

## Development of an autotuning magnet girder with high stability and accuracy (Postprint)

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### Abstract

With the development of high-performance photon sources which have extremely low emittance, autotuning magnet girders have drawn more and more attention, especially for diffraction-limited storage rings and free-electron lasers. The biggest challenge is to simultaneously obtain high stability and high flexibility. In this paper, an autotuning magnet-girder prototype has been designed and developed. Topological optimization, multipoint supports, and locking systems have been applied for magnet-girder design to improve the stability. The modal analysis accords with the vibration test well. The natural frequency of the magnet-girder assembly is deduced as high as 45.6 Hz, which demonstrates good stability. Ball-cam movers have been chosen as adjustment mechanisms, and a closed-loop control scheme has been used to pursue high accuracy. The kinematic resolution is better than 1  $\mu$ m, and the accuracy is better than 1  $\mu$ m within the adjusting range of  $\pm 5$  mm. Besides, it can eliminate most of the calibration, which can save much manpower and time. The tests demonstrate that the magnet girder can be used for beam-based girder alignment with high stability and high accuracy.

### Full Text

### Preamble

#### Development of an Auto-Tuning Magnet Girder with High Stability and Accuracy

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**Abstract:** With the development of high-performance photon sources featuring extremely low emittance, auto-tuning magnet girders have drawn increasing attention, particularly for diffraction-limited storage rings (DLSRs) and free-electron lasers (FELs). The greatest challenge lies in simultaneously achieving high stability and high flexibility. This paper presents the design and development of an auto-tuning magnet girder prototype that addresses this challenge. Topological optimization, multi-point supports, and locking systems have been applied to improve stability. Modal analysis results agree well with vibration test data, yielding a natural frequency of 45.6 Hz for the magnet-girder assembly, which demonstrates excellent stability. Ball-cam movers serve as the adjustment mechanism, and a closed-loop control scheme achieves high accuracy. The kinematic resolution is better than 1  $\mu$ m, with positioning accuracy better than 1  $\mu$ m within an adjustment range of  $\pm 5$  mm. Furthermore, the system eliminates most calibration requirements, saving substantial manpower and time.

Test results demonstrate that the magnet girder can be used for beam-based girder alignment with both high stability and high accuracy.

## I. INTRODUCTION

The demand for extremely high brilliance and low emittance has become increasingly urgent in photon source development, especially for DLSRs and FELs [1]. Nearly all newly designed or upgraded synchrotron radiation facilities require emittance at the sub-nanometer level or lower—for example, 42 pm $\cdot$ rad for APS-U [2] and 34 pm $\cdot$ rad for HEPS [3-4]. This necessitates that magnets achieve ultrahigh stability and alignment accuracy simultaneously, posing a significant challenge for magnet girder design.

Vibration and motion of lattice elements can lead to substantial beam emittance growth and brilliance degradation. Vibrations occurring primarily above 1 Hz can be mitigated through stable girders and beam-based feedback systems, while slow position drift necessitates re-alignment, for which flexible and accurate kinetic mechanisms are preferred.

Natural frequency represents a critical stability parameter due to the rapid drop in ground vibration amplitude at higher frequencies, as observed at APS [5], HEPS [6], and other accelerator sites [7], although values are strongly site-dependent. Table I summarizes the natural frequency and misalignment tolerance requirements for several new or upgraded photon sources. Except for Sirius, which employs small, lightweight girders to achieve a much higher natural frequency, most facilities use relatively long girders. Almost all new synchrotron radiation sources have selected non-automatic or semi-automatic adjusting mechanisms such as wedge jacks to obtain high magnet stability.

TPS utilized ball-cam movers for automatic alignment, but this came at the cost

of a required natural frequency higher than 30 Hz, with a tested value of 33 Hz using a locking system—lower than other facilities. The misalignment tolerance between girders can reach 50  $\mu\text{m}$  using laser trackers and accurate wedge jacks, but can hardly be improved further. The auto-tuning girder represents an essential and urgent challenge for beam-based girder alignment due to its excellent adjusting flexibility and accuracy. It may assume most static orbit correction responsibilities, leaving corrector strength available for dynamic correction (active orbit feedback) and local bump creation for matching to beamline acceptances or for machine studies [12].

Existing auto-tuning magnet girders primarily employ ball-cam movers as the kinetic mechanism, first applied in the Final Focus Test Beam (FFTB) at SLAC [13] and subsequently adopted in photon sources such as SLS [12], Diamond [14], and TPS [11]. The SLS and Diamond mechanisms are similar, using five ball-cam movers to adjust five degrees of freedom (excluding translation along the beam direction). TPS improved the design to adjust all six degrees of freedom, enabling complete control of girder position. While auto-tuning girders can achieve better positioning accuracy and save considerable manpower and time, they struggle to reach high natural frequencies—for example, above 50 Hz—as shown in Table II.

The requirements for high stability and high flexibility conflict to some extent. Stability correlates positively with support stiffness, which demands larger contact areas and locking forces, while flexibility requires kinetic mechanisms with appropriate constraints and minimal interference such as friction. Using a locking system after adjustment, as implemented in TPS, represents a practical approach, though uneven locking forces can degrade alignment accuracy. This may be improved through locking system calibration to balance force distribution on the girder or through fine-tuning with the locking forces applied.

Magnet girder design requires comprehensive consideration of magnet stability, adjustment accuracy and flexibility, convenient installation, lattice periodicity, cost, and other factors, among which stability, adjustment accuracy/flexibility, and cost are paramount. Many light sources have not adopted auto-tuning magnet girders due to relatively lower stability and higher costs. However, as a technical research direction, auto-tuning magnet girders with high stability and accuracy have drawn increasing attention in the pursuit of extremely high brilliance and low emittance.

This paper describes the development and testing of an auto-tuning magnet girder prototype with excellent performance. The natural frequency of the magnet-girder assembly reaches 45.6 Hz or higher, comparable to new photon sources using manual wedge jacks. The adjustment ranges exceed  $\pm 10$   $\mu\text{m}$  with improved accuracy better than 1  $\mu\text{m}$  and 5  $\mu\text{rad}$  without the locking system, and better than 8  $\mu\text{m}$  and 15  $\mu\text{rad}$  after locking.

## II. STRUCTURE DESIGN

The girder prototype was designed for the straight multiplet of the HEPS-TF

(Test Facility of High Energy Photon Source) lattice [19], which contains nine magnets. The assembly consists of a girder body, ball-cam movers, pedestals, locking systems, fake magnets (as gravity loads), sensors, and a control system, as shown in Fig. 1 [Figure 1: see original paper].

The girder body measures 3300 mm in length and is welded from steel plates to reduce weight and improve manufacturability. The upper plate is 50 mm thick and mounts nine steel blocks weighing approximately 6.5 tons total as fake magnets. Six ball units are mounted at the upper boundaries of the girder body, each matching a corresponding cam mover mechanism. The six cam movers are fixed atop three pedestals, which are also steel plate weldments designed for fixation at the lower surface. Six locking systems are placed symmetrically between the girder body and the three pedestals to lock the girder after adjustment, thereby improving the natural frequency of the magnet-girder assembly. Following the TPS design [17], each locking system includes a wedge block for locking, disc springs for buffering, and a motor-driven screw. Eight length gauges monitor the translation and rotation of the girder body, with gauge blocks mounted to the girder body or pedestals and measuring contacts touching corresponding references.

In this paper, X denotes the horizontal direction, Y the vertical direction, Z the beam direction, Pitch rotation about the X-axis, Yaw rotation about the Y-axis, and Roll rotation about the Z-axis.

### III. STABILITY STUDY

#### A. Topological Optimization of the Girder Body

Topological optimization was first applied to this girder prototype design [20] and subsequently used in APS-U and Sirius girder designs [8,21]. This method determines the optimal material distribution to maximize or minimize an objective criterion subject to given constraints. Both static and dynamic topological optimizations were performed for the girder body using ANSYS Parametric Design Language (APDL) code. The static optimization aimed to minimize compliance, indicating highest static stiffness and minimum deformation, while the dynamic optimization aimed to maximize natural frequency, indicating highest dynamic stiffness.

The optimization model is shown in Fig. 2 [Figure 2: see original paper]. The six ball unit housings were set as fixed constraints, and the girder body (excluding the upper plate) was designated as the optimization domain. For static optimization, magnet weights of approximately 6.5 tons were applied as force loads to reduce node count and accelerate calculation, with compliance specified as the objective function and an 80% volume reduction from the initial value as the constraint. For dynamic optimization, fake magnet models were retained due to their importance for the mass matrix, while other settings remained similar to the static case.

Both optimizations converged successfully. The girder body's static stiffness

dominates the uneven flatness of the entire assembly, while its dynamic stiffness plays a relatively weaker role due to the even lower stiffness of other components such as the ball-cam movers. Therefore, the static optimization result was primarily referenced, with the dynamic optimization serving as supplementary guidance. Figure 3 [Figure 3: see original paper] shows the convergence curves and element pseudo-density distribution from the static optimization. Post-optimization improvements in deformation and natural frequency are summarized in Table III.

## B. Modal Analyses and Vibration Test

The magnet-girder assembly can be modeled as a multi-degree-of-freedom system with motion described by the differential equation:

$$M\ddot{X} + C\dot{X} + KX = F,$$

from which the natural circular frequency  $\omega_p$  can be derived from  $|K - \omega_p^2 M| = 0$ , where  $M$  is the mass matrix,  $C$  the damping matrix,  $K$  the stiffness matrix,  $F$  the excitation force vector, and  $X$  the displacement vector.

The ball-cam movers can be modeled as springs with stiffness estimated from force and deformation. Deformation of the ball-cam mover can be decomposed into several major factors, as shown in Fig. 4 [Figure 4: see original paper].  $D_1$  and  $D_2$  represent deformations at contact positions calculable using Hertz theory [22];  $D_3$  and  $D_4$  represent bearing deformations calculable using empirical equations [23];  $D_5$  represents shaft deformation; and  $D_6$  represents elastic material deformation. Calculated deformations from gravity for each factor are listed in Table IV.

Unlike ball-cam mover stiffness, the stiffness of the locking system and the pedestal-ground interface is difficult to estimate and cannot be considered as rigid joints. Three operational cases were investigated to study the roles of the locking system and pedestal stability. FEM modal analyses using ANSYS and experimental modal analysis were conducted. Case 1: pedestals fixed by anchor bolts without locking systems. Case 2: pedestals fixed by anchor bolts with six locking systems. Case 3: pedestals grouted to the ground with six locking systems. In the tests, pedestals were mounted to the ground using anchor bolts only, as grouting was not permitted in the experimental hall. However, the natural frequency could be deduced if the calculated results for the first two cases agreed well with test results, which they did, as shown in Table V. The natural frequencies and mode shapes from FEA match the test results very well. Figure 5 [Figure 5: see original paper] shows the first-order mode shapes from FEA for the three cases, while Fig. 6 [Figure 6: see original paper] shows the tested mode shapes and transfer functions for Case 2.

The tests demonstrate that the locking system definitively improves natural frequency. The assembly's natural frequency increases with locking force and

becomes constant when forces exceed 2 tons, indicating that contacts have stabilized and the locking systems are exhibiting their inherent stiffness. Pedestal stability also plays a crucial role in assembly stability. The natural frequencies of the pedestals in Cases 1 and 2 are identical: 56.1 Hz from testing and 63.1 Hz assumed in FEA. However, frequency can be significantly enhanced by grouting pedestals to the ground. Similar grouted pedestals have achieved over 200 Hz in TPS [24] and 420 Hz in HEPS [25]. The analyzed natural frequency of the magnet-girder assembly reaches 45.6 Hz when the pedestal frequency is 190 Hz, and could be even higher with greater pedestal stability.

Figure 7 [Figure 7: see original paper] compares harmonic responses in the X direction for the three cases. Inputs are harmonic nodal displacements at the pedestal base with 10 nm amplitude and frequencies from 2 to 100 Hz. Response curves are recorded at the magnet center on the girder side. Lower natural frequencies clearly produce larger vibrations at lower ground motion frequencies. Considering that ground vibration amplitude is higher at lower frequencies, the actual vibration would be even more severe. Responses in other directions are similar, which explains why stable girders with higher natural frequencies are preferred for photon sources and similar DC magnet facilities.

#### IV. KINETIC STUDY

##### A. Kinetic Mechanism

Ball-cam movers were selected for the prototype due to their high accuracy, compact structure, and multi-degree-of-freedom capability. The Boyes Clamp principle [26] was adopted for kinematic support, as also used in TPS [11]. Figure 8 Figure 8: see original paper illustrates how a kinematic three-point footing defines the upper body's position when balls are mounted to a rigid upper body. The girder has six support points that can be considered as three pairs. Each pair functions as a large, rigid ball, with the two corresponding shafts acting as a groove, as shown in Fig. 8(b). The girder position is thereby determined by these three assumed balls and their three angled grooves. Rotating the cam shafts changes the grooves, enabling girder position control.

Figure 9 [Figure 9: see original paper] shows a diagram of one ball-cam mover. The main dimensions always satisfy the relation:

$$d = \sqrt{(x + r \cos(\theta + \varphi) - u)^2 + (y + r \sin(\theta + \varphi) - v)^2},$$

where  $d$  is the center-to-center distance between ball and cam,  $x$  and  $y$  are the origin relative positions between ball and cam in the two directions,  $u$  and  $v$  are the ball adjustment values in two directions,  $r$  is the eccentricity between cam center and fixed axis,  $\theta$  is the cam starting angle, and  $\varphi$  is the adjusted angle.

Values of  $d = 110$  mm,  $x = 82$  mm, and  $y = 73$  mm were selected based on MATLAB optimization to achieve large, symmetrical adjustment ranges. Required adjustment ranges are  $\pm 8$  mm horizontally and  $\pm 10$  mm vertically,

accounting for machining tolerances, uneven ground, and long-term position drift. Neglecting machining and assembly errors, each mechanism's adjustment ranges are -9.43 mm to 9.33 mm horizontally and -10.67 mm to 10.46 mm vertically, as shown in Fig. 10 [Figure 10: see original paper]. Girder adjustment ranges can then be derived through simple coordinate transformation and the Pythagorean theorem. Open-loop tests using dial indicators verified these ranges, with results listed in Table VI.

## B. Kinetic Algorithms

The girder position can be described by a  $3 \times 6$  matrix representing the three coordinate positions of the six balls. Girder translation and rotation are monitored by length gauges, as shown in Fig. 11 [Figure 11: see original paper]. The adjustment algorithm's purpose is to convert desired girder adjustments into motor steps.

In TPS, the neighboring girder served as the length gauge reference, causing rotations to affect translations that required elimination [27]. For our prototype, the pedestals serve as length gauge references and can be considered motionless during short-term adjustment, eliminating coupling between the six degrees of freedom.

The target position girder matrix  $G_t$  can be calculated as:

$$G_t = R_t G_0 + T_t,$$

where  $G_0$  is the girder matrix when each ball and cam are at their origin relative positions, and  $T_t$  is the target position translation matrix. These matrices are defined as:

$$G_0 = \begin{bmatrix} x_{10} & \cdots & x_{60} \\ y_{10} & \ddots & y_{60} \\ z_{10} & \cdots & z_{60} \end{bmatrix},$$

$$T_t = \begin{bmatrix} x_{Tt} & \cdots & x_{Tt} \\ y_{Tt} & \ddots & y_{Tt} \\ z_{Tt} & \cdots & z_{Tt} \end{bmatrix}.$$

$R_t$  is the target position rotation matrix, which for small-angle rotations simplifies to:

$$R_t = \begin{bmatrix} 1 & -\sigma_t & \eta_t \\ \sigma_t & 1 & -\chi_t \\ -\eta_t & \chi_t & 1 \end{bmatrix}.$$

Similarly, the current position girder matrix  $G_c$  is:

$$G_c = R_c G_0 + T_c.$$

The adjustment matrix  $dG$  is then:

$$dG = G_t - G_c,$$

where  $x_{n0}$ ,  $y_{n0}$ , and  $z_{n0}$  are the original coordinate positions of ball  $n$ ;  $x_{Tt}$ ,  $y_{Tt}$ , and  $z_{Tt}$  are girder translations at the target position in X, Y, and Z directions; and  $\chi_t$ ,  $\eta_t$ ,  $\sigma_t$  are girder rotations at the target position in Pitch, Yaw, and Roll directions.

The calculated  $dG$  is then transformed from assembly coordinates to ball-cam coordinates to obtain each shaft's adjustment angle. The kinetic algorithm is implemented in MATLAB/Simulink, with a Beckhoff controller and EtherCAT used for the control system.

### C. Kinetic Tests

Following assembly, open-loop tests were performed using all calibrated dimensions as algorithm parameters, followed by closed-loop tests to verify accuracy. Length gauge measurement ranges are 12 mm, so all kinetic tests were conducted within  $\pm 5.3$  mm adjustment range to provide margins.

#### 1. Resolution and Repeatability Tests

Adjusting resolution is determined by a 5-phase stepping motor and reducer with a 160:1 ratio. The theoretical resolution is below 0.8  $\mu\text{m}$  without motor subdivision, as shown in Fig. 12 [Figure 12: see original paper]. Resolution was tested at five origin positions for all six directions using 1  $\mu\text{m}$  step sizes with ten steps per position. Both resolution and coupling errors are better than 1  $\mu\text{m}$  and 1  $^\circ$  rad. Detailed results for three translation directions are shown in Table VII, matching theoretical predictions well.

Repeatability was tested at four target positions for all six directions. The girder moved from  $G_0$  to each target position  $G_t$  five times, with standard deviations calculated as the repeatability metric. All results are better than 2  $\mu\text{m}$  and 2  $^\circ$  rad. Figure 13 [Figure 13: see original paper] shows repeatability in the X direction.

#### 2. Adjusting Error Tests

With good resolution and repeatability established, adjusting errors were tested across 16 target positions per translation direction, 12 target positions per rotation direction, and several combined translation-rotation positions.

Open-loop tests measured machining and assembly accuracy and verified stiffness. Open-loop errors varied with adjustment values following particular pat-

terns, indicating systematic errors. To identify error sources, systematic and random errors were added to each main algorithm dimension. Back-calculation revealed that modifying ball-cam mover center-to-center distances ( $d$ ) by gravity-induced deformation produced calculated errors matching test results well, as shown in Fig. 14 [Figure 14: see original paper]. After eliminating this gravity-induced systematic error, maximum open-loop adjusting error decreased from 26 mm to 14 mm within  $\pm 5$  mm adjustment range. This gravity-induced error also validates the ball-cam mechanism stiffness calculations described in Section III.B.

Closed-loop tests verified accuracy. The difference between  $G_t$  and  $G_c$  was added as compensation to  $G_t$  at each step until within tolerance. Target positions matched those used for open-loop tests. All adjusting errors are better than 1 mm and 5 rad within three iterations.

For all kinetic tests, nearly all dimension parameters were calibrated using coordinate measuring machines, laser trackers, or other devices, requiring substantial manpower and time. If only length gauge positions are calibrated while other dimensions use design values, open-loop errors increase significantly while closed-loop errors remain acceptable with few additional iterations. Figure 15 [Figure 15: see original paper] compares iteration numbers for translation along X and rotation in Roll. For X-direction translation with 12 target positions from -5 mm to 5 mm, the average iteration number is 2.17 for the calibrated-dimension case versus 2.33 for the designed-dimension case. This indicates that over 90% of calibration manpower and time can be saved with only slight increases in adjustment time, which is highly attractive for batch alignment.

### 3. Coupling Error from Locking System

Locking systems enhance stability but can introduce coupling errors due to uneven forces during the locking process. Fine-tuning with locking forces can reduce these errors. Nine target positions were tested, each repeated three times. Results demonstrate that coupling errors can be reduced to better than 8 mm and 15 rad within ten iterations, which remains far better than alignment requirements for all new or upgraded photon sources. Table VIII shows one example for a 2.5 mm Y-direction target translation.

## V. CONCLUSION

A new auto-tuning magnet girder prototype has been developed and tested to achieve high stability and accuracy for future synchrotron radiation photon sources, particularly diffraction-limited storage ring light sources. Motor-driven ball-cam movers provide adjustment capability. Compared with existing auto-tuning magnet girders, both stability and flexibility have been improved through topological optimization and locking systems. Ball-cam mover stiffness has been calculated and validated through FEM analyses that agree well with test results, yielding a magnet-girder assembly natural frequency of 45.6 Hz or higher with improved pedestal fixation. Through ingenious algorithm design and closed-

loop control, substantial calibration manpower and time can be saved with only minor increases in adjustment time, while achieving attractive accuracy. The kinematic resolution is better than 1  $\mu$ m, with accuracy better than 1  $\mu$ m and 5  $\mu$ rad within  $\pm 5$  mm adjustment range without locking, and better than 8  $\mu$ m and 15  $\mu$ rad after locking. The concepts and methods presented here demonstrate a viable approach for beam-based girder alignment and provide valuable experience for similar applications.

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