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Abstract

Neutron computed tomography (NCT) is widely used as a noninvasive measurement technique in nuclear engineering, thermal hydraulics, and cultural heritage. The neutron source intensity of NCT is usually low and the scan time is long, resulting in a projection image containing severe noise. To reduce the scanning time and increase the image reconstruction quality, an effective reconstruction algorithm must be selected. In CT image reconstruction, the reconstruction algorithms used can be divided into three categories: analytical algorithms, iterative algorithms, and deep learning. Because the analytical algorithm requires complete projection data, it is not suitable for reconstruction in harsh environments, such as strong radiation, high temperature, and high pressure. Deep learning requires large amounts of data and complex models, which cannot be easily deployed, as well as has a high computational complexity and poor interpretability. Therefore, this paper proposes the OS-SART-PDTV iterative algorithm, which uses the ordered subset simultaneous algebraic reconstruction technique (OS-SART) algorithm to reconstruct the image and the first-order primal-dual algorithm to solve the total variation (PDTV), for sparse-view NCT three-dimensional reconstruction. The novel algorithm was compared with other algorithms (FBP, OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV) by simulating the experimental data and actual neutron projection experiments. The reconstruction results demonstrate that the proposed algorithm outperforms the FBP, OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV algorithms in terms of preserving edge structure, denoising, and suppressing artifacts.

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Full Text

First-Order Primal-Dual Algorithm for Sparse-View Neutron Computed Tomography-Based Three-Dimensional Image Reconstruction

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Abstract

Neutron computed tomography (NCT) is widely used as a noninvasive measurement technique in nuclear engineering, thermal hydraulics, and cultural heritage. However, the neutron source intensity in NCT systems is typically low and scan times are long, resulting in projection images containing severe noise. To reduce scanning time while improving image reconstruction quality, an effective reconstruction algorithm must be selected. In CT image reconstruction, algorithms can be divided into three categories: analytical, iterative, and deep learning-based methods. Because analytical algorithms require complete projection data, they are unsuitable for reconstruction in harsh environments characterized by strong radiation, high temperature, and high pressure. Deep learning approaches require large amounts of data and complex models that cannot be easily deployed, and they suffer from high computational complexity and poor interpretability. Therefore, this paper proposes the OS-SART-PDTV iterative algorithm for sparse-view NCT three-dimensional reconstruction, which combines the ordered subset simultaneous algebraic reconstruction technique (OS-SART) for image reconstruction with a first-order primal-dual algorithm for solving total variation (PDTV). The novel algorithm was compared against FBP, OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV using both simulated experimental data and actual neutron projection experiments. The reconstruction results demonstrate that the proposed algorithm outperforms these comparative methods in preserving edge structure, denoising, and suppressing artifacts.

Keywords: NCT; First-order primal-dual algorithm; OS-SART; Total variation; Sparse-view

I. Introduction

Neutron tomography is a noninvasive measurement method that differs from X-ray imaging and muon tomography, and it has been applied in various fields with high accuracy and reliability [?]. The strong transmission of neutrons through metallic elements makes neutron tomography extremely useful for examining hydrogen in heavy elements under harsh experimental conditions such as strong radiation, high-temperature, and high-pressure two-phase flow systems. Additionally, neutrons are employed in boron neutron therapy owing to their ability to detect and reconstruct the three-dimensional (3D) distribution of boron concentration [?]. Neutron tomography systems consist of two components: a neutron radiography system and a computed tomography system capable of measuring and displaying the 3D void fraction in boiling flow within heated rod bundles. These systems have also been applied to simulate advanced nuclear reactor cores [?]. Asano and Takenaka [?] used neutron tomography to determine the void fraction distribution in two-phase air-water flows. Kureta et al. (2008) visualized toroidal streaming in a heated fuel rod bundle using 3D neutron tomography [?]. Tremsin et al. (2013) employed neutron imaging to assess the attachment, deposition, and structural integrity of fuel core blocks [?]. Andersson et al. (2015) developed a portable fast NCT system to obtain the void fraction distribution in fuel rod bundles in a boiling water reactor [?]. Although portable NCT systems can perform measurements at any location, their neutron generators exhibit extremely low neutron yields. Therefore, promoting the rapid development of neutron tomography to improve the efficiency and safety of nuclear fuels represents an important research direction.

The most crucial component of a neutron tomography system is the 3D reconstruction algorithm, which directly determines image quality and measurement reliability. Because neutron CT scanning times are long (ranging from 30 minutes to several hours) and practical application environments are complex, obtaining complete projection data is difficult. Consequently, reconstruction algorithms that can efficiently utilize sparse-view projection data are required to obtain accurate measurement results. Furthermore, sparse-view projection data reconstruction is valuable for investigations in harsh environments such as those with strong radiation. Currently, various algorithms are used for reconstructing sparse-view and highly undersampled projection data. Therefore, this study focuses on developing a 3D reconstruction algorithm for sparse-view NCT.

Sparse-view reconstruction is a highly complex mathematical problem and represents a significant research direction in the image reconstruction field. Because prior information can be embedded in iterative reconstruction algorithms, they are considered the most likely candidates to replace the FBP algorithm for sparse-view reconstruction. With the development of deep learning, it has been applied to the field of tomographic reconstruction, and many researchers have achieved significant results in sparse-view tomography. As proposed in [?], the DEAR model, which adds a priori information and data in the image domain to the compressed sensing-based variational model, can eliminate reconstruction artifacts while improving image quality. A multiple adversarial learning angiography image reconstruction framework has been proposed in the literature [?], which addresses the challenge of low-intensity aortic reconstruction by introducing dual-correlation constrained adversarial learning; its application to clinical data demonstrates feasibility and effectiveness.

However, due to practical limitations, this study focuses on neutron CT 3D reconstruction, and it was not possible to obtain a large number of neutron projection images for supervised training. Therefore, this study concentrates on an iterative algorithm for neutron CT 3D reconstruction. Currently, mainstream iterative reconstruction algorithms include ART [?], SIRT [?], and SART [?]. Although these iterative algorithms can reconstruct high-quality images, when projection data are extremely sparse and no a priori information is introduced, the reconstructed images exhibit significant artifacts. Donoho [?] proposed compressive sensing (CS) theory, which has promoted rapid development in sparse-view image reconstruction. According to the mathematical implications of CS theory, an image may be reconstructed exactly if it is sparse or can be sparsely represented through spatial transformation. The evolution of CS theory has facilitated the application of total variation algorithms in sparse-view image reconstruction [?]. Rudin proposed the ROF denoising model, which reduces noise while preserving image edge and detailed structural information by using total variation as a constraint for image denoising [?]. Several TV-based techniques and variations have been proposed to improve algorithm performance, including directional TV [?], weighted TV [?], edge-preserving TV [?], and weight TV [?]. The total variation model is a typical regularization model. Total variation minimization is particularly important in CT imaging because of its ability to resolve acute discontinuities, which is critical for many imaging problems since sample edges contain essential structural information about the object. However, due to the non-smooth character of total variation, achieving total variation regularization minimization is challenging. Furthermore, one drawback of these models is their assumption that the image is piecewise constant, which can destroy significant image information. The total variation image denoising problem belongs to a class of convex optimization problems. To solve convex optimization problems involving total variation, second-order methods exhibit good convergence and require only a small number of iterations. However, each iteration of second-order methods is extremely complicated, making them difficult to apply to large-scale problems. In contrast, first-order methods involve only function values and gradient information, and each iteration is relatively simple. In cases of gradient magnitude sparsity, minimizing the total variation of intermediate images yields high-quality reconstructed images. Therefore, this study employs a first-order method to address the total variation image denoising problem and proposes an algorithm applicable to 3D NCT reconstruction from sparse views.

The goal of this study was to identify an efficient 3D reconstruction algorithm for sparse-view NCT. To address artifacts and noise in images reconstructed from sparse-view projections, we propose the ordered subset simultaneous algebraic reconstruction technique (OS-SART-PDTV) algorithm, which uses OS-SART for image reconstruction and a first-order primal-dual algorithm to solve total variation (PDTV). This novel algorithm achieves a blurred version of a piecewise constant object through phenomenological modeling and reconstructs high-quality 3D images by minimizing the total variation of intermediate images through gradient-amplitude sparsity, which allows the interior of reconstructed images to change quickly and smoothly. The proposed method, including the algorithm and NCT fundamentals, is presented in Section 2. Simulation experiments and a real neutron projection experiment are discussed in Section 3. Section 4 discusses the research process and presents conclusions. Finally, a brief conclusion is presented in Section 5.

II. Principles and Algorithms of NCT

2.1 NCT System

This section describes the fundamentals of CT image reconstruction. In general, CT image reconstruction generates a 3D volume from a 2D collection of projected images using mathematical algorithms. This requires developing a measurement model using mathematical methods to relate measured data to desired physical properties. Image reconstruction consists of two processes: forward projection and back projection. Forward projection enables calculation of model measurements corresponding to actual measurements that match physical properties. Model measurements corresponding to actual measurements are calculated using forward projection, which refers to mapping an object onto the projection domain. Back projection is the opposite operation, determining physical properties from measurements. These two operations largely determine image reconstruction precision. The geometry of forward projection depends on the radiation beam geometry. Based on the selected system, radiation source, and available data acquisition systems, the appropriate beam geometry is selected to obtain reliable measurements. The NCT system primarily includes a collimator and shield, rotating sample stage, and neutron imaging system. The neutron imaging system includes a 2D neutron image detector, conversion screen, and 3D reconstruction software. The algorithm used for 3D reconstruction directly determines the imaging quality and measurement reliability of the system and represents its core component. A schematic of the NCT scanning system is shown in Figure 1 [Figure 1: see original paper].

Fig. 1 Schematic of the NCT system

In NCT systems, the mass of the neutron source and noise generated by electronics can cause projection data to contain noise. Therefore, Poisson noise and Gaussian noise are added to numerical projection data during reconstruction to validate the denoising capability of the algorithm while testing its reliability [**?**]. Consequently, these two noise components must be considered during NCT. The noise model is given by:

$$I_{\text{noisy}} = \text{Poisson}(I_0 \exp(-p)) + \mathcal{N}(\mu_{\text{dark}}, \sigma^2)$$

where I_{noisy} denotes noisy transmission data, I_0 represents the mean photon number, μ_{dark} indicates the average value of the electronic noise, and σ^2 is the noise variance. System calibration typically estimates μ_{dark} via dark current measurements [**?**].

2.2.1 CT Projection Acquisition

In NCT, the geometry of the neutron beam is primarily parallel. Among the mathematical concepts, the parallel geometric beam is the simplest. An object can be represented by a function $f(x, y)$ in the plane. The projection represents the decay integral of the neutron beam as it passes through the object from the neutron source, which is considered to be the line integral along each ray. The expression of the projection is given by the Radon transform:

$$b(s, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos\theta + y \sin\theta - s) \, dx \, dy$$

where L represents a neutron beam passing straight through an object, s represents the detector coordinates in the projection domain, and θ denotes the counterclockwise rotation angle of the x-axis in the projection domain. The projection consists of a group of integrals of the neutron attenuation coefficients of the sample measured at the detector for every given angle.

2.2.2 Central Slice Theorem

The Central Slice Theorem establishes a connection between the 2D projection of an object and its 2D Fourier transform. The 1D Fourier transform of the projection function $b(s, \theta)$ in a certain direction is the value of the 2D Fourier transform of the density function $f(x, y)$ in Fourier space along the same direction on a straight line passing through the origin. The 1D Fourier transform of the projection is defined as:

$$B(\omega, \theta) = \int_{-\infty}^{\infty} b(s, \theta) e^{-i\omega s} \, ds$$

According to the Central Slice Theorem, this equals the 2D Fourier transform of $f(x, y)$ evaluated at $(\omega \cos\theta, \omega \sin\theta)$:

$$B(\omega, \theta) = F(\omega \cos\theta, \omega \sin\theta)$$

where $F(u, v)$ is the 2D Fourier transform of $f(x, y)$. Several problems arise when the central slice theorem is used for transform reconstruction. First, the theorem produces data in Fourier space that are inconsistent with Cartesian space, requiring interpolation into Cartesian coordinates. However, interpolation in Fourier space before the inverse Fourier transform can have a significant influence on the reconstruction. The second problem is the difficulty in performing targeted reconstruction and identifying fine structures within small areas.

2.3.1 TV Algorithm

In NCT, measurement time can be drastically reduced by decreasing the number of projection views; however, this leads to sparse sampling. Based on CS theory, a signal can be reconstructed exactly if it is sparse or can be represented sparsely. Under ideal conditions, NCT reconstruction can be converted to solving a linear system problem; thus, the discrete model of CS theory may be represented as:

$$\mathbf{b} = \mathbf{A}\mathbf{g}$$

where $\mathbf{A}$ stands for the system matrix, $\mathbf{b}$ denotes the projection data obtained at each angle, and $\mathbf{g}$ represents the image to be reconstructed. Mathematically, solving this system usually converts it into a least squares problem:

$$\arg\min_{\mathbf{g}} \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2$$

However, in sparse-view image reconstruction, the incompleteness of projection data leads to reconstructed images containing noise and artifacts. Therefore, various regularization models have been proposed to obtain approximate solutions:

$$\arg\min_{\mathbf{g}} \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2 + \lambda R(\mathbf{g})$$

where $\|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2$ represents the data fidelity term, $R(\mathbf{g})$ denotes the regularization term, and λ is the regularization parameter used to balance the regularization and data terms. The total variation regularization term is defined as:

$$\mathrm{TV}(\mathbf{g}) = \sum_{i,j,k} \|\nabla \mathbf{g}_{ijk}\| = \sum_{i,j,k} \sqrt{(\nabla_x \mathbf{g}_{ijk})^2 + (\nabla_y \mathbf{g}_{ijk})^2 + (\nabla_z \mathbf{g}_{ijk})^2}$$

where $\mathbf{g}$ denotes the 3D image, $\nabla \mathbf{g}_{ijk}$ represents the gradient at voxel (i, j, k), and $\| \cdot \|$ denotes the Euclidean norm.

2.3.2 First-Order Primal-Dual Algorithm

Owing to the piecewise smoothness of the regularization term in the variational model, the total variational model can effectively preserve image edges during image recovery and achieve improved recovery results. Therefore, the total variational model, particularly the total variational regularization model [**?**] proposed by Rudin, Osher, and Fatemi, has gained popularity. According to the ROF model, the discrete total variational model of 3D images can be formulated as follows [**?**]:

$$\min_{\mathbf{g}} \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2 + \lambda \mathrm{CTV}(\mathbf{g})$$

where λ is the regularization parameter, and $\mathrm{CTV}(\mathbf{g})$ is the regularization term of the discrete total variational model. To obtain a stable solution to the minimization problem, we consider the constrained formulation:

$$\min_{\mathbf{g}} \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2 \quad \text{subject to} \quad \mathrm{CTV}(\mathbf{g}) \leq \tau$$

The expression of the regularization term of the total variational model using finite differences is:

$$\mathrm{CTV}(\mathbf{g}) = \sum_{i,j,k} \sqrt{(\nabla_x \mathbf{g}_{ijk})^2 + (\nabla_y \mathbf{g}_{ijk})^2 + (\nabla_z \mathbf{g}_{ijk})^2}$$

where $\nabla_x, \nabla_y, \nabla_z$ denote the forward difference operators in the x, y, and z directions, respectively. In this study, the above model was improved using the finite difference matrix representation. Thus, the expression of the model investigated in the present study can be obtained as follows:

$$\min_{\mathbf{g}} \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2 + \lambda \|\nabla \mathbf{g}\|_1$$

Although the total variational model efficiently preserves image edges, it is difficult to obtain the minimum value directly because of the non-smooth nature of the total variational model [**?**]. Because the first-order primal-dual algorithm is an efficient method for solving non-smooth convex optimization problems in imaging, we employed it to solve this problem.

Definition 1: A point $(\mathbf{g}^*, \mathbf{p}^*)$ is a saddle point of the functional $J(\mathbf{g}, \mathbf{p})$ if for all $(\mathbf{g}, \mathbf{p})$ we have:

$$J(\mathbf{g}^*, \mathbf{p}) \leq J(\mathbf{g}^*, \mathbf{p}^*) \leq J(\mathbf{g}, \mathbf{p}^*)$$

Chambolle and Pock suggested a first-order primal-dual algorithm to solve the following saddle-point problem [**?**]:

$$\min_{\mathbf{g}} \max_{\mathbf{p}} \langle \nabla \mathbf{g}, \mathbf{p} \rangle + F(\mathbf{g}) - G^*(\mathbf{p})$$

where F and G^* are convex functions, and ∇ denotes the gradient operator. The first-order primal-dual iterations are as follows [**?**, **?**]:

$$\mathbf{p}^{n+1} = \text{prox}_{\sigma G^*}(\mathbf{p}^n + \sigma \nabla \bar{\mathbf{g}}^n)$$

$$\mathbf{g}^{n+1} = \text{prox}_{\tau F}(\mathbf{g}^n - \tau \nabla^* \mathbf{p}^{n+1})$$

$$\bar{\mathbf{g}}^{n+1} = \mathbf{g}^{n+1} + \theta(\mathbf{g}^{n+1} - \mathbf{g}^n)$$

where $\tau, \sigma > 0$ denote the iteration steps for the primal and dual variables, respectively; θ is the combination parameter used to ensure algorithm convergence.

Definition 2: The convex conjugate of a function f is defined as:

$$f^*(\mathbf{y}) = \sup_{\mathbf{x}} \langle \mathbf{x}, \mathbf{y} \rangle - f(\mathbf{x})$$

According to this definition, the problem can be converted into a min-max problem [**?**] as follows:

$$\min_{\mathbf{g}} \max_{\mathbf{p}} \mathcal{L}(\mathbf{g}, \mathbf{p}) = \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2 + \lambda \langle \nabla \mathbf{g}, \mathbf{p} \rangle - \delta_P(\mathbf{p})$$

where δ_P is the indicator function of the set $P = \{\mathbf{p} : \|\mathbf{p}_{ijk}\| \leq 1\}$. The iterative formula can be obtained by using a first-order primal-dual algorithm:

$$\mathbf{p}^{n+1} = \text{proj}_P(\mathbf{p}^n + \sigma \nabla \bar{\mathbf{g}}^n)$$

$$\mathbf{g}^{n+1} = (\mathbf{I} + \tau \mathbf{A}^T \mathbf{A})^{-1}(\mathbf{g}^n - \tau \nabla^* \mathbf{p}^{n+1} + \tau \mathbf{A}^T \mathbf{b})$$

$$\bar{\mathbf{g}}^{n+1} = \mathbf{g}^{n+1} + \theta(\mathbf{g}^{n+1} - \mathbf{g}^n)$$

The minimization problem for the subproblem $\mathbf{g}$ can be converted into solving a linear system:

$$(\mathbf{I} + \tau \mathbf{A}^T \mathbf{A})\mathbf{g}^{n+1} = \mathbf{g}^n - \tau \nabla^* \mathbf{p}^{n+1} + \tau \mathbf{A}^T \mathbf{b}$$

For the dual subproblem, the projection onto set P is given by:

$$\text{proj}_P(\mathbf{p})_{ijk} = \frac{\mathbf{p}_{ijk}}{\max(1, \|\mathbf{p}_{ijk}\|)}$$

Table 1 First-order primal-dual algorithm

Input: Projection data $\mathbf{b}$, system matrix $\mathbf{A}$, parameters $\lambda, \tau, \sigma, \theta$
Initialize: $\mathbf{g}^0 = 0$, $\mathbf{p}^0 = 0$, $\bar{\mathbf{g}}^0 = \mathbf{g}^0$
Repeat:
1. Update dual variable: $\mathbf{p}^{n+1} = \text{proj}_P(\mathbf{p}^n + \sigma\nabla\bar{\mathbf{g}}^n)$
2. Update primal variable: $\mathbf{g}^{n+1} = (\mathbf{I} + \tau\mathbf{A}^T\mathbf{A})^{-1}(\mathbf{g}^n - \tau\nabla^*\mathbf{p}^{n+1} + \tau\mathbf{A}^T\mathbf{b})$
3. Extrapolation: $\bar{\mathbf{g}}^{n+1} = \mathbf{g}^{n+1} + \theta(\mathbf{g}^{n+1} - \mathbf{g}^n)$
4. Calculate the deviation value RD

Until reaching the maximum number of iterations
Output: Reconstructed image $\mathbf{g}$

where s is the iteration step of the primal variable, t is the iteration step of the dual variable, and k is the number of iterations.

2.3.3 Adaptive Weighted Total Variation Algorithm

The conventional TV term is based on the assumption that the reconstructed image is piecewise constant, which can cause excessive smoothing of reconstructed image edges. To alleviate this over-smoothing problem, many researchers have studied weighted adaptive total variation [**?**]; therefore, in this study, an adaptive weighted total variation (AwTV) minimization image model was used for comparison with the proposed algorithm:

$$\min_{\mathbf{g}} \|\mathbf{A}\mathbf{g} - \mathbf{b}\|^2 + \lambda \text{AwTV}(\mathbf{g})$$

subject to $\mathbf{g} \geq 0$, where AwTV denotes the adaptive weight of the total variance of the reconstructed image. The AwTV is defined as:

$$\text{AwTV}(\mathbf{g}) = \sum_{i,j,k} w_{ijk} \|\nabla\mathbf{g}_{ijk}\|$$

with the weight factor w_{ijk} defined as:

$$w_{ijk} = \frac{1}{\|\nabla\mathbf{g}_{ijk}\| + \epsilon}$$

where ϵ is a small constant to avoid division by zero.

2.3.4 Fast Gradient Projection Algorithm

Beck and Teboulle proposed a fast gradient projection algorithm [**?**], derived as follows. The model expression to solve the TV-based denoising problem is:

$$\min_{\mathbf{g}} \|\mathbf{g}-\mathbf{f}\|^2 + \lambda \mathrm{TV}(\mathbf{g})$$

where $\mathbf{f}$ is the noisy image. The TV function is nonsmooth, leading to significant difficulties in solving this problem. To address this challenge, Chambolle proposed a dual formulation. The dual problem is constructed by introducing the dual variable $\mathbf{p} \in P$, where P is defined as:

$$P = \{\mathbf{p} : \|\mathbf{p}_{ijk}\| \leq 1 \text{ for all } i, j, k\}$$

The dual problem becomes:

$$\max_{\mathbf{p}\in P} -\|\mathrm{div}\mathbf{p} - \mathbf{f}\|^2$$

The gradient projection algorithm is an effective method for solving this denoising problem. The projection onto set P can be computed simply as:

$$\mathrm{proj}_P(\mathbf{p})_{ijk} = \frac{\mathbf{p}_{ijk}}{\max(1, \|\mathbf{p}_{ijk}\|)}$$

The fast gradient projection algorithm uses an accelerated scheme to achieve faster convergence.

2.3.5 OS-SART Algorithm

In 1984, Andersen and Kak proposed an improved SART. The SART improves upon ART algorithms by calculating the error of all rays passing through a pixel at the same projection angle. While ART uses only one ray per iteration, SART corrects all rays at the same angle, thereby reducing noise introduced by the ART algorithm. Its formula is:

$$\mathbf{g}^{k+1} = \mathbf{g}^k + \lambda \mathbf{A}^T \frac{\mathbf{b} - \mathbf{A}\mathbf{g}^k}{\mathbf{A}\mathbf{1}}$$

The OS-SART is a combination of ordered subsets and the SART algorithm, and its formula is:

$$\mathbf{g}^{k+1} = \mathbf{g}^k + \lambda \mathbf{A}_s^T \frac{\mathbf{b}_s - \mathbf{A}_s\mathbf{g}^k}{\mathbf{A}_s\mathbf{1}}$$

where s denotes the subset index, $\mathbf{A}_s$ is the system matrix for subset s, $\mathbf{b}_s$ is the projection data for subset s, k denotes the number of iterations, and λ is the relaxation factor.

2.3.6 Proposed Algorithm

Based on the OS-SART iterative algorithm and first-order primal-dual algorithm with total variation denoising, we propose a new algorithm for sparse-view NCT 3D image reconstruction (OS-SART-PDTV). The iterative process of this algorithm contains two loops: the outer and inner loops correspond to OS-SART for image reconstruction and PDTV for image denoising.

Table 2 OS-SART-PDTV algorithm

1. **While** the stop criterion is not met
   Step 1: OS-SART reconstruction
   2. Initialize: $\mathbf{g}^0 = 0$
   3. **For** each subset s
   4. $\mathbf{g}^{n+1} = \mathbf{g}^n + \lambda \mathbf{A}_s^T \frac{\mathbf{b}_s - \mathbf{A}_s \mathbf{g}^n}{\mathbf{A}_s \mathbf{1}}$
   5. Apply non-negativity constraint: $\mathbf{g}^{n+1} = \max(\mathbf{g}^{n+1}, 0)$
   6. **End for**
   Step 2: PDTV denoising
   7. Initialize dual variable $\mathbf{p}^0 = 0$
   8. **While** the stop criterion is not met
   9. Update dual variable: $\mathbf{p}^{k+1} = \text{proj}_P(\mathbf{p}^k + \sigma \nabla \mathbf{g}^n)$
   10. Update primal variable: $\mathbf{g}^{n+1} = (\mathbf{I} + \tau \mathbf{A}^T \mathbf{A})^{-1} (\mathbf{g}^n - \tau \nabla^* \mathbf{p}^{k+1} + \tau \mathbf{A}^T \mathbf{b})$
   11. **End while**
2. **End while**
3. **Output:** Final reconstructed image $\mathbf{g}$

III. Experiment

3.1 Quantitative Evaluation Index

In this study, we used four image evaluation metrics to quantitatively analyze the quality of the reconstructed images for each algorithm. The accuracy of image reconstruction was quantitatively analyzed using the correlation coefficient (CC) metric, which is defined as follows:

$$\mathrm{CC} = \frac{\sum_{v=1}^{M}(\mathrm{true}_v - \overline{\mathrm{true}})(\mathrm{rec}_v - \overline{\mathrm{rec}})}{\sqrt{\sum_{v=1}^{M}(\mathrm{true}_v - \overline{\mathrm{true}})^2}\sqrt{\sum_{v=1}^{M}(\mathrm{rec}_v - \overline{\mathrm{rec}})^2}}$$

where true_v is the reference image to be reconstructed, M denotes the number of voxels, and rec_v represents the reconstructed value at voxel v. When the reconstructed image matches the reference image exactly, the CC value equals 1.

The mean structural similarity index (MSSIM) provides a comprehensive evaluation of the reconstructed image by comparing differences between the reference image and reconstructed image in terms of brightness, contrast, and structure:

$$\mathrm{MSSIM} = \frac{1}{N}\sum_{i=1}^{N}\mathrm{SSIM}(\mathbf{x}_i, \mathbf{y}_i)$$

where the SSIM for each window is:

$$\mathrm{SSIM}(\mathbf{x}, \mathbf{y}) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

RMSE is defined by:

$$\mathrm{RMSE} = \sqrt{\frac{1}{M}\sum_{v=1}^{M}(\mathrm{true}_v - \mathrm{rec}_v)^2}$$

A higher RMSE value indicates a larger error between the reconstructed and reference images.

Universal quality image (UQI) is a widely used image evaluation index, defined as:

$$\mathrm{UQI} = \frac{4\sigma_{\mathrm{true,rec}}\mu_{\mathrm{true}}\mu_{\mathrm{rec}}}{(\sigma_{\mathrm{true}}^2 + \sigma_{\mathrm{rec}}^2)(\mu_{\mathrm{true}}^2 + \mu_{\mathrm{rec}}^2)}$$

where $\mu_{\mathrm{true}}, \mu_{\mathrm{rec}}$ are the means, $\sigma_{\mathrm{true}}^2, \sigma_{\mathrm{rec}}^2$ are the variances, and $\sigma_{\mathrm{true,rec}}$ is the covariance. The UQI value ranges from 0 to 1 and increases with similarity.

3.2 Digital 3D Shepp–Logan Model Experiment

In the digital simulation experiment, we compared and analyzed the performance of five algorithms—FBP, OS-SART-TV, OS-SART-AwTV, OS-SART-FGPTV, and OS-SART-PDTV—by reconstructing projection data from the 3D Shepp–Logan model. The dimensions of the 3D Shepp–Logan model were $256 \times 256 \times 256$. An iterative algorithm must use reasonable iterative parameters to obtain an ideally reconstructed image. The following parameters were used:

- **OS-SART-TV:** $\lambda = 2.7$, iterations = 300
- **OS-SART-AwTV:** $\lambda = 2.5$, iterations = 300, $\epsilon = 0.01$
- **OS-SART-FGPTV:** $\lambda = 2.2$, iterations = 100
- **OS-SART-PDTV:** $\lambda = 0.006$, iterations = 150, $\tau = 0.0032$, $\sigma = 2.2$, $\theta = 1$

To verify the denoising capability, Poisson noise with incident flux $I_0 = 300$ and Gaussian noise with $\sigma = 0.01$ were added according to the NCT system noise model.

Fig. 2 shows the relationship between the RMSE of the reconstructed images using the OS-SART-PDTV algorithm and the number of iterations for different sparse views (28 and 38 projection views), demonstrating that the OS-SART-PDTV algorithm converges to a stable solution after approximately 50 iterations.

Fig. 2 RMSE versus iteration steps for the OS-SART-PDTV algorithm from different sparse views: (a) 28; (b) 38.

Fig. 3 [Figure 3: see original paper] shows the reconstruction results with different numbers of projected views using the five algorithms. As illustrated in Fig. 3, the smoothness and sharpness of images reconstructed using all five algorithms improved rapidly as the number of projection views increased. Compared to the other three iterative algorithms, images reconstructed using the FBP algorithm showed severe streak artifacts. The reconstructed images from the four iterative algorithms were smoother and had sharper boundaries than those from the FBP algorithm. As shown in the enlarged ROI in Fig. 3, the OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV algorithms can effectively reduce artifacts; however, part of the edge structure is not accurately reconstructed. In contrast, the novel algorithm can not only maintain good reconstruction performance but also reconstruct finer structures.

Fig. 3. Sliced images at z=130 for four algorithms reconstructing images with different numbers of projection views.

Additionally, we compared and analyzed the error images of the reconstructed images using different algorithms, which reflect the difference between pixel values of the reconstructed and reference images. According to the error image in Fig. 4 [Figure 4: see original paper], the image reconstructed using the FBP algorithm lost many features and produced numerous artifacts, indicating that this algorithm is unsuitable for sparse-view NCT 3D reconstruction. The other four iterative algorithms performed better at suppressing artifacts and reconstructing detailed structural information. The OS-SART-PDTV algorithm exhibits much less detail loss in the error image than the other algorithms.

We plotted the profile of the reconstructed images for each algorithm in Fig. 5 [Figure 5: see original paper] to further evaluate performance. The number of projection views from left to right was 28, 38, and 38+, respectively. Here, 38+ indicates that the noise model was added to the 3D Shepp-Logan model. As shown in Fig. 5, there was large fluctuation in the FBP profile line with the greatest deviation from the true pixel value. Although the OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV algorithms performed better than the FBP algorithm, there remained a gap between their amplitudes and true pixel values. The profile of OS-SART-PDTV matches the reference value best and is closest to the true value, indicating that the OS-SART-PDTV algorithm is well-suited for sparse-view NCT 3D reconstruction.

Fig. 4. Slice of error images at z=130

Fig. 5. Horizontal profiles of the 3D Shepp–Logan model at z =130th slices. (a) 28; (b) 38; (c) 38+.

As shown in Fig. 6 [Figure 6: see original paper], we used four image evaluation metrics to quantitatively analyze the reconstructed images from each algorithm. First, the evaluation indices of images reconstructed using noiseless projection data were analyzed. From the CC, MSSIM, and UQI histograms shown in Figure 6, the values of these three metrics for reconstructed images from each algorithm gradually increased as the number of projection views increased, exhibiting similar characteristics. According to the RMSE histogram in Fig. 6c, the reconstructed images from OS-SART-PDTV had the smallest RMSE values compared to the other algorithms for the same projection views. The RMSE of reconstructed images for each algorithm gradually decreased as the number of projection views increased. The RMSE value of images reconstructed using the OS-SART-PDTV algorithm at 28 projection views was 0.03352, indicating that the reconstructed image was closest to the real image and that the quality of reconstructed images was highest. As shown in the UQI histogram in Fig. 6d, the UQI values of reconstructed images for each algorithm gradually increased as the number of projected views increased. The trend of UQI was opposite to that of RMSE. The reconstructed images from the OS-SART-PDTV algorithm had the highest UQI values for the same number of projection views, demonstrating its superiority. The UQI value of the image reconstructed using the OS-SART-PDTV algorithm was 0.99039 for 28 projection views. We then quantitatively analyzed four evaluation metrics for images reconstructed using data containing noisy projections. From the CC, MSSIM, and UQI histograms, it can be seen that the OS-SART-PDTV algorithm achieved the largest evaluation metrics for these three reconstructed images when reconstructing the same number of noise-containing projections. According to the RMSE histogram, the reconstructed image from the OS-SART-PDTV algorithm had the lowest

RMSE value. Therefore, based on both quantitative and visual analyses of each algorithm, the OS-SART-PDTV algorithm produces the highest-quality reconstructed images for the same number of projection views.

Fig. 6. Histogram of reconstructed image evaluation metrics for the 3D Shepp-Logan model. (a) CC; (b) MSSIM; (c) RMSE; (d) UQI.

3.3 Digital Head Model Experiment

We further analyzed the performance of these five algorithms by reconstructing a digital head model. The reconstructed images from the five algorithms using different numbers of projection views are shown in Fig. 7 [Figure 7: see original paper]. Based on the reconstructed images, the smoothness and sharpness of all five algorithms improved significantly as the number of projected views increased. Compared to the other four algorithms, images reconstructed using the FBP algorithm showed severe streak artifacts. The reconstructed images from the four iterative algorithms were smoother and had cleaner image boundaries than those from the FBP algorithm. As shown in the enlarged ROI in Fig. 7, the OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV algorithms are effective in reducing artifacts, but the edge structure of the image is not accurately reconstructed. In contrast, the proposed OS-SART-PDTV algorithm can not only reduce artifacts but also reconstruct more detailed structural information.

To further evaluate algorithm performance, the contours of reconstructed images for each algorithm are plotted in Fig. 8 [Figure 8: see original paper]. As shown in Fig. 8, the numbers of projected views from left to right are 28, 38, and 38+, where 38+ indicates that noise has been added to the projection data. As shown in Fig. 8, there was large fluctuation in the FBP profile, which had the largest gap from the true pixel values. Although the OS-SART-TV, OS-SART-AwTV, and OS-SART-FGPTV algorithms performed better than the FBP algorithm, there remained a large deviation between actual and true pixel values. The profile of OS-SART-PDTV matches the reference value best and is closest to the true value, indicating that the OS-SART-PDTV algorithm is well-suited for sparse-view NCT 3D reconstruction.

According to the RMSE histogram of reconstructed images from each algorithm shown in Fig. 9 [Figure 9: see original paper], the OS-SART-PDTV algorithm had the smallest RMSE values. As the number of projected views gradually increased, the RMSE of reconstructed images for each algorithm gradually decreased. When the number of projected views was 28, the RMSE value of the OS-SART-PDTV algorithm-reconstructed image was 0.02404, indicating that the reconstructed image from this algorithm was closest to the real image and that the quality of reconstructed images was highest.

The UQI histogram in Fig. 9d shows that the UQI values of reconstructed images for each algorithm increased as the number of projected views increased. The UQI trend was opposite to that of RMSE. The OS-SART-PDTV algorithm reconstructs images with the largest UQI value for the same number of projected views, indicating that this algorithm outperforms the other three iterative algorithms. When the number of projected views was 28, the UQI of the image reconstructed by the PDTV algorithm was 0.98594. We then quantitatively evaluated images containing noisy projections reconstructed using the four algorithms. The OS-SART-PDTV algorithm reconstructs images with the largest CC, MSSIM, and UQI values for the same number of projected views. According to the RMSE histogram, the RMSE value of the reconstructed image obtained using OS-SART-PDTV was the smallest. Therefore, through quantitative and visual analyses of each algorithm, OS-SART-PDTV exhibited superior performance in sparse-view reconstruction.

To analyze the relationship between the OS-SART-PDTV algorithm and regularization parameters, we present reconstructed images with different regularization parameters in Fig. 10 [Figure 10: see original paper]. Based on the reconstructed images, an optimal image was obtained by adjusting the regularization parameters when reconstructing the head model. According to the reconstructed images with different regularization parameters, it can be seen that reconstructed image quality reaches its best when the regularization parameters reach a certain value as the reconstruction parameters gradually increase. As the regularization parameter continued to increase, the quality of reconstructed images decreased.

Fig. 7. Sliced images at z=70 for four algorithms reconstructing images with different numbers of projection views.

Fig. 8. Horizontal profiles of the digital head model at z=250th slices. (a) 28; (b) 38; (c) 38+.

Fig. 9. Histogram of reconstructed image evaluation metrics for the digital head model. (a) CC; (b) MSSIM; (c) RMSE; (d) UQI.

Fig. 10. Reconstructed images with different regularization parameters for the OS-SART-PDTV algorithm 38+

3.4 Real Neutron Projection Experiment

To further validate the performance of this novel algorithm in practical applications, the algorithm's performance was verified using projection data from an NCT-based clock model (Fig. 11) provided by Burkhard Schillinger of the Technical University of Munich, Germany [?].

Fig. 11. Clock Model

Schillinger et al. provided 201 neutron projection images captured at equal angles in the range of 0° to 180°. They also provided two dark field and two open field background images. The background images were obtained using the CCD camera with the neutron beam turned off, which contained readout noise as well as electronic noise caused by dark currents. Open-field images were obtained using a CCD camera with neutrons turned on and without the samples.

As shown in Fig. 12 [Figure 12: see original paper], the artifacts in images reconstructed by the FBP algorithm gradually increase as the number of projected views decreases. When the number of projected views is 45, the four iterative algorithms outperform the FBP algorithm in suppressing artifacts and reconstructing fine structural information. Although the OS-SART-TV reconstructs images with fewer artifacts, the edges of reconstructed images are too smooth, resulting in loss of detailed structural information. While OS-SART-AwTV and OS-SART-FGPTV algorithms can also reconstruct many image details, their reconstruction results are poorer compared with the OS-SART-PDTV algorithm that reconstructs the same number of sparse views. Visually, the OS-SART-PDTV algorithm outperforms other algorithms in terms of noise removal, artifact suppression, and retention of detailed structural information.

The performances of the three iterative algorithms were quantitatively analyzed using four image-evaluation metrics, as shown in Fig. 13 [Figure 13: see original paper]. In Fig. 13, the RMSE of the image reconstructed by the OS-SART-PDTV algorithm is the smallest, indicating minimal error between the reference image and reconstructed image, whereas the image reconstructed by the OS-SART-PDTV algorithm has the highest CC, MSSIM, and UQI values, indicating it is most similar to the reference image. In summary, the OS-SART-PDTV algorithm performed well in denoising, suppressing artifacts, and reconstructing fine structures when reconstructing projection data from sparse NCT scans, demonstrating that this novel algorithm can be applied to 3D reconstruction of sparse-view NCT.

Fig. 12. Sliced images at z=201 for four algorithms reconstructing images with different numbers of projection views.

Fig. 13. Histogram of reconstructed image evaluation metrics for the clock model. (a) CC; (b) MSSIM; (c) RMSE; (d) UQI.

Fig. 14 [Figure 14: see original paper] presents the profiles of reconstructed images. According to the profile, the OS-SART-PDTV algorithm has an advantage over the other algorithms, and its profile is closest to the reference.

Fig. 14. The profile of reconstructed images by FBP, OS-SART-TV, OS-SART-AwTV, OS-SART-FGPTV, and OS-SART-PDTV at z=201. The number of projection views is (a) 45; (b) 90; (c) 135.

IV. Discussion

Herein, we proposed a 3D reconstruction algorithm for sparse-view NCT. Because the analytical algorithm (FBP) has inherent defects that make it unsuitable for sparse-view reconstruction, we employed an iterative algorithm applied to 3D reconstruction of sparse-view NCT. Furthermore, iterative algorithms can embed a priori information to regularize the image. Therefore, TV-based iterative algorithms are widely used in sparse-view CT image reconstruction because they are superior to other algorithms in artifact suppression and denoising. By reconstructing projection data from the 3D Shepp-Logan, head, and clock models, the OS-SART-PDTV algorithm outperformed the other algorithms in both quantitative and visual analyses. Therefore, OS-SART-PDTV is suitable for sparse-view NCT.

In addition, deep learning represents an important research direction in the field of image reconstruction and has led to many significant results in sparse-view reconstruction, such as in [**?**], which used short C-arm CT scans. This study proposed the DRONE model, which uses a codec network in the embedding module to extract deep features in the data and image domains, combined with a Wasserstein distance generative adversarial network to maintain details and features in the image domain. It combines data residual and image residual networks in the refinement module to restore fine structural features from the output of the embedding module. According to the CS iterative reconstruction model, the deep prior of the data and image domains is regularized, thus ensuring the robustness of the DRONE network and making the DRONE model effective at retaining image edge information, recovering image features, and reconstructing accurate image information in practical applications.

Generally, in practical NCT scanning systems, projected images are affected by photon statistical noise and electrical noise. Therefore, several iterative reconstruction algorithms cannot achieve satisfactory results in NCT scanning systems. To further test our algorithm's performance, we reconstructed projection data obtained from the NCT scan clock model using the OS-SART-PDTV algorithm. Based on the results of reconstructed neutron projection data, OS-SART-PDTV exhibited good performance in denoising, suppressing artifacts, and reconstructing fine structures.

According to the results of the 3D Shepp-Logan model reconstruction, the OS-SART-PDTV algorithm converged monotonically to a stable solution (Fig. 3). Iterative reconstruction algorithms generally need to optimize several parameters to obtain a stable solution, which is a common problem encountered by all iterative reconstruction algorithms. The OS-SART-PDTV algorithm must optimize the following parameters: $\lambda, \tau, \sigma, \theta$. We found that the quality of reconstructed images was significantly improved when λ varied within a small range. In other words, the OS-SART-PDTV algorithm was more sensitive to this parameter. The number of iterations is usually set in the range of 50–100 because the improvement rate of image quality gradually decreases as the number of iterations increases, and after reaching a certain number of iterations, image quality no longer improves. According to our experimental results, the OS-SART-PDTV algorithm obtained better reconstructed images after 50 iterations. In addition, the two parameters τ and σ must be tuned several times to obtain high-quality reconstructed images. When reconstructing projection data of a new sample, these two parameters must be readjusted.

V. Conclusion

NCT has proven to be a highly effective noninvasive measurement method and has been widely applied in nuclear engineering, thermodynamics, and other fields. However, due to the complexity of the NCT application environment, obtaining complete projection data is challenging. The selection of an efficient sparse-view reconstruction algorithm is directly related to system imaging quality and measurement result reliability. To address this need, we propose the OS-SART-PDTV algorithm for sparse-view NCT 3D reconstruction. The OS-SART-PDTV algorithm consists of two steps: OS-SART for image reconstruction and PDTV for solving total variation using a first-order primal-dual algorithm.

Based on reconstruction results from projection data obtained from the NCT scan clock model, the OS-SART-PDTV algorithm demonstrates significant advantages over other algorithms in preserving edge structure, denoising, and suppressing artifacts. Therefore, the OS-SART-PDTV algorithm has great potential for application in NCT 3D reconstruction.

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Author Contributions

All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Yang Liu, Tengfei Zhu, Zhi Luo, and Xiaoping Ouyang. The first draft of the manuscript was written by Yang Liu and Tengfei Zhu, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Data Availability Statement

The data that support the findings of this study are openly available in Science Data Bank at https://www.scidb.cn/en/datalist?tID=journalOne&dataSetType=journal&cssCode=j00186&co

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