Density and temperature of fermions and bosons from quantum fluctuations postprint

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Date: 2023-06-18T00:00:00+00:00

Abstract

A novel method to determine the density and temperature of a system constituted by fermions and/or bosons is proposed based on quantum fluctuations. For fermions system, the results in the limit where the reached temperature T is small and where there is no constraint for the reached temperature T compared to the Fermi energy f at a given density   are given, respectively. Quadrupole and multiplicity fluctuation relations are derived in terms of T/ f. We compared the two set results in the limit when T is much smaller compared to Fermi energy f and they are consistent, as expected. The classical limit is also obtained for high temperatures and low densities. For bosons system, quadrupole and multiplicity fluctuations using Landau' s theory of fluctuations near the critical point for a Bose-Einstein condensate (BEC) at a given density   are derived. As an example, we apply our approach to heavy ion collisions using the Constrained Molecular Dynamics model (CoMD) which includes the fermionic statistics. The multiplicity fluctuation quenching for fermions is found in the model and confirmed by experimental data. To reproduce the available experimental data better, we propose a modification of the collision term in the approach to include the possibility of $\alpha-\alpha$collisions. The relevant Bose-Einstein factor in the collision term is properly taken into account. This approach increases the yields of bosons relative to fermions closer to data. Boson fluctuations become larger than one as expected.

Full Text

Preamble

Density and temperature of fermions and bosons from quantum fluctuations

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Abstract

A novel method to determine the density and temperature of a system constituted by fermions and/or bosons is proposed based on quantum fluctuations. For fermionic systems, results are presented both in the limit where the reached temperature T is small and where there is no constraint for the reached temperature T compared to the Fermi energy _f at a given density . Quadrupole and multiplicity fluctuation relations are derived in terms of _f and they are consistent, as expected. The classical limit is also obtained for high temperatures and low densities. For bosonic systems, quadrupole and multiplicity fluctuations using Landau' s theory of fluctuations near the critical point for a Bose-Einstein condensate (BEC) at a given density   are derived. As an example, we apply our approach to heavy ion collisions using the Constrained Molecular Dynamics model (CoMD) which includes fermionic statistics. The multiplicity fluctuation quenching for fermions is found in the model and confirmed by experimental data. To reproduce the available experimental data better, we propose a modification of the collision term in the approach to include the possibility of α-α collisions. The relevant Bose-Einstein factor in the collision term is properly taken into account. This approach increases the yields of bosons relative to fermions closer to data. Boson fluctuations become larger than one as expected.

Key words: Density, Temperature, Fermions, Bosons, Quantum fluctuations, Bose-Einstein condensate

Introduction

In recent years, the availability of heavy-ion accelerators providing colliding nuclei from a few MeV/nucleon to GeV/nucleon, together with new high-performance 4π detectors, has fueled a field of research loosely referred to as Nuclear Fragmentation. The characteristics of the fragments produced depend on the beam energy and the target-projectile combinations, which can be externally controlled [1-3]. Fragmentation experiments could provide information about nuclear matter properties to constrain the equation of state (EOS) [4]. To date, a method does not exist to determine the densities and temperatures reached during collisions that takes into account the genuine quantum nature of the system, which has been well known in other fields [5-7]. Long ago, Bauer stressed the crucial influence of Pauli blocking in the momentum distributions of nucleons emitted in heavy ion collisions near the Fermi energy [8]. We have recently proposed a method based on fluctuations estimated from an event-by-event determination of fragments arising after the energetic collision [9-11]. A similar approach has also been applied to observe multiplicity fluctuations experimentally in a trapped Fermi gas [12-14] and the enhancement of multiplicity fluctuations in a trapped boson gas [15]. We go beyond Refs. [12-15] by including quadrupole fluctuations as well to enable direct measurement of densities and temperatures for subatomic systems where it is difficult to obtain such information directly.

We apply the proposed method to the microscopic CoMD approach [16-22] which includes fermionic statistics. The quenching of multiplicity fluctuations resulting from the quantum nature of the system is found. The energy densities and temperatures calculated using protons and neutrons display a rapid increase around 3 MeV temperature, which is an indication of a first-order phase transition. This result is confirmed by the rapid increase of the entropy per unit volume in the same temperature region.

Recent experimental data on low density clustering in nuclear collisions, and comparisons to microscopic quantum statistical models, suggest that to reproduce the data, a Bose condensate may be needed [23,24]. We know that light nuclei display an α-cluster structure, exemplified by the so-called 'Hoyle' state in ^{12}C, i.e., the first excited state of such a nucleus which decays into 3α' s [25]. The fact that the ground state of nuclei could be made of α clusters could justify their copious production in heavy ion collisions near the Fermi energy. Preliminary experimental results on ^{40}Ca+^{40}Ca performed at the Cyclotron Institute at Texas A&M University show that events with large multiplicity of α-like (i.e., ^{12}C, ^{16}O, etc.) or d-like (i.e., ^{6}Li, ^{10}B, etc.) fragments are found [26]. At the same time, these effects raise the natural question of whether α clustering and production could be a signature of a BEC [27-29]. In fragmentation reactions, CoMD predicts large yields of α clusters, but the experimental yield is largely underestimated [16-22]. We think the role of bosons in the model has been missed. Therefore, we add boson correlations in the collision term and the boson yields are largely increased and closer to data. These features should be kept in mind when discussing a possible BEC in the model.

2. Determining density and temperature from fluctuations

A method for measuring temperature based on momentum quadrupole fluctuations of detected particles was proposed in Ref. [30]. A quadrupole moment in a direction transverse to the beam axis (z-axis) is defined to minimize non-equilibrium effects [9-11]. The average Q_{xy} is zero for a given particle type in the center of mass of the equilibrated emitting source. Its variance is given by the simple formula:

$$\langle Q_{xy}^2 \rangle = \int d^{3p}\, p_x^2 p_y^2 f(p)$$

where f(p) is the momentum distribution of particles. In Ref. [30], a classical Maxwell-Boltzmann distribution of particles with temperature T and mass m was assumed, which gives:

$$\langle Q_{xy}^2 \rangle = \frac{m^2 T^2 N}{4}$$

where N is the average number of particles. In heavy ion collisions, the produced particles do not follow classical statistics because of their quantum nature; the correct distribution function must be used in Eq. (1). Protons (p), neutrons (n), tritium (t), etc., follow Fermi-Dirac statistics [9-10], while deuterons (d), alphas (α), etc., should follow Bose-Einstein statistics [11].

Using the Fermi-Dirac distribution f(p) in Eq. (1), we obtain:

$$\langle Q_{xy}^2 \rangle = \frac{m^2 T^2 N}{4} \cdot \mathrm{FQC}$$

where FQC is the quantum correction factor. When _f is the Fermi energy of the nuclear matter and is the density, one can perform the low temperature approximation and expand FQC to order (T/ _f)^2. A detailed derivation can be found in Ref. [9]. Initially, we expected this to be sufficient when T/ _f $<$ 0.3 is fulfilled. It turns out that higher order terms are needed when T/ _f $>$ 0.3. Therefore, we parameterized the numerical result of FQC as a function of T/ _f, which is indistinguishable from the numerical result. Details can be found in Ref. [10]. We outline the results as:

$$\mathrm{FQC} = \begin{cases} 1 + \frac{\pi^2}{3}\left(\frac{T}{\varepsilon_f}\right)^2 & \text{(low order)} \\ \frac{1+a_1(T/\varepsilon_f)+a_2(T/\varepsilon_f)^2}{1+b_1(T/\varepsilon_f)} & \text{(higher order)} \end{cases}$$

In the extreme case T/ _f 1, the quantum correction factor FQC shows similar behavior in the low temperature approximation and when including higher order corrections. At high temperature T, FQC for higher order corrections converges to unity, where the classical limit is recovered as expected. The momentum quadrupole fluctuations in Eq. (2) depend on both T and _f, thus we need more information to determine both quantities.

Within the same framework we can calculate the multiplicity fluctuations of fermions [7,31-32]. Similar to the momentum quadrupole fluctuations, the low temperature approximation and results including higher order corrections are derived in Refs. [7,31-32], respectively. Since Eq. (3) is a function of T/ _f and in experiments or models one recovers the normalized multiplicity fluctuation, we express T/ _f as a function of the normalized multiplicity fluctuation for convenience. In the following, we will use x to replace T/ _f to simplify equations. Thus we have:

$$\frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle} = \begin{cases} \frac{\pi^2}{3} x & \text{(low order)} \\ \frac{x(1-0.5x)}{1+0.2x} & \text{(higher order)} \end{cases}$$

For x 1, the higher order correction result for T/ _f becomes 0.635x, which recovers the low temperature approximation result as expected. Once the normalized multiplicity fluctuation of fermions is measured from experimental data or models, one can easily derive the value of T/ _f from Eq. (4). Then one can substitute T/ _f into Eq. (3) to obtain FQC and solve Eq. (2) for T, where the momentum quadrupole fluctuation can also be measured in experimental data or models. Knowing T, we obtain the Fermi energy from Eq. (4). Then one can derive the density using _f = (ħ^2/2m)(3π^2)$^{2\ 3}$. Till now, the scenario for fermions is completed. The multiplicity fluctuation is the first quantity we should investigate when studying the properties of fermions.

For bosons, we need to use the Bose-Einstein distribution in Eq. (1). There is a difference from fermions. We need to consider temperatures below or above the critical temperature T__c. The critical temperature is given by:

$$T_c = \frac{2\pi\hbar^2}{mk_B}\left(\frac{\rho}{\zeta(3/2)}\right)^{2/3}$$

where is the Riemann zeta function and is the density. For T > T__c, we obtain:

$$\langle Q_{xy}^2\rangle = \frac{m^2T^2N}{4}\cdot \mathrm{BQC}$$

where BQC is the quantum correction factor for bosons. For T < T__c, the momentum quadrupole fluctuations are:

$$\langle Q_{xy}^2\rangle = \frac{m^2T_c^2N}{4}\cdot \mathrm{BQC}'$$

The quantum correction factor for bosons is always less than 1 above the critical temperature, thus the same quadrupole fluctuation implies a higher temperature in a Bose gas than in a classical gas. These features are in contrast to the behavior of fermion systems, for which the temperature is always smaller than the classical limit. The momentum quadrupole fluctuations depend on temperature and density through T__c in Eq. (5), thus we need more information to determine both quantities. We stress that Eqs. (6,7) are derived under the assumption of a non-interacting Bose gas. Interactions will change the results somewhat. However, from superfluid ^{4}He we know that the experimental critical temperature is not much different from the ideal gas result.

Within the same framework we can calculate the multiplicity fluctuations of bosons numerically. The multiplicity fluctuations diverge when T > T__c because the isothermal compressibility diverges for ideal bosons [7,31-32]. This phenomenon is not observed in experiments. Therefore, we need to include interactions between bosons (and fermions if present) near the critical point. We use Landau' s phase transition theory near the critical point. More details of Landau' s phase transition theory can be found in Ref. [11]. We obtain the normalized multiplicity fluctuations for bosons:

$$\frac{\langle(\Delta N)^2\rangle}{\langle N\rangle} = \begin{cases} 0.155|\tau|^{-1} & \text{for } \tau < 0 \\ 0.62|\tau|^{-1} & \text{for } \tau > 0 \end{cases}$$

where τ = (T - T_c)/T_c is the reduced temperature. For practical purposes, we parameterized the functions in Eq. (7) in terms of normalized multiplicity fluctuations x through:

$$\frac{\langle(\Delta N)^2\rangle}{\langle N\rangle} = 1.5963|\tau|^{-1} - 1.0077$$

where the quantum correction factor for bosons is BQC = (z^{-1}e^{ /kT} - 1)$^{-1}$, with z = e^{ /kT} being the fugacity which depends on temperature T and chemical potential connecting with T_c. Below T_c, z = 1. Therefore, similar to the fermions case, the multiplicity fluctuation of bosons is the first quantity to investigate. When T > T_c, one can use Eqs. (7,10,11) to calculate the temperature T and then use Eq. (9) to calculate the critical temperature T_c. It is straightforward to calculate the density of bosons using Eq. (5). When T < T_c, one can use Eqs. (5,6,8) to calculate the temperature and density of bosons.

3. Results and discussion

To illustrate the strength of our approach, we simulated ^{40}Ca+^{40}Ca heavy ion collisions at fixed impact parameter b = 1 fm and beam energies E_{lab}/A ranging from 4 MeV/A up to 100 MeV/A. Collisions were followed up to a maximum time of 1000 fm/c in order to accumulate sufficient statistics. The choice of central collisions was dictated by the desire to obtain full equilibration. However, this did not occur, especially at the highest beam energies, due to partial transparency for some events. For this reason, the quadrupole in the transverse direction, Eq. (1), was chosen. Furthermore, in order to correct for collective effects as much as possible, we defined a 'thermal' energy for protons as:

$$E_{\text{thermal}} = \frac{3}{2}E_{\perp} + \left[\frac{E_{\text{total}}}{A} - \frac{3}{2}\frac{E_{\perp}}{Z} - 8\text{ MeV}\right]$$

where E_{total} and E_ are the average total and transverse kinetic energies (per particle) of protons, 8 MeV is the average binding energy of a nucleon, Z is the total charge of the system, and N_p is the average number of protons emitted at each beam energy. For other particles, we use the same definition to calculate thermal energies. For a completely equilibrated system, the transverse kinetic energy (times 3/2) equals the total kinetic energy and the term in square brackets cancels. All the center-of-mass energy, E_{cm}/A, is converted into thermal energy (plus the Q-value). In the opposite case, say an almost complete transparency of the collision, transverse energy would be negligible and the resulting thermal energy would be small. Our approximation accounts for some corrections, and this becomes more exact when many fragment types are included in Eq. (12). However, this approximation might be important in experiments where only some fragment types are detected or if, because of the time evolution of the system, different particles are sensitive to different excitation energies, for instance if some particles are produced early or late in the collision.

[Figure 1: see original paper] Normalized multiplicity fluctuation versus excitation energy per particle. (Top panel) CoMD results for d and α particles. (Bottom panel) CoMD results for p, n, t and ^{3}He. Notice the change of scales in the two panels.

In Fig. 1, we show the normalized multiplicity fluctuations of particles from CoMD. The multiplicity fluctuation quenching for fermions is observed, analogous to Refs. [12-14]. Recently, Stein et al. examined experimental data and found similar multiplicity fluctuation quenching for fermions. More details can be found in Ref. [33]. These results are also confirmed in Mabiala' s experimental data [34]. Since the multiplicity fluctuations are obtained, we can use Eqs. (2-4) to extract the temperature and density of the system. Meanwhile, in the same framework, it is straightforward to derive other thermodynamical quantities. One such quantity is the entropy S. Details can be found in Ref. [11].

[Figure 2: see original paper] (Top panel) Excitation energy versus temperature. The full triangles refer to quantum temperatures; the open stars refer to classical temperatures from fluctuations; the open crosses refer to experimental data using double ratio thermometer from Ref. [35] obtained for mass number A = 60–100. (Middle panel) Energy density versus temperature. Full symbols refer to the higher order correction results and the open symbols refer to the low temperature approximation results. (Bottom panel) Entropy density versus temperature. The full symbols refer to the results from Ref. [10] and the open symbols refer to the results from particle ratio of the number of d to p(n) [4,36].

To better summarize the results, we plot in Fig. 2 the excitation energy per particle and the entropy density versus temperature. The so-called caloric curve is well studied in the literature and shows a well-defined mass dependence. In Fig. 2 we report the experimental data (open symbols) from Ref. [35], obtained in the mass region A = 60–100, which is closest to our system. Recall that the experimental temperatures were obtained using classical approximations, thus it is no surprise that they agree well with our classical results (open star). The classical calculation clearly shows a region of constant temperature (less than 6 MeV) which would indicate a phase transition. However, notice that the density changes with changing temperature. For this reason, one might wonder about the physical meaning of the caloric curve, and it could be better to investigate the energy density (middle panel). A rapid variation of the energy density is observed around 2 MeV for neutrons and 3 MeV for protons, which indicates a first-order phase transition. As we can see from the figure, the higher order correction results give small corrections while keeping intact the relevant features obtained in the lowest approximation. This again suggests that in the simulations the system is fully quantal. We also notice that Coulomb effects become negligible at T 3 MeV where the phase transition occurs. The smaller role of the Coulomb field in the phase transition has recently been discussed experimentally in the framework of Landau' s description of phase transitions [37-39]. The rapid increase of the entropy per unit volume (bottom panel) is due to the sudden increase of the number of degrees of freedom (fragments) with increasing T.

Comparing the charge particle distribution with experimental data shows that we cannot reproduce the experimental data completely. This is not surprising since we only have one fixed impact parameter in the model while the experimental data includes all possible impact parameters. The experimental filter should be taken into account as well, but these features are not relevant to our goals. The α yield is underestimated, a feature which cannot be corrected by the experimental filter. The important ingredient missing in the model is the possibility of boson-boson collisions (α-α, d-d, etc.) and correlations. Therefore, we propose a modification of the collision term in CoMD to include the possibility of α-α collisions. We refer to the modified version as CoMDα. We use the Minimum Spanning Tree method (MST) to identify α particles at each time step, same as the cluster identification in CoMD. First, one particle is chosen, then the three closest particles with the correct values of spin and isospin (i.e., two protons and two neutrons with opposite spin, respectively) are selected within a radius of 2.4σ (the value used in cluster identification) in coordinate space. If all conditions are fulfilled, we identify the four particles as an α. We run over all particles and determine all possible α particles. Each particle can belong only to one α. At each time step, we search for α-α pairs whose distance is smaller than 2.5 fm. We follow the mean free path method [1,40-41] and define a collision probability for the α-α pair:

$$P_{\text{coll}} = \frac{\sigma \cdot \Pi \cdot f_1 \cdot f_2 \cdot \rho(r_i) \cdot v \cdot dt}{\exp(V_c/E_k)}$$

where σ is the cross section, Π is the Bose-Einstein factor, f_i is the average occupation probability for α (i = 1, 2), (r_i) is the local density, v is the relative velocity of the two α particles, dt is the time step, and exp(V_c/E_k) is the Coulomb barrier correction factor where V_c is the Coulomb energy between the two αs and E_k is their relative kinetic energy. For simplicity, we take σ as the α-α geometric cross section in this study. Notice that in such an approximation, the strong resonances which lead to the formation of ^{8}Be are not included. We expect that such resonances will increase the α yields from ^{8}Be decay; however, we have not been able to implement this effect in the present model. If an α-α

collision occurs, we calculate the Bose-Einstein factor Π before the collision and Π' after the collision. If Π' > Π, the collision will be accepted; otherwise, it will be rejected. Thus, the Bose factors (1 + f) increase the probability of collision in contrast to the Pauli blocking factors [1-2]. This will produce fluctuations larger than Poissonian, which is a signature of a BEC. Meanwhile, if the α particle does not suffer any collision in that time step, one of its nucleons can collide with another nucleon subject to Pauli blocking. This might break the α into nucleons. We repeat the same simulations as before using CoMDα.

[Figure 3: see original paper] Normalized multiplicity fluctuation versus excitation energy per particle. (Top panel) CoMDα results for d and α particles. (Bottom panel) CoMDα results for p, n, t and ^{3}He. Notice the change of scales in the two panels. The d fluctuations keep increasing at high energies because they are produced from the decay of excited α clusters. Similarly for the large fluctuations observed for p and n.

Similar to Fig. 1, we plot the normalized multiplicity fluctuations of particles versus excitation energy per particle in Fig. 3. As we can see in the top panel, d- and α-normalized fluctuations are generally larger than one. The multiplicity fluctuations of fermions (bottom panel) are less than one for most thermal energies. These results are what we expect. Since we consider Pauli blocking for fermions and the Bose-Einstein factor for bosons, the quantum effects for fermions and bosons should show up through the multiplicity fluctuations even if the system is a mixture of fermions and bosons. When the thermal energy is very high, the normalized fluctuations of fermions are larger than one as well, suggesting that α particles are so excited as to emit nucleons or d which carry large fluctuations of the parent. We also notice that the thermal energy of CoMDα in Fig. 3 is larger than that of CoMD in Fig. 1 at the same beam energy. This simply tells us that we have more thermalization in CoMDα than in CoMD because of the large number of collisions in CoMDα, including the α-α collisions.

[Figure 4: see original paper] (Top panel) Reduced density versus reduced temperature for bosons assuming T < T_c; (Bottom panel) Reduced density versus reduced temperature for bosons assuming T > T_c.

In Fig. 4, we plot the reduced densities for d and α versus reduced temperatures, assuming the temperature is below the critical temperature (top panel) and above the critical temperature (bottom panel). From Fig. 4, one can see that below the critical temperature, the α densities are too high and unphysical. However, the densities of bosons are reasonable assuming the temperature is above critical temperature.

4. Conclusion

In conclusion, we have addressed a general approach for deriving densities and temperatures of fermions or bosons from quantum fluctuations (momentum quadrupole fluctuations and multiplicity fluctuations). For fermions, the higher order correction results are consistent with the low temperature approximation results at very low temperature. We have shown that for high temperatures and low densities, the classical result is recovered as expected. For bosonic systems, quadrupole and multiplicity fluctuations using Landau' s theory of fluctuations near the critical point for a Bose-Einstein condensate (BEC) at a given density are derived. We apply our approach to the simulation data of CoMD which includes fermionic statistics. The multiplicity fluctuation quenching for fermion particles, due to the quantum nature, is found. These results are also confirmed by recent experimental data investigations. We derived the energy densities and entropy densities at different excitation energies for p and n. Both quantities show a rapid variation in the same temperature region, indicating a possible first-order phase transition.

Considering that the possibility of boson-boson collisions and correlations is missing in CoMD, the alpha production is underestimated compared to experimental data. We proposed a modified version of the model, CoMDα, to include the possibility of α-α collisions. The relevant Bose-Einstein factor in the collision term is properly taken into account. This approach increases the yields of bosons relative to fermions closer to data. In the framework of CoMDα, we discussed the multiplicity fluctuations for particles and obtained the temperatures and densities for d and α. We suggest that multiplicity fluctuations larger than one for bosons, in contrast to fermion multiplicity fluctuations which are smaller than one, is a signature of a BEC in nuclei.

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Note: Figure translations are in progress. See original paper for figures.