Gravitational waves from the axial oscillation of neutron star considering non-Newtonian gravity: Postprint

Authors: Wen Dehua, ZHOU Ying

Date: 2023-06-18T00:00:00+00:00

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Full Text

Preamble

Gravitational Waves from the Axial Oscillation of Neutron Stars Considering Non-Newtonian Gravity

WEN Dehua*, ZHOU Ying
Department of Physics, South China University of Technology, Guangzhou 510640, China

Abstract

The eigen-frequencies of the axial w-modes of neutron stars described by a super-soft equation of state (EOS) are investigated, taking into account non-Newtonian gravity. The results show that for a given stellar mass, the frequencies of wI and wI2 for our model are lower than those of typical EOSs (such as APR), and the frequencies increase with stellar mass, which is contrary to the behavior predicted by typical EOSs. These characteristics may provide a probe to test super-soft symmetry energy and non-Newtonian gravity in the

future. Moreover, our model also exhibits the universal behavior of mass-scaled eigen-frequencies as a function of compactness.

Key words: Neutron stars, Oscillation, Non-Newtonian gravity

Introduction

Neutron stars (NSs) provide an ideal laboratory for studying gravitational waves and the equation of state (EOS) of dense matter [?, ?], which are two important topics in modern physics. For both subjects, the w-mode—a type of oscillation mode that exists only in general relativity—is believed to be an important mechanism for producing gravitational waves [?] and will contain abundant information about the EOS of super-dense matter [?]. The w-mode is associated with space-time curvature and exists for all relativistic stars, including black holes, while the motion of the fluid is negligible [?, ?, ?]. The standard axial w-mode is categorized as wI.

At present, however, the EOS of matter under extreme conditions remains highly uncertain. One of the main sources of this uncertainty is the poorly known density dependence of the nuclear symmetry energy, $E_{sym}(\rho)$ [?]. Many theories predict that the symmetry energy increases continuously at all densities [?], but many other models predict different behavior at supra-saturation densities [?]. The latter type of symmetry energy function is generally regarded as soft above saturation density. Among the soft models, the original Gogny-Hartree-Fock (GHF) model [?] and group III [?] are super-soft, in which the pressure drops quickly to zero around three times the saturation density ρ_0.

Super-soft models either cannot maintain stable NSs or predict maximum NS masses significantly below $1.4M_\odot$ (where $M_\odot$ is the solar mass). On the other hand, recent experimental results support the super-soft prediction through analysis of FOPI/GSI experimental data on the π^-/π^+ ratio in relativistic heavy-ion collisions [?] within a transport model using the MDI (Momentum-Dependent-Interaction) EOS [?]. To enable super-soft EOS to support observed neutron star masses, non-Newtonian gravity was introduced [?]. In this work, we investigate the w-mode using a super-soft EOS with non-Newtonian gravity taken into account.

The paper is organized as follows: In Section 2, we describe the EOS and the non-Newtonian gravity formalism used to calculate w-modes. In Section 3, we present our numerical results. Section 4 provides concluding remarks.

2 Equation of State and Non-Newtonian Gravity

The MDI interaction is a phenomenological effective interaction based on a modified finite-range Gogny interaction [?, ?, ?]. Using the MDI EOS, circumstantial evidence for super-soft symmetry energy was recently found from analyzing FOPI/GSI experimental data [?] on the π^-/π^+ ratio in relativistic heavy-ion collisions [?]. For a neutron star consisting of neutrons, protons, and electrons (npe), the internal pressure is given by Eq.(1) [?, ?, ?], with the chemical equilibrium condition $\mu_e = \mu_n - \mu_p = 4\delta E_s(\rho)$ and charge neutrality requirement $\rho_e = (1-\delta)\rho/2$.

The energy per nucleon for symmetric nuclear matter can be well approximated as [?, ?]:

$$\frac{E}{A} = \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} + \frac{B}{2}\left(\frac{\rho}{\rho_0}\right)^{\sigma} + C_l \frac{\rho}{\rho_0}\arctan(\Lambda) + C_u\left(\frac{\rho}{\rho_0}\right)^{4/3}$$

and the symmetry energy can be expressed as [?]:

$$E_{sym}(\rho) = \frac{\hbar^2 k_F^2}{6m} + \frac{C_l}{2}\left(\frac{\rho}{\rho_0}\right)^{\sigma} + C_u\left(\frac{\rho}{\rho_0}\right)^{4/3}$$

where k_F is the Fermi momentum for symmetric nuclear matter. The parameter values are $\sigma = 4/3$, $B = 106.35$ MeV, $C_l = -11.70$ MeV, and $C_u = -103.40$ MeV, with ρ_0 being the saturation density. The symmetry energy at saturation density is $E_{sym}(\rho_0) = 31$ MeV [?]. The parameter x was introduced to vary the density dependence of the symmetry energy without changing any properties of symmetric nuclear matter. It has been shown that $x = 1$ can reproduce the FOPI/GSI pion production data [?] within transport model analysis, corresponding to a super-soft symmetry energy. The resulting EOS is denoted as MDIx1.

In the traditional framework, the MDIx1 EOS cannot support observed pulsar masses because of its super-soft symmetry energy. To reconcile this with neutron star observations, non-Newtonian gravity was introduced [?]. Non-Newtonian gravity can be described by adding a Yukawa term to the conventional gravitational potential between objects [?]:

$$\Phi(r) = -\frac{Gm_{1m}2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

where G is the universal gravitational constant, α is a dimensionless strength parameter, and λ is a length scale. In the scalar/vector boson exchange picture, $\alpha = \pm g^2/(4\pi G m_b^2)$ and $\lambda = 1/\mu$, where g, μ, and m_b are the boson-baryon coupling constant, boson mass, and baryon mass, respectively. Within the mean-field approximation, non-Newtonian gravitational effects can be described by modifying the EOS of dense matter while leaving Einstein's equations unchanged. The extra energy density due to non-Newtonian gravity can be calculated by [?, ?]:

$$\epsilon_{UB} = \frac{g^2}{2\mu^2}\rho^2$$

and the corresponding additional pressure is:

$$P_{UB} = \frac{g^2}{2\mu^2}\rho^2$$

where a constant boson mass independent of density is assumed [?, ?]. Including non-Newtonian gravity, the EOS becomes $P = P_{npe} + P_{UB}$, where P_{npe} is the conventional pressure for a neutron star composed mainly of neutrons, protons, and electrons.

Shown in [Figure 1: see original paper] are the EOSs versus reduced baryon number density ρ/ρ_0, where the non-Newtonian gravitational parameter g^2/μ^2 is taken as 75 GeV^{-2} or 100 GeV^{-2} (denoted as MDIx1-75 and MDIx1-100). To illustrate the effect of non-Newtonian gravity, the typical APR EOS [?] is used for comparison. The mass-radius relation is displayed in [Figure 2: see original paper]. It is evident that for the MDIx1 EOS, the neutron star radius is significantly larger than that for the typical APR EOS.

3 Gravitational Waves from the Axial w-Modes of Neutron Stars with Super-Soft EOS and Non-Newtonian Gravity

We first briefly introduce the formalism for axial w-modes, using geometrical units ($G = c = 1$). According to Chandrasekhar & Ferrari [?], the axial perturbation equations for a static neutron star can be simplified by introducing a function $Z(r)$ constructed from the radial part of the perturbed axial metric components. This function satisfies the differential equation:

$$\frac{d^{2Z}}{dr_*^2} + [\omega^2 - V(r)]Z = 0$$

where $\omega(= \omega_0 + i\omega_i)$ is the complex eigen-frequency of the axial w-mode and r_* is the tortoise coordinate [?]. Inside the star, the potential function V is defined by:

$$V(r) = \frac{l(l+1)}{r^2}e^{-\nu} + \frac{1}{r^3}e^{-\lambda}\left[\frac{d}{dr}(r^{2e^{(\nu-\lambda)/2}})\right]$$

where l is the spherical harmonics index, ρ is the density, p is the pressure, m is the mass inside radius r, and e^ν and e^λ are the metric functions given by the line element for a static neutron star. Outside the neutron star, the equation reduces to the Regge-Wheeler equation for black holes.

The solutions are subject to boundary conditions requiring regularity at the center and purely outgoing waves at infinity. Numerical calculations are performed using the continued fraction method [?, ?] together with the EOSs described in Section 2.

[Figure 3: see original paper] shows the frequency (a) and damping time (b) of the wI-mode as a function of neutron star mass M. To demonstrate the effect of non-Newtonian gravity on the w-mode, we also plot results for the typical APR EOS, which is commonly used as a benchmark in neutron star studies. [FIGURE:3(a)] reveals that at the same stellar mass, the frequency of the wI-mode for neutron stars described by MDIx1 with non-Newtonian gravity is smaller than that for the typical APR EOS. Moreover, for our model, the frequencies increase with stellar mass, contrary to the behavior predicted by typical EOSs. The frequency differences between our model and typical EOSs may arise from the neutron star' s global structure; as shown in [Figure 2: see original paper], neutron stars described by our model have larger radii. Additionally, a larger non-Newtonian gravitational parameter g^2/μ^2 corresponds to a smaller frequency. The damping time of the wI-mode shown in [FIGURE:3(b)] indicates no notable distinguishability among the three considered EOSs for neutron stars with canonical mass $1.4M_{\odot}$.

Previous work has shown that the eigen-frequency of w-modes scaled by stellar mass exhibits universal behavior independent of the EOS when expressed as a function of compactness M/R, allowing accurate determination of gravitational wave eigen-frequencies from w-modes if neutron star masses and radii are precisely observed [**?**]. [Figure 4: see original paper] displays the frequency (a) and damping time (b) of the wI-mode scaled by stellar mass M as a function of compactness M/R. Our results also exhibit this universal behavior, with the scaled frequencies for the two models MDIx1-75 and MDIx1-100 showing perfect universal behavior.

We also investigate the eigen-frequencies and scaled properties of the wI2-mode. [Figure 5: see original paper] shows the frequencies (a) and damping times (b) of the wI2-mode as a function of neutron star mass M, while [Figure 6: see original paper] shows these quantities scaled by stellar mass M as a function of compactness M/R. [FIGURE:5(a)] demonstrates that the wI2-mode frequencies for MDIx1 are nearly constant for a fixed non-Newtonian gravitational parameter (e.g., 12.6 kHz for $g^2/\mu^2 = 75$ GeV^{-2} and 10.9 kHz for $g^2/\mu^2 = 100$ GeV^{-2}). In contrast, the frequencies for the APR model decrease significantly as stellar mass increases. Similarly, the wI2-mode also exhibits the scaled universal property, as shown in [Figure 6: see original paper].

4 Conclusion

In this work, we calculated the eigen-frequencies of the axial w-modes of neutron stars described by a super-soft equation of state, with non-Newtonian gravity taken into account. We found that for a given stellar mass, the frequencies of wI and wI2 for our model are lower than those of typical EOSs (such as APR), and the frequencies increase with stellar mass, contrary to the behavior predicted by typical EOSs. These characteristics of the wI and wI2 frequencies can be used as a probe to test super-soft symmetry energy and non-Newtonian gravity in the future. Moreover, we have shown that our model also exhibits the previously discovered universal behavior of mass-scaled eigen-frequencies as a function of compactness.

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Note: Figure translations are in progress. See original paper for figures.