Study of recursive model for pole-zero cancellation circuit (postprint)

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Date: 2023-06-18T00:00:00+00:00

Abstract

The output of charge sensitive amplifier (CSA) is a negative exponential signal with long decay time which will result in undershoot after C-R differentiator. Pole-zero cancellation (PZC) circuit is often applied to eliminate undershoot in many radiation detectors. However, it is difficult to use a zero created by PZC circuit to cancel a pole in CSA output signal accurately because of the influences of electronic components inherent error and environmental factors. A novel recursive model for PZC circuit is presented based on Kirchhoff' s Current Law (KCL) in this paper. The model is established by numerical differentiation algorithm between the input and the output signal. Some simulation experiments for a negative exponential signal are carried out using Visual Basic for Application (VBA) program and a real x-ray signal is also tested. Simulated results show that the recursive model can reduce the time constant of input signal and eliminate undershoot.

Full Text

Preamble

Study of Recursive Model for Pole-Zero Cancellation Circuit
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(Received July 5, 2013; accepted in revised form January 15, 2014; published online February 20, 2014)

Abstract: The output of a charge sensitive amplifier (CSA) is a negative exponential signal with a long decay time that results in undershoot after C-R differentiation. Pole-zero cancellation (PZC) circuits are commonly employed to eliminate this undershoot in many radiation detectors. However, due to inherent component errors and environmental factors, it is difficult to accurately cancel a pole in the CSA output signal using a zero created by a PZC circuit. This paper presents a novel recursive model for PZC circuits based on Kirchhoff' s Current Law (KCL). The model is established using a numerical differentiation algorithm between the input and output signals. Simulation experiments for a negative exponential signal are carried out using a Visual Basic for Applications (VBA) program, and a real X-ray signal is also tested. The results demonstrate that the recursive model can reduce the time constant of the input signal and eliminate undershoot.

Keywords: Pole-zero cancellation, Numerical analysis in time domain, Numerical simulations
DOI: 10.13538/j.1001-8042/nst.25.010403

Introduction

In radiation detection techniques, detector signals must be amplified before digitization. The amplification chain consists of two stages: a preamplifier and a main amplifier. The preamplifier is positioned as close as possible to the detector to maximize the signal-to-noise ratio while providing a low-impedance source for the main amplifier and a high-impedance load for the detector. The main amplifier then further amplifies and shapes the preamplifier output.

Charge sensitive amplifiers (CSAs) are extensively used as preamplifiers in X-ray fluorescence analyzers and well logging systems due to their low noise performance and efficient operation at high counting rates. The CSA output is a two-component signal characterized by a rapidly rising edge followed by a slow trailing edge that decays back to baseline with the CSA' s long time constant, rather than an ideal step function. At high counting rates, CSA output pulses can pile up on the tails of previous pulses, causing baseline drift, characteristic peak shifting, and degraded energy resolution. In severe cases, piled-up pulses may even block the subsequent amplifier stage. In multi-channel analyzers (MCA), pole-zero cancellation (PZC) techniques are employed to reduce the width of negative exponential signals, thereby decreasing pulse pile-up probability while maintaining good energy resolution at high counting rates.

Various PZC circuit implementations have been reported. One design utilized a single 24 pF capacitor with twenty parallel PMOS transistors to mitigate pile-up effects. Pawel Grybos proposed a continuous CSA feedback reset system with a novel PZC circuit architecture for high-rate input pulse conditions, which reduced the influence of DC voltage shifts caused by high pulse rates on the CSA feedback resistance and PZC circuit. Seino et al. employed an alternative technique to avoid energy resolution degradation at high counting rates without using a PZC circuit. However, accurately canceling a pole in the CSA output signal using a zero from the PZC circuit transfer function expressed in Laplace domain remains challenging due to component tolerances and environmental factors. This paper presents a novel recursive model for PZC circuits that eliminates undershoot and reduces the width of CSA output signals.

II. Pole-Zero Cancellation Circuit

Figure 1 [Figure 1: see original paper] shows the CSA analog circuit, which consists of a feedback capacitance C_f, a feedback resistance R_f, and an operational amplifier. A current signal generated in the detector is integrated on C_f, while R_f discharges C_f to prevent amplifier saturation when a series of charge pulses arrives at the CSA input.

When $G \cdot C_f \gg C_i + C_f$, where G is the amplifier open-loop gain and C_i is the CSA input capacitance, the CSA input voltage signal is given by Eq. (1), where Q represents the total charge collected in the detector (proportional to the deposited energy), and τ_f is the time constant with $\tau_f = R_f \cdot C_f$:

$$\nu_i(t) = \frac{Q}{C_f} e^{-t/\tau_f}$$

The feedback resistance R_f has intrinsic Johnson noise that can be minimized by selecting a larger R_f value. For applications requiring small signal extraction and noise minimization, R_f is often chosen in the MΩ range, resulting in a relatively long time constant τ_f. This long time constant induces prolonged undershoot after C-R shaping, potentially overloading subsequent amplifier stages and driving them into nonlinear operation, which compromises small-signal amplification capability. It also causes pulse pile-up that deteriorates energy resolution.

PZC circuits can eliminate undershoot resulting from the CSA' s long time constant. Figure 2 [Figure 2: see original paper] illustrates a PZC circuit. The input signal is defined as v_i and the output as v_o. R_{PZ} is a variable resistor that can eliminate undershoot when properly adjusted.

Equation (1) can be expressed in Laplace domain as Eq. (2):

$$V_i(s) = \frac{Q}{C_f} \frac{1}{s + 1/\tau_f}$$

The transfer function of the PZC circuit in Figure 2 is:

$$H(s) = \frac{s + 1/\tau_1}{s + 1/\tau_2}$$

where $\tau_1 = R_{PZ} \cdot C$ and $\tau_2 = (R \parallel R_{PZ}) \cdot C$. The output signal is therefore:

$$V_o(s) = V_i(s)H(s) = \frac{Q}{C_f}\frac{1}{s+1/\tau_f}\frac{s+1/\tau_1}{s+1/\tau_2}$$

The pole-zero cancellation condition requires $\tau_1 = \tau_f$. Under this condition, Eq. (4) simplifies to:

$$V_o(s) = \frac{Q}{C_f}\frac{1}{s+1/\tau_2}$$

The time-domain output $v_o(t)$ is obtained through inverse Laplace transform:

$$v_o(t) = \frac{Q}{C_f}e^{-t/\tau_2}$$

Thus, when $\tau_1 = \tau_f$, the CSA output becomes a negative exponential signal with a shorter decay time (time constant τ_2).

III. Numerical Analysis and Simulations

While Laplace transform facilitates signal analysis, it can be difficult to convert complex signals from time domain to Laplace domain, and time-domain characteristics may be lost in the process. This novel recursive model for PZC circuits is implemented based on analysis of digital C-R shaping methods and digital Sallen-Key low-pass filters.

A. Numerical Recursive Root

According to Kirchhoff' s Current Law (KCL), which states that the sum of currents entering a junction equals the sum leaving it, the voltage transmission in Figure 2 is described by:

$$\frac{v_i - v_o}{R_{PZ}} + C\frac{d(v_i - v_o)}{dt} = \frac{v_o}{R} + C\frac{dv_o}{dt}$$

Rearranging:

$$C\frac{d(v_i - v_o)}{dt} = \frac{v_o}{R} - \frac{v_i - v_o}{R_{PZ}} + C\frac{dv_o}{dt}$$

Analog signals can be converted to discrete series with small time intervals using high-speed ADC. A first-order numerical differentiation method solves Eq. (8). Letting $v_i = x[n]$, $v_o = y[n]$, and $dt = \Delta t$, Eq. (8) becomes:

$$R_{PZ} \cdot C \cdot (x[n] - y[n]) + (x[n] - x[n-1] - (y[n] - y[n-1])) = R \cdot C \cdot y[n]$$

where Δt is the sampling time period of the high-speed ADC, and $x[n]$ and $y[n]$ are the input and output data series, respectively. Defining $k_1 = \Delta t/(R_{PZ} \cdot C)$ and $k_2 = \Delta t/(R \cdot C)$, Eq. (9) simplifies to:

$$(1 + k_1 + k_2) \cdot y[n] = (1 + k_1) \cdot x[n] - x[n-1] + y[n-1]$$

Through mathematical transformation, Eq. (10) yields the numerical recursive root corresponding to the PZC circuit:

$$\begin{cases} y[n] = \frac{(1+k_1)\cdot x[n]-x[n-1]+y[n-1]}{1+k_1+k_2} & n \geq 1 \\ y[n] = 0 & n \leq 0 \end{cases}$$

This is the recursive model for the PZC circuit. Processing CSA output signals involves recursive application of Eq. (11). Different output signals can be obtained by varying the shaping parameters k_1 and k_2.

To verify the model theoretically, consider a standard negative exponential input signal $v_i = A \cdot e^{-t/\tau}$, where A is amplitude and τ is the time constant. Substituting into Eq. (7) yields:

$$\frac{A}{\tau}e^{-t/\tau} - \frac{dv_o}{dt} - \frac{A}{R_{PZ} \cdot C}e^{-t/\tau} - \frac{v_o}{R \cdot C} - \frac{v_o}{R_{PZ} \cdot C} = 0$$

Adjusting R_{PZ} such that $R_{PZ} \cdot C = \tau$ simplifies Eq. (13) to:

$$\frac{dv_o}{dt} + \frac{R_{PZ} + R}{R \cdot R_{PZ} \cdot C}v_o = 0$$

Letting $v_o = y$ and defining $b = (R_{PZ} + R)/(R \cdot R_{PZ} \cdot C)$, Eq. (14) becomes:

$$y' + b \cdot y = 0$$

This first-order linear homogeneous equation has the solution:

$$y = C \cdot e^{-bt}$$

where C is a constant. Thus, the PZC circuit output in time domain is a negative exponential signal without undershoot, with time constant $(R \parallel R_{PZ}) \cdot C$, equivalent to the conclusion from Eq. (6). This confirms that the time-domain model provides the same signal processing function as an actual PZC circuit.

B. Computer Simulations

A simulation platform was developed using VBA. The recursive PZC model and input signals were implemented programmatically. The simulations comprised four phases:

1. Standard Negative Exponential Signal Simulation

A standard negative exponential signal was generated using:

$$v_i = A \cdot e^{-t/\tau}$$

where A is amplitude and τ is the decay constant. Figure 3 [Figure 3: see original paper] shows the signal with $A = 2000$, $\tau = 200$ (time constant = 10 μs with $\Delta t = 50$ ns).

2. Digital C-R Shaping for Standard Negative Exponential Signal

A C-R shaping circuit was simulated (Figure 4 [Figure 4: see original paper]). Its numerical recursive root is:

$$\begin{cases} y[n] = \frac{x[n]-x[n-1]+y[n-1]}{1+k} & n \geq 1 \\ y[n] = 0 & n \leq 0 \end{cases}$$

where $k = \Delta t/(R \cdot C)$. Figure 4 shows input and output signals at $k = 0.01$ ($\Delta t = 50$ ns), demonstrating undershoot after C-R differentiation.

3. Recursive Model Processing for Negative Exponential Signal

The same standard signal from Figure 3 was processed using the recursive PZC model, with C-R shaping output as comparison. Figure 5 [Figure 5: see original paper] shows input and output signals at shaping parameters $k_1 = 0.005$, $k_2 = 0.05$ (equivalent to $R_{PZ} = 10$ kΩ, $R = 1$ kΩ, $C = 1$ nF, $\Delta t = 50$ ns). The results show that the recursive model eliminates undershoot and reduces the input signal' s time constant.

4. Recursive Model Processing for Nuclear Signal

A two-component exponential signal representing nuclear detector output was simulated:

$$\nu_i = A \cdot (e^{-t/\tau_1} - e^{-t/\tau_2})$$

where A is amplitude, τ_1 is the pre-CSA decay constant, and τ_2 is the CSA decay constant. A signal with $A = -2000$, $\tau_1 = 10$, $\tau_2 = 1000$ (CSA time constant = 50 μs at $\Delta t = 50$ ns) was generated with an added baseline. Setting $k_1 = 0.001$ according to cancellation requirements, Figure 6 [Figure 6: see original paper] shows minimal amplitude difference between original and output signals, with reduced width and no undershoot.

A real X-ray signal test validated the recursive model. The input pulse came from an SDD X-ray detector (Amptek), sampled by a 10-bit ADC at 20 MHz after CSA preamplification and C-R shaping. The digital output was transmitted to a PC for recursive model implementation. With input signal time constant of 3.2 µs, k_1 was set to 0.0156. Figure 7 [Figure 7: see original paper] shows that the real X-ray signal width was reduced and undershoot eliminated at different shaping parameters.

IV. Conclusion

This paper derives a numerical recursive solution for PZC circuits through time-domain numerical analysis and establishes the corresponding recursive model. Theoretical verification based on KCL confirms the model's accuracy. Computer simulations using a standard negative exponential signal and tests with real X-ray signals demonstrate that the recursive PZC model overcomes the limitation of losing time-domain characteristics inherent in Laplace domain analysis. The model enriches digital nuclear signal analysis methods and can be applied to real-time preamplifier signal processing and digital pulse shaping analysis, which are key techniques in digital nuclear instrumentation.

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Note: Figure translations are in progress. See original paper for figures.