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Date: 2023-06-18T00:00:00+00:00

Abstract

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Full Text

Preamble

Nuclear Science and Techniques 24 (2013) 050502

Probing the density dependence of the symmetry energy with central heavy ion collisions

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Abstract

An improved isospin-dependent Boltzmann-Langevin model, incorporating inelastic channels and momentum-dependent interactions, is used to investigate the high-density behavior of nuclear symmetry energy. By employing several forms of nuclear symmetry energy, we calculate the time evolution of the neutron-to-proton ratio, pion multiplicity and π^-/π^+ ratio, as well as the kinetic energy and transverse momentum spectra of the π^-/π^+ ratio in heavy ion collisions at 400A MeV. We find that both the neutron-to-proton ratio and the π^-/π^+ ratio are highly sensitive to the nuclear symmetry energy, with π^- being more sensitive than π^+ . A supersoft symmetry energy yields a larger π^-/π^+ ratio.

Keywords: Improved isospin-dependent Boltzmann-Langevin model, Nuclear symmetry energy, Neutron-to-proton ratio, Pion production

Introduction

Constraints on the nuclear symmetry energy $E_{\text{sym}}(\rho)$ have been a subject of intense interest in recent years. The symmetry energy, which represents the energy cost to convert all protons to neutrons at fixed density in symmetric nuclear matter, is crucial for understanding both nuclear structure—such as binding energies—and neutron star properties, including the nature and stability of phases within stellar interiors [?, ?]. The energy per nucleon in asymmetric nuclear matter can typically be expressed as $E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4)$, where $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ represents the isospin asymmetry, and ρ_n and ρ_p are the neutron and proton densities, respectively. The first term represents the symmetric contribution, while the second term is the symmetry energy contribution, with higher-order terms in δ generally being negligible [?].

At densities exceeding saturation density ($\rho > \rho_0$), the density dependence of $E_{\text{sym}}(\rho)$ remains highly controversial, with even its basic trend unconstrained. Various theoretical models predict two distinct forms: one where $E_{\text{sym}}(\rho)$ increases monotonically with nucleon density, and another where it increases up to ρ_0 then decreases at higher densities. Numerous probes have been proposed to constrain $E_{\text{sym}}(\rho)$ at supra-saturation densities, including the neutron-to-proton ratio of squeezed-out nucleons [?], neutron-proton differential flow [?], the π^-/π^+ ratio [?], the K^0/K^+ ratio [?], the Σ^-/Σ^+ ratio [?], and others. Among these, the π^-/π^+ ratio has emerged as particularly promising, motivated by the $\Delta(1232)$ resonance model [?] which predicts a fundamental relationship between π^-/π^+ and N/Z in the participant region of the reaction. This N/Z ratio is determined by dynamical isospin fractionation [?], enabling the use of π^-/π^+ as a constraint on $E_{\text{sym}}(\rho)$.

However, significant uncertainties remain when using the π^-/π^+ ratio to probe $E_{\text{sym}}(\rho)$. Comparisons with FOPI data [?] have yielded contradictory results: the isospin-dependent Boltzmann-Uehling-Uhlenbeck model (IBUU04) [?] and improved isospin-dependent Boltzmann-Langevin model (ImIBL) [?] suggest

a very soft $E_{\text{sym}}(\rho)$, while the improved isospin-dependent quantum molecular dynamics model (ImIQMD) [?] favors a hard symmetry energy. At sub-saturation densities ($\rho < \rho_0$), the qualitative trend of $E_{\text{sym}}(\rho)$ is better constrained, though quantitative precision remains elusive. Proposed probes in this regime include isospin diffusion [?, ?], neutron skin thickness [?], single and double neutron-to-proton ratios [?, ?], heavy fragment mass distributions [?], and others. While IBUU04 [?] and the constrained molecular dynamics II model (CoMDII) [?] suggest nearly linear $E_{\text{sym}}(\rho)$, the improved quantum molecular dynamics model (ImQMD) [?] and isospin-dependent quantum molecular dynamics model (IQMD) [?] predict softer symmetry energies from double neutron-to-proton ratio analyses. Recently, a quantum statistical approach has predicted $E_{\text{sym}}(\rho)$ at low temperature and density limits [?].

As an ensemble-averaged theory, the conventional BUU model has successfully described various heavy ion collision phenomena but lacks fluctuations crucial for pre-equilibrium particle emission and subthreshold multifragmentation. Conversely, QMD-like models simulate reactions event-by-event and include fluctuations, but inter-event correlations remain uncertain, and QMD fails to reproduce subthreshold kaon production [?]. In this work, we employ the ImIBL model to investigate $E_{\text{sym}}(\rho)$ using both the neutron-to-proton ratio and π^-/π^+ ratio in $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV. The paper is organized as follows: Section 2 introduces the model, Section 3 presents results, and Section 4 provides conclusions.

2 Theoretical Model

The ImIBL model used in this study is an updated version of the Boltzmann-Langevin (BL) model [?]. In the BL framework, the fluctuating phase-space single-particle density satisfies the following equation [?]:

$$\frac{\partial f}{\partial t} + \nabla_p \hat{K} \cdot \nabla_r f - \nabla_r \hat{U} \cdot \nabla_p f = \delta K(r, p, t)$$

where the left-hand side describes Vlasov propagation determined by the nuclear mean field, and $\delta K(r, p, t)$ represents the fluctuating collision term that acts as a stochastic force. This collision term has the correlation function:

$$\langle \delta K(r, p, t) \delta K(r', p', t') \rangle = \delta(r - r') \delta(t - t') C(p, p')$$

where angle brackets denote a local average over fluctuating densities generated during a time interval δt . In the weak-coupling limit, this correlation function can be reduced and expressed in terms of one-body properties of the locally averaged distribution [?].

The BL model employs the projection method [?, ?], which projects fluctuations onto a set of low-order local multipole moments Q_{LM} of order L with magnetic quantum numbers M of the momentum distribution. These multipole moment fluctuations are characterized by a diffusion matrix that can be deduced from the correlation matrix [?]:

$$D_{LM,L'M'} = \int dp dp' Q_{LM}(p) C(p, p') Q_{L'M'}(p')$$

According to Refs. [?], the stochastic component of the dynamics can be reduced to two equations:

$$\begin{aligned} Q_{20}(r, t) &= \bar{Q}_{20}(r, t) + \sqrt{2\Delta t} C_{20}(r, t) W_1 \\ Q_{30}(r, t) &= \bar{Q}_{30}(r, t) + \sqrt{2\Delta t} C_{30}(r, t) W_2 \end{aligned}$$

where W_1 and W_2 are Gaussian random numbers, and Q_{20} and Q_{30} represent fluctuating quadrupole and octupole moments, respectively. Fluctuations are inserted into momentum space through a scaling procedure [?].

[Figure 1: see original paper] shows the time evolution of the ensemble-averaged quadrupole moment of the momentum distribution in the ImIBL model (upper panel) and the associated variance σ_{20} (bottom panel) for central $^{40}\text{Ca}+^{48}\text{Ca}$ collisions at 100A MeV. The ImIBL model exhibits a larger variation range for the Q_{20} value compared to the BUU model. The bottom panel reveals a bump in the early collision stage for the BL model that is absent in BUU simulations, indicating a greater difference between real and averaged quadrupole moments during early collisions—a phenomenon we consider important for dynamical evolution.

In the Vlasov part of Eq. (3), we employ an isospin- and momentum-dependent single-nucleon potential:

$$U(\rho, \delta, \mathbf{p}) = A_u(x) \frac{\rho}{\rho_0} + A_l(x) \frac{\rho}{\rho_0} \tau_z \delta + V_{\text{MDI}}$$

with bulk parameters $\alpha = -390$ MeV, $\beta = 320$ MeV, and $\gamma = 1.14$ for the soft EOS with MDI (SM), and $\alpha = -130$ MeV, $\beta = 59$ MeV, and $\gamma = 2.09$ for the hard EOS with MDI (HM) [?]. The corresponding compressibilities K are 200 MeV and 380 MeV for SM and HM, respectively.

The local part of the symmetry energy $E_{\text{sym}}(\rho)$ adopts two forms:

$$E_{\text{sym}}(\rho) = 12.5 \left(\frac{\rho}{\rho_0} \right)^{\gamma_s} \quad (8)$$

$$E_{\text{sym}}(\rho) = a_{\text{sym}} \left(\frac{\rho}{\rho_0} \right) + b_{\text{sym}} \left(\frac{\rho}{\rho_0} \right)^2 \quad (9)$$

In Eq. (8), $\gamma_s = 0.5, 1.0,$ and 2.0 correspond to soft, linear, and hard symmetry energies, respectively. The parameters in Eq. (9) are $a_{\text{sym}} = -18.9$ MeV and $b_{\text{sym}} = -3.8$ MeV, yielding a supersoft symmetry energy. The curvature parameters are $275.2, -25.4, -62.9,$ and -403.6 MeV for hard, linear, soft, and supersoft symmetry energies, respectively. [Figure 2: see original paper] compares $E_{\text{sym}}(\rho)$ for these cases with results from Refs. [?, ?].

The momentum-dependent potential is expressed as [?]:

$$V_{\text{MDI}} = \int dp' f(r, p') v_d(\mathbf{p} - \mathbf{p}')$$

3.1 Neutron-to-Proton Ratio

[Figure 3: see original paper] displays the time evolution of the free neutron-to-proton ratio and the corresponding central point density in central $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV, calculated using supersoft and hard symmetry energies. The solid and dashed lines show results in the rapidity interval $|y_0| < 0.5$ and emission angle interval $60^\circ < \theta_{\text{lab}} < 120^\circ$, while dotted and dotted-dashed lines show results without cuts. The supersoft symmetry energy leads to reduced neutron emission in high-density regions, while an inversion of this trend occurs in low-density regions due to contributions from nucleon emission during the expansion phase when fragments are forming. These results are consistent with Ref. [?].

3.2 Pion Multiplicity

The inelastic channels and parameterized cross sections for each channel are taken from Ref. [?], using free-space cross sections for elastic channels and in-medium cross sections for inelastic channels. In-medium effects include medium corrections to the ρ meson mass [?].

The left panel of [Figure 4: see original paper] shows the time evolution of π^\pm multiplicity in central $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV for four different forms of $E_{\text{sym}}(\rho)$. Together with the bottom panel of [Figure 3: see original paper], this demonstrates that pions are predominantly produced in high-density regions, making pion production a valuable probe of high-density phase space. Notably, produced π^- are more sensitive to $E_{\text{sym}}(\rho)$ than π^+ . To further illustrate this, the right panel of [Figure 4: see original paper] shows π^\pm multiplicity as a function of the curvature parameter K_{sym} . The π^- yield varies strongly with different $E_{\text{sym}}(\rho)$ from soft to stiff cases because π^- production occurs primarily through neutron-neutron collisions. In high-density regions, a softer $E_{\text{sym}}(\rho)$

results in reduced neutron emission, creating a more neutron-rich environment compared to the hard case. We conclude that π^- yield plays a crucial role in investigating $E_{\text{sym}}(\rho)$.

3.3 π^-/π^+ Ratio

To further understand pion production, [Figure 5: see original paper] presents the kinetic energy and transverse momentum spectra of the differential π^-/π^+ ratio in $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV with impact parameter $b = 1$ fm. In low kinetic energy and low transverse momentum regions, the π^-/π^+ ratio is highly sensitive to $E_{\text{sym}}(\rho)$. Therefore, measuring high-energy and high-momentum pions is unnecessary for constraining the density dependence of symmetry energy—a finding useful for designing experimental facilities. The super-soft symmetry energy yields a larger π^-/π^+ ratio, consistent with Refs. [?, ?] but inconsistent with Refs. [?, ?]. Meson production as a probe of $E_{\text{sym}}(\rho)$ remains an open question, with threshold effects known to be important for K meson production. For pions, threshold effects not included in the present calculations may also significantly influence the evolution, and we plan to incorporate them in future ImIBL model developments.

[Figure 4: see original paper] (Color online) (Left) Time evolution of π^\pm multiplicity and (Right) π^\pm multiplicity as a function of K_{sym} in $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV with impact parameter $b = 1$ fm.

[Figure 5: see original paper] (Color online) (Left) Kinetic energy spectra and (Right) transverse momentum spectra of the π^-/π^+ ratio for $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV with impact parameter $b = 1$ fm.

4 Conclusion

In summary, we have investigated $E_{\text{sym}}(\rho)$ using the free neutron-to-proton ratio and π production in $^{48}\text{Ca}+^{48}\text{Ca}$ collisions at 400A MeV within the ImIBL model framework. Both the free neutron-to-proton ratio and the π^-/π^+ ratio exhibit sensitivity to $E_{\text{sym}}(\rho)$. A soft $E_{\text{sym}}(\rho)$ results in...

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