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Full Text

Preamble

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Density Isomer of Nuclear Matter in an Equivalent Mass Approach

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Abstract

The equation of state of symmetric nuclear matter is studied with an equivalent mass model. The equivalent mass of a nucleon has been expanded to order 4 in density. We first determine the first-order expansion coefficient in the quantum hadron dynamics, then calculate the coefficients of the second to fourth

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Key words: Density isomer, Equivalent mass, Nuclear equation of state

Introduction

Modern nuclear physics has recently made substantial progress in understanding numerous interesting phenomena, such as the monotonic decrease of direct flow in the collective motion of relativistic heavy ion collisions [?], the extension of symmetry energy from heavy-ion collisions at intermediate energies suggesting a soft symmetry energy [?], the mass reduction of the pressure and transition temperature in single-superconductivity [?], analytic flavor expression of nuclear symmetry energy to the fourth order [?], and the first direct observation of the deexcitation of the low-lying isomeric state ^{229m}Th from photon emission [?]. These advances highlight the importance of higher-order contributions in the nuclear equation of state (EoS).

In principle, the fundamental theory of strong interactions is Quantum Chromodynamics (QCD). However, due to the difficulty in treating finite density within QCD, particularly the consistent implementation of chemical potential, phenomenological models have been widely applied. On the other hand, a non-interacting system is exactly solvable, and one can therefore use an equivalent particle mass to mimic interaction effects.

The equivalent mass approach has been extensively applied in various contexts, including studies of quark matter [?], quark strangelets [?], chiral condensates [?], strange quark stars [?], and QCD phase transitions [?]. Isomeric states are particularly interesting and important for the nuclear equation of state. It is commonly understood that there is a minimum in the density dependence of the average energy per nucleon, with the corresponding density normally called the nuclear saturation density. As early as the 1950s, researchers realized that a second minimum—the so-called density isomer—could exist in nuclear matter [?]. This density isomer could arise from phase transitions [?], pion condensation [?], or other mechanisms [?].

As an application to nuclear matter, we have previously shown that the equivalent-mass approach produces nuclear saturation when the equivalent mass is expanded as a Taylor series to third order in density [?]. The purpose of the present paper is to investigate whether the equivalent-mass model can produce a density isomer by extending the expansion of the nucleon equivalent mass to fourth order. We first determine the first-order expansion coefficient within quantum hadron dynamics, then calculate the coefficients of the second to fourth order for given binding energy and incompressibility at normal nuclear saturation density. It is found that a density isomeric state appears if the incompressibility is smaller than a critical value. The model dependence of this conclusion has also been checked by varying the first-order coefficient.

The paper is organized as follows. In Section 2, we describe the equivalent-mass model suitable for symmetric nuclear matter, while the relevant thermodynamic treatments are presented in Section 3. Numerical results and discussions are provided in Section 4, and finally a summary is given in Section 5.

2 Equivalent-Mass Model for Symmetric Nuclear Matter

As mentioned in the introduction, our model incorporates particle interactions through an equivalent mass that varies properly with density. With this definition of equivalent mass, the energy density of symmetric nuclear matter can be written in the same form as that of free particles:

$$\varepsilon = g\int_0^{\nu} \frac{d^{3p}}{(2\pi)^3}\sqrt{p^2+M^2}$$

where $g = 2\times 2 = 4$ is the degeneracy factor, M is a density-dependent nucleon mass to be determined through expansion, and the Fermi momentum ν is connected to the nucleon number density ρ by

$$\rho = g\frac{\nu^3}{6\pi^2}$$

The integration can be carried out explicitly to give

$$\varepsilon = \frac{g}{8\pi^2}\left[\nu\sqrt{\nu^2+M^2}\left(\frac{M^2}{2}+\nu^2\right)-\frac{M^4}{2}\ln\left(\frac{\nu+\sqrt{\nu^2+M^2}}{M}\right)\right]$$

From Eq. (1), one can obtain the partial derivative

$$\frac{\partial\varepsilon}{\partial M} = \frac{gM}{2\pi^2}\int_0^{\nu}\frac{p^2dp}{\sqrt{p^2+M^2}} = \frac{gM}{4\pi^2}\left[\nu\sqrt{\nu^2+M^2}-M^2\ln\left(\frac{\nu+\sqrt{\nu^2+M^2}}{M}\right)\right]$$

The second derivative can also be obtained similarly. A glance at Eq. (4) immediately reveals that $\partial\varepsilon/\partial M > 0$; that is, ε is a monotonically increasing function of the nucleon mass at any fixed density. Therefore, one can obtain any value of ε with an equivalent density-dependent nucleon mass, provided the energy density is not smaller than that obtained by setting $M = 0$ in Eq. (1) or taking the limit $M \to 0$ in Eq. (3).

[Figure 1: see original paper] demonstrates this clearly, showing the energy density as a function of nucleon mass at different fixed values of nucleon number density. The energy density is generally a monotonically increasing function of mass.

Therefore, if one can obtain the energy density from other models or even from QCD in the future, we can solve the equation $\varepsilon = E_{\mathrm{mod}}$ for the equivalent mass.

In the covariant meson-baryon effective field theories of the nuclear many-body problem (often called quantum hadron dynamics or QHD [?, ?]), for example, we have [?]

$$E_{\mathrm{QHD}} = \frac{g}{8\pi^2}\left[\nu\sqrt{\nu^2+M_N^{*2}}\left(\frac{M_N^{*2}}{2}+\nu^2\right)-\frac{M_N^{*4}}{2}\ln\left(\frac{\nu+\sqrt{\nu^2+M_N^{*2}}}{M_N^*}\right)\right]+\frac{M_\sigma^2(M_N-M_N^*)^2}{2g_\sigma^2}+\frac{g_\omega^2\rho^2}{2M_\omega^2}$$

where $M_N = 938.926$ MeV is the average mass of nucleons in free space, M_σ and M_ω are respectively the masses of the sigma and omega mesons, and g_σ and g_ω are the corresponding coupling constants. Notably, this equivalent mass contains contributions not only from the scalar σ meson but also from the vector ω meson, which differs from conventional effective masses that depend solely on the scalar meson.

The first derivative of the energy per nucleon with respect to density satisfies

$$\frac{\partial(E/\rho)}{\partial\rho} = \frac{1}{\rho}\frac{\partial\varepsilon}{\partial\rho} - \frac{\varepsilon}{\rho^2}$$

where the expression for the partial derivative $\partial\varepsilon/\partial M$ has been given in Eq. (5). Similarly, for the second derivative we have

$$\frac{\partial^2(E/\rho)}{\partial\rho^2} = \frac{1}{\rho}\frac{\partial^2\varepsilon}{\partial\rho^2} - \frac{2}{\rho^2}\frac{\partial\varepsilon}{\partial\rho} + \frac{2\varepsilon}{\rho^3}$$

3 Thermodynamic Treatment

Special attention must be paid to the consistency of thermodynamic formulas when the particle mass depends explicitly on density. At zero temperature, the main expressions are the energy density and pressure. In the literature, there are three kinds of treatments. In the first treatment, both the energy density and pressure expressions have the same form as in the constant-mass case [?]. In the second treatment, both the energy density and pressure have an additional term due to the density dependence of the particle mass [?]. Finally, in the third treatment, the additional term is added only to the pressure, but not to the energy density [?, ?, ?]. These treatments were originally developed for quark matter with density-dependent quark masses.

In the present context of nuclear matter, the energy density and pressure expressions for the first treatment are

$$P_1 = \frac{g}{24\pi^2} \frac{\nu^4}{\sqrt{\nu^2 + M^2}}$$

For the second treatment, these become

$$P_2 = \frac{g}{24\pi^2} \frac{\nu^4}{\sqrt{\nu^2 + M^2}} - \rho \frac{dM}{d\rho} \frac{\partial \varepsilon}{\partial M}$$

For the third treatment, they are

$$P_3 = \frac{g}{24\pi^2} \frac{\nu^4}{\sqrt{\nu^2 + M^2}} - \rho \frac{dM}{d\rho} \frac{\partial \varepsilon}{\partial M}$$

In Eqs. (11-13), the ε expression is given by Eq. (1) or Eq. (3), while the quantity Ω_0 has the same form as the thermodynamic potential density of a free system, but with the constant particle mass replaced by a density-dependent one.

Replacing p with ν in the square root on the right-hand side of Eq. (1), one immediately finds $P_1 > 0$; that is, the pressure in the first treatment can never be less than zero. This means the equation of state is always monotonic, which is obviously incorrect. In the second treatment, the pressure can be zero or negative. However, zero pressure is not consistent with the minimum of the energy per nucleon, as we will see shortly that the pressure should be exactly zero at the energy minimum. One can verify that the third treatment satisfies the thermodynamic consistency requirement.

In fact, the chemical potential can be obtained from $d\varepsilon/d\rho$, giving

$$\mu = \frac{d\varepsilon}{d\rho} = \sqrt{\nu^2 + M^2} + \frac{dM}{d\rho} \frac{\partial \varepsilon}{\partial M}$$

In normal thermodynamic formulas, one has only the first term on the right. In the present case, we have an additional term—the second term in Eq. (15)—which arises due to the density dependence of the nucleon mass. This term also appears in the pressure expression to ensure thermodynamic consistency [**?**]:

$$P = \rho \frac{d\varepsilon}{d\rho} - \varepsilon = \rho \sqrt{\nu^2 + M^2} - \varepsilon + \rho \frac{dM}{d\rho} \frac{\partial \varepsilon}{\partial M}$$

From Eqs. (9), (15), and (16), we can easily verify that

$$\frac{\partial (E/\rho)}{\partial \rho} = \frac{P}{\rho^2}$$

This expression can also be derived directly from the fundamental differential equality

$$d(\varepsilon V) = -PdV + \mu dN$$

where V is the volume, N is the particle number, ε is the energy density, and $V\varepsilon$ is the system energy. Eq. (18) is simply a combination of the first and second laws of thermodynamics at zero temperature, with $\rho = N/V$.

The density isomer appears at about three times the nuclear saturation density. In [Figure 3: see original paper], the energy per nucleon, ε/ρ, and the corresponding pressure are shown simultaneously. It is evident that the pressure at the second minimum is zero, indicating that the isomer is mechanically stable. Its energy per nucleon is only slightly higher than that at normal saturation density.

In the above calculation, we determined the coefficient a_1 using Eq. (27) from QHD. We now examine how the value of a_1 influences the isomer, as different models can easily yield different a_1 values. For this purpose, we discuss the value of the incompressibility K.

[Figure 4: see original paper] shows the variation of the density isomer with the first-order expansion coefficient a_1 for the values indicated in the legend. The density isomer exists only for sufficiently small a_1 values.

The incompressibility represents the curvature of the equation of state and thus measures the stiffness of nuclear matter at saturation density. Its value is not yet completely determined experimentally, although the isoscalar giant monopole resonance provides a direct experimental tool for studying nuclear incompressibility in finite nuclei [?]. Early calculations with relativistic models that included contributions from the negative-energy sea gave values of K in the range of 250–270 MeV [?].

Note: Figure translations are in progress. See original paper for figures.

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