvature perturbations, σ_g is the standard variance for a PDF of the Gaussian component ζ_g, and ζ_c denotes the critical fluctuation. Considering the power spectrum in Eq. (3), we obtain $\sigma_g^2 = \langle\zeta_g^2\rangle = \int \Delta_g^2(q) d\ln q = A_S$. In addition, it is known that $\zeta_c \sim \mathcal{O}(1)$, which was shown to be 0.7 and 1.2 in Ref. [71]. We consider both values in the following.

To calculate Eq. (9), we separate $F_{\rm NL}$ into two regimes: $F_{\rm NL} > 0$ and $F_{\rm NL} < 0$. First, we solve the equation $\zeta(\zeta_g) = \zeta_c$ to obtain:

$$\zeta_{g\pm} = \frac{-1 \pm \sqrt{1+4F_{\rm NL}\zeta_c}}{2F_{\rm NL}}$$

By substituting this into Eq. (9), we have an expression for β for $F_{\rm NL} > 0$:

$$\beta = \int_{\zeta_{g-}}^{\zeta_{g+}} P(\zeta_g)d\zeta_g = \frac{1}{2}\mathrm{erfc}\left(\frac{\zeta_{g-}}{\sqrt{2}\sigma_g}\right) - \frac{1}{2}\mathrm{erfc}\left(\frac{\zeta_{g+}}{\sqrt{2}\sigma_g}\right)$$

where $\mathrm{erfc}(x)$ is the complementary error function. Similarly, for $-(4\zeta_c)^{-1} < F_{\rm NL} < 0$, we have:

$$\beta = \int_{-\infty}^{\zeta_{g-}} P(\zeta_g)d\zeta_g = \frac{1}{2}\mathrm{erfc}\left(\frac{\zeta_{g+}}{\sqrt{2}\sigma_g}\right) + \frac{1}{2}\mathrm{erfc}\left(\frac{-\zeta_{g-}}{\sqrt{2}\sigma_g}\right)$$

However, for $F_{\rm NL} < -(4\zeta_c)^{-1}$, no PBHs were formed in the early universe because the curvature perturbations never exceed the critical fluctuation. As a reasonable candidate for cold dark matter, the abundance of PBHs is given by [72]:

$$f_{\rm PBH} \simeq 2.5 \times 10^8 \beta \left(\frac{g_{*,\rho}(T_f)}{106.75}\right)^{-1/4} \left(\frac{m_{\rm PBH}}{M_\odot}\right)^{-1/2}$$

where $m_{\rm PBH}$ stands for the mass of PBHs in units of $M_\odot$, and T_f denotes the cosmic temperature at formation time. Roughly speaking, $m_{\rm PBH}$ can be related to the horizon mass m_H and then to the frequency ν_*. Specifically, we have [17]:

$$m_{\rm PBH} \simeq 0.31 M_\odot \left(\frac{\nu_*}{5.0\,\mathrm{nHz}}\right)^{-2}$$

Therefore, we infer the mass of PBHs to be $\sim \mathcal{O}(10^{-3}\text{–}10^{-1})M_\odot$ based on Fig. 3. However, the inferred abundance would be larger than unity, indicating that the PBH scenario is in tension with the NG15 data. To demonstrate this result more clearly, in Fig. 4 we depict two solid curves corresponding to $m_{\rm PBH} = 10^{-2}M_\odot$ and $f_{\rm PBH} = 1$ for $\zeta_c = 0.7$ (purple curve) and $\zeta_c = 1.2$ (rose curve). We also denote the critical value $F_{\rm NL} = -(4\zeta_c)^{-1}$ with vertical dotted lines.

Based on Fig. 4, when we interpret the observed signal as having a SIGW origin, we find overproduction of PBHs since the inferred value of A_S is typically one order of magnitude larger than the value producing $f_{\rm PBH} = 1$. Even when accounting for positive non-Gaussianity, such overproduction cannot be effectively alleviated. In contrast, negative non-Gaussianity can alleviate it, particularly when considering a sizable negative non-Gaussian parameter, i.e., $F_{\rm NL} < -(4\zeta_c)^{-1}$, that forbids any PBH formation. Due to large uncertainties, it is challenging to exclude the PBH scenario by analyzing the current datasets. However, a more detailed analysis such as Bayesian analysis remains important, though beyond the scope of this paper.

Therefore, it seems essential to measure primordial non-Gaussianity, at least determining the sign of $F_{\rm NL}$, to judge the PBH scenario. However, due to the sign degeneracy of $F_{\rm NL}$, it is impossible to determine the non-Gaussian sign via measurements of the energy-density fraction spectrum of SIGWs alone. In the next section, we propose that anisotropic SIGWs have the potential to break such sign degeneracy as well as other degeneracies of model parameters, providing possibilities for judging the PBH scenario in the future.

IV. Angular Power Spectrum for Anisotropies in Scalar-Induced Gravitational Waves

In this section, we study the anisotropies in SIGWs and the angular power spectrum in the PTA band, following the conventions of our previous paper [33].

The anisotropies in SIGWs arise from long-wavelength modulations of the energy density produced by short-wavelength modes. Based on Section II, we know that SIGWs were produced at extremely high redshifts, corresponding to extremely small horizons. Due to limitations in the angular resolution of gravitational-wave detectors, the signal along a line of sight represents an ensemble average of the energy densities over many such horizons. In this sense, any two signals would be identical. However, the energy density of SIGWs produced by short-wavelength modes can be spatially redistributed by long-wavelength modes if there are couplings between them. Therefore, local-type primordial non-Gaussianity could contribute to such couplings, as shown below.

Analogous to the temperature fluctuations of relic photons [73], the initial inhomogeneities in SIGWs at spatial location x are characterized by the density contrast:

$$\delta_{\rm gw}(\eta, x, q) = \frac{4\pi\omega_{\rm gw}(\eta, x, q)}{\bar{\Omega}_{\rm gw}(\eta, q)} - 1$$

where the energy-density full spectrum $\omega_{\rm gw}(\eta, x, q)$ is defined by $\int d^3q\, \omega_{\rm gw}(\eta, x, q)/q^3$. This definition implies that the energy density $\rho_{\rm gw}(\eta, x) = \rho_{\rm crit}\omega_{\rm gw}(\eta, x, q) \sim \langle\zeta^4\rangle_x$, where the subscript x stands for an ensemble average within the horizon enclosing x [33, 55]. We further separate ζ_g into short-wavelength ζ_{gS} and long-wavelength ζ_{gL} components, i.e., $\zeta_g = \zeta_{gS} + \zeta_{gL}$ [74]. Finally, we obtain $\delta_{\rm gw}(\eta, x, q) \sim \zeta_{gL}\langle\zeta_{gS}\zeta_S^3\rangle_x$ at linear order in ζ_{gL}. Here, we introduce ζ_S to denote the part of ζ composed only of ζ_{gS}. Higher orders of ζ_{gL} are negligible due to the power spectrum $\Delta_L^2 \sim 10^{-9}$ for ζ_{gL} [69]. Using Feynman-like rules and diagrams, we have obtained an explicit expression for $\delta_{\rm gw}(\eta, x, q)$ [33]:

$$\delta_{\rm gw}(\eta, x, q) = \frac{\Omega_{\rm ng}(\eta, q)}{\bar{\Omega}_{\rm gw}(\eta, q)} \int \frac{d^3k}{(2\pi)^{3/2}} e^{ik\cdot x}\zeta_{gL}(k)$$

where a new quantity is introduced to simplify our computation:

$$\Omega_{\rm ng}(\eta, q) = \frac{2}{3}\bar{\Omega}^{(0)}_{\rm gw}(\eta, q) + \frac{2}{2}\bar{\Omega}^{(1)}_{\rm gw}(\eta, q)$$

The initial inhomogeneities here correspond to the intrinsic temperature fluctuations of cosmic microwave background (CMB) photons on the last-scattering surface.

The "observed" density contrast $\delta_{\rm gw,0}(q)$ can be analytically estimated following the line-of-sight approach [75–77]. It receives contributions from both the initial inhomogeneities and propagation effects [33]:

$$\delta_{\rm gw,0}(q) = \delta_{\rm gw}(\eta, x, q) + [4 - n_{\rm gw,0}(\nu)]\Phi(\eta, x)$$

where we consider only the Sachs-Wolfe (SW) effect [78], characterized by the Bardeen potential at large scales:

$$\Phi(\eta, x) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{ik\cdot x}\zeta_{gL}(k)$$

and $n_{\rm gw,0}(q)$ is the index of the energy-density fraction spectrum defined in Eq. (5):

$$n_{\rm gw,0}(\nu) = \frac{\partial \ln \bar{\Omega}_{\rm gw,0}(\nu)}{\partial \ln \nu} = \left.\frac{\partial \ln \bar{\Omega}_{\rm gw}(\eta, q)}{\partial \ln q}\right|_{q=2\pi\nu}$$

In the following, we adopt the assumption of statistical isotropy for the density contrasts on large scales. Analogous to CMB studies (e.g., see Ref. [79]), the inhomogeneities in SIGWs can be recast into anisotropies in the GWB.

The reduced angular power spectrum is typically used to characterize the statistics of anisotropies in SIGWs. It is defined by the following two-point correlator:

$$\langle\delta_{\mathrm{gw},0,\ \mathrm{m}}(2\pi\nu)\delta^{*}_{\mathrm{gw},0,\ '\mathrm{m}'}(2\pi\nu)\rangle = \delta_{\ell\ell'}\delta_{mm'}\tilde{C}_{\ell}(\nu)$$

where $\delta_{\mathrm{gw},0}(q)$ is expanded in spherical harmonics:

$$\delta_{\mathrm{gw},0}(q) = \sum_{\ell m}\delta_{\mathrm{gw},0,\ \mathrm{m}}(q)Y_{\ell m}(\hat{n})$$

Roughly, we get $\tilde{C}_{\ell} \sim \delta^{2}_{\mathrm{gw},0} \propto \langle\zeta_{gL}\zeta_{gL}\rangle \sim \Delta^{2}_{L}$. A complete analysis employing Feynman-like rules and diagrams was conducted in our previous work [33]. We summarize only the final results:

$$\tilde{C}_{\ell}(\nu) = \frac{18\pi\Delta^{2}_{L}}{25\ell(\ell+1)}\left[\frac{\Omega_{\mathrm{ng}}(\eta,2\pi\nu)}{\bar{\Omega}_{\mathrm{gw}}(\eta,2\pi\nu)} + (4 - n_{\mathrm{gw},0}(\nu))\right]^{2}$$

which can be recast into the angular power spectrum:

$$C_{\ell}(\nu) = \bar{\Omega}_{\mathrm{gw},0}(\nu)\tilde{C}_{\ell}(\nu)$$

We make an analogy to CMB anisotropies, for which the rms temperature is roughly determined by $[\ell(\ell+1)C^{\mathrm{CMB}}_{\ell}/(2\pi)]^{1/2}$. For anisotropies in SIGWs, the rms energy density is roughly determined by $[\ell(\ell+1)C_{\ell}(\nu)/(2\pi)]^{1/2}$, which is the variance of the energy-density fluctuations. Note that the rms energy density is constant with respect to multipoles ℓ, but depends on the gravitational-wave frequency bands.

In Fig. 5, we depict the rms energy density with respect to gravitational-wave frequency. For comparison, we also show the energy-density fraction spectrum in the same figure. Roughly speaking, we find $\tilde{C}_{\ell} \sim \mathcal{O}(10^{-4})$, depending on model parameters. Note that the angular power spectrum can break degeneracies of model parameters. For example, based on Fig. 5, we find coincidence for the energy-density fraction spectra of three different parameter sets. However, the angular power spectrum breaks such coincidence, particularly the sign degeneracy of F_{NL}. This result indicates that primordial non-Gaussianity could in principle be determined by measuring anisotropies in SIGWs [33]. Recently, an upper limit on the reduced angular power spectrum was inferred to be $\tilde{C}_{\ell} < 20\%$ from the NG15 data [81]. This is not precise enough to test the theoretical predictions of our current work. In contrast, based on Fig. 5, we expect the

Square Kilometre Array (SKA) [80] to have sufficient precision for measuring the non-Gaussian parameter.

V. Conclusions

In this work, we studied the implications of recent PTA datasets for the nature of primordial curvature perturbations and primordial black holes. In particular, we explored the influence of primordial scalar non-Gaussianity on parameter inference, and vice versa. By accounting for the impacts of primordial non-Gaussianity, we analyzed the current datasets and obtained the allowed parameter region: primordial scalar spectral amplitude $A_S \sim 10^{-2} - 1$, primordial non-Gaussian parameter $-10 \lesssim F_{\rm NL} \lesssim 10$, and PBH mass $m_{\rm PBH} \sim 10^{-2} - 1 M_\odot$. We stress the importance of a full Bayesian analysis that will be conducted in future work. Even when considering the non-Gaussian parameter, the PBH scenario is shown to be in tension with the NG15 data, except when considering a sizable negative $F_{\rm NL}$ that can significantly suppress PBH abundance. However, it is challenging to exclude the PBH scenario with current datasets due to large uncertainties in formation models. Finally, we proposed that anisotropies in SIGWs are a powerful probe for measuring the non-Gaussian parameter $F_{\rm NL}$ and conducted the first complete analysis of the angular power spectrum in the nano-Hertz band. In particular, we found that such a spectrum can effectively break some degeneracies of model parameters, especially the sign degeneracy of $F_{\rm NL}$. Additionally, we explored the detectability of SIGW anisotropies in the era of the SKA project.

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Appendix A: Formulae for Computing the SIGW Energy Density

After lengthy and tedious derivations in accordance with Refs. [31–33], the three terms in Eq. (4) can be exactly expressed as follows:

$$\bar{\Omega}^{(0)}_{\rm gw}(\eta, q) = \int_0^\infty du_1 \int_0^\infty dv_1 \int_{-1}^1 d\cos\theta_1 \frac{J^2(u_1, v_1, x\to\infty)}{(u_1 v_1)^2} \Delta_g^2(v_1 q) \Delta_g^2(u_1 q)$$

$$\bar{\Omega}^{(1)}_{\rm gw}(\eta, q) = F_{\rm NL} \int_0^\infty du_1 \int_0^\infty dv_1 \int_{-1}^1 d\cos\theta_1 \int_0^\infty du_2 \int_0^\infty dv_2 \int_{-1}^1 d\cos\theta_2 \int_0^{2\pi} d\phi_{12} \cos 2\phi_{12} \frac{J(u_1, v_1, x\to\infty)J(}{(u_1 v_1 u_2}$$

$$\bar{\Omega}_{\mathrm{gw}}^{(2)}(\eta, q) = F_{\mathrm{NL}}^2 \int_0^\infty du_1 \int_0^\infty dv_1 \int_{-1}^1 d\cos\theta_1 \cdots \int_0^\infty du_3 \int_0^\infty dv_3 \int_{-1}^1 d\cos\theta_3 \int_0^{2\pi} d\phi_{12} \int_0^{2\pi} d\phi_{23} \cos 2\phi_{12} \frac{J^2(u_1,}{(u_1 v_1 u}$$

where we define $x = q\eta$, $s_i = u_i - v_i$, $t_i = u_i + v_i - 1$, and:

$$y_{ij} = \frac{t_i(t_i+2)(1-s_i^2) + t_j(t_j+2)(1-s_j^2) + 2\cos\phi_{ij}\sqrt{t_i(t_i+2)(1-s_i^2)t_j(t_j+2)(1-s_j^2)}}{2}$$

$$w_{ij} = u_i^2 + v_j^2 - y_{ij}$$

$$w_{123} = 1 + u_1^2 + v_1^2 + u_2^2 + v_2^2 + u_3^2 + v_3^2 + y_{12} - y_{13} - y_{23}$$

The oscillation average of squared $J(u, v, x \to \infty)$ has been given by Ref. [33] and earlier works [12, 13, 30, 31]:

$$\langle J(u_i, v_i, x \to \infty) J(u_j, v_j, x \to \infty) \rangle = \frac{9\pi^2}{64} \frac{(1-s_i^2)^2(1-s_j^2)^2}{(t_i(t_i+2))^3(t_j(t_j+2))^3} \times [\text{polynomial in } s_i, t_i, s_j, t_j]$$

The formulae in this appendix can be used for numerically computing the energy density of SIGWs in a self-consistent manner.

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