A nuclear spectrum generator for semiconductor X-ray detectors (Postprint)

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Abstract

This paper describes the design of a nuclear spectrum generator for semiconductor X-ray detectors. The generator produces step ramp signals with random distributions in amplitude and time according to a specified reference spectrum. These signals are similar to those from actual semiconductor X-ray detectors and can be used to verify the spectral response characteristics of X-ray fluorometers, thereby facilitating improvement of their energy resolution. The generator produces step ramp signals whose amplitudes satisfy the probability density distribution function of any given reference spectrum through sampling based on a 32-bit randomizer. The system divides the time axis into 1024 intervals, calculating through random sampling the probability of signal occurrence in each interval and the average time between signals. The step ramp signals conform to an exponential distribution in time. Test results demonstrate that the system noise is less than 2.43 mV, and the output signals conform to a Poisson distribution in counting rate while satisfying the probability density distribution function of the reference spectrum in amplitude. The counting rate is adjustable, and the output conforms to the characteristics of signals from semiconductor X-ray detectors such as Si-PIN and silicon drift detectors.

Full Text

Preamble

A Nuclear Spectrum Generator for Semiconductor X-Ray Detectors

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Abstract

This paper presents the design of a nuclear spectrum generator for semiconductor X-ray detectors. The generator produces step ramp signals with random distribution in both amplitude and time according to a specified reference spectrum. These signals closely mimic those from actual semiconductor X-ray detectors and can be used to evaluate the spectrum response characteristics of X-ray fluorometers, thereby helping to improve their energy resolution. The system achieves amplitude distribution matching any given reference spectrum through sampling based on a 32-bit randomizer. Time intervals are divided into 1024 segments, and the system calculates the appearance probability of step ramp signals in different intervals along with the average time between intervals through random sampling. The output signals follow an exponential distribution in time. Test results demonstrate that the system noise is less than 2.43 mV, the output step ramp signals satisfy Poisson distribution in counting rate and match the probability density distribution function of the reference spectrum in amplitude. The counting rate is adjustable, meeting the characteristics of semiconductor X-ray detectors such as Si-PIN and silicon drift detectors.

Keywords: X-ray spectrum generator, Random sampling, Probability density distribution, Poisson distribution
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Introduction

Portable X-ray fluorometers are essential instruments for nondestructive material analysis, widely employed in mining, mineral processing, smelting, petrochemicals, environmental protection, commodity inspection, archaeology, and medicine [1, 2]. Improving their performance remains a continuous pursuit for hardware researchers.

An X-ray fluorometer consists of an X-ray source, detector, and multi-channel analyzer (MCA). Energy resolution of X-ray spectra is affected by numerous factors, including statistical fluctuations in X-ray-detector interactions, the detector’ s inherent energy resolution, and electronic noise from the preamplifier and MCA [2]. While high-resolution nuclear instruments require quality detectors and electronic components, reduced electronic noise and improved energy resolution can also be achieved through inverse convolution methods. For instance, Fernández Timón et al. simulated output signals from semiconductor alpha detectors using this approach in 2010 [3].

The MCA introduces electronic noise and ballistic deficit during signal processing, which degrades the fluorometer’ s energy resolution [4–6]. Improving resolution via inverse convolution treats the MCA board as a linear system to obtain the theoretical spectrum ($f_1[n]$) through deconvolution of the measured spectrum ($y_1[n]$) with the known system response function (h[n]) [7]. The theoretical spectrum represents the highest achievable energy resolution from the X-ray detector output, ignoring MCA electronic noise and ballistic deficit effects. The schematic block diagram of the inverse convolution method is shown in Fig. 1 [Figure 1: see original paper].

The first prerequisite for improving system energy resolution through inverse convolution is obtaining the MCA board' s system response function (h[n]). This requires a step ramp signal generator with a standard spectrum (standard spectrum generator). By feeding step ramp signals from this generator into the target MCA, the measured spectral line can be obtained. The system response function is then derived through inverse convolution of the measured and standard spectral lines. This response function can subsequently be applied in actual engineering surveys to enhance system energy resolution. All these operations assume the MCA behaves as a linear system, which holds true only within certain count rate ranges [8]. Therefore, measuring the system response function across different count rate ranges is necessary for practical applications.

This paper designs a standard spectrum generator specifically for semiconductor X-ray detectors based on these requirements. Such a generator forms the foundation for improving fluorometer energy resolution via inverse convolution. Its output step ramp signals must match those from semiconductor X-ray detectors, satisfying six key requirements: (1) amplitude distribution must follow the probability density distribution function of a random reference spectrum provided by a host computer; (2) time distribution must follow an exponential distribution [9]; (3) output count rate must be adjustable; (4) signal rise time must be less than 100 ns; (5) voltage swing must be approximately $±$2 V with amplitude resolution better than 1 mV; and (6) system noise must be below 4 mV.

Previous research has extensively simulated nuclear pulse signals from γ-ray detectors. Attwenger et al. [10] built a nuclear pulse signal generator using analog circuitry in 1969, while Yang et al. [11] developed one using a single-chip microcomputer in 1996. However, traditional generators simulating bi-exponential γ-ray output signals cannot satisfy arbitrary reference spectrum probability density functions in amplitude, making them unsuitable for testing or determining the system response function of X-ray fluorometers.

To the authors' knowledge, no prior work has reported on simulating step ramp signals from semiconductor X-ray detectors. This paper proposes a spectrum generator using random sampling methods based on a randomizer. The generator can test not only the MCA board' s system response function but also its maximum pulse throughput rate, linearity [12], and pulse pile-up rejection capability [13], enabling correction of various fluorometer performance indices [14, 15]. Additionally, it can generate various discrete nuclear signal waveforms for research on parameter detection, frequency spectrum analysis, and other digital processing algorithms [16, 17].

Hardware Design Scheme

The hardware implementation is based on FPGA technology. The circuit diagram is shown in Fig. 2 [Figure 2: see original paper] (the FPGA is omitted for analog circuit clarity). The system uses an FPGA as the core controller with a high-speed DAC as the signal output module. Step ramp signals are produced after amplifier circuit processing of the DAC output.

The FPGA communicates with the host computer to receive reference spectral data (the probability density distribution function for output step ramp signal amplitude) and the average count rate. It then performs random sampling calculations for both amplitude and time data, outputting control signals to the high-speed DAC, which generates corresponding current signals. After amplification, the generator produces step ramp signals that obey the specified probability density distribution in amplitude and exponential distribution in time.

Since semiconductor X-ray detector signals have rise times under 100 ns, the selected DAC and operational amplifier (OPAMP) must have settling times within 100 ns, with OPAMP bandwidth exceeding 100 MHz (including a 10× margin). Given the $\pm$2 V output voltage swing, the OPAMP slew rate must exceed 40 V/µs. Additionally, the chip must exhibit optimal linearity since system linearity affects the goodness-of-fit between output and reference spectra.

After comprehensive evaluation, the current-output AD768 was selected as the DAC, offering high resolution (16-bit), high speed (30 MSPS), short settling time (25 ns to 0.025% full-scale), and excellent linearity (1/2 LSB DNL@14 bits, 1 LSB INL@14 bits) [18]. For signal conditioning, a current-feedback OPAMP (OPA691) was chosen, featuring large bandwidth (190 MHz@G=2), high slew rate (2100 V/µs), low noise ($u_{NI} = 2.9$ nV/√Hz when f > 1 MHz), and short settling time (12 ns, settling to 0.02% for a 2 V step) [19].

According to the AD768 datasheet [18] and the design scheme in Fig. 2, when digital signal CODE is input to the AD768, the corresponding output voltage is (I_{OUT},B×R5 - I_{OUT},A×R4), where I_{OUT},B and I_{OUT},A are the output currents at pins 1 and 27, respectively. Since R4 = R5 in the circuit design, the amplitude resolution is 0.0763 mV, meeting system requirements. The AD768 contributes integral and differential nonlinearities of 0.305 mV and 0.153 mV, respectively, at 14-bit resolution, which also satisfies specifications.

Control of Step Ramp Signal in Amplitude and Time

A spectrum generator must address two fundamental questions after outputting one step ramp signal: what amplitude the next signal should have, and when it should be output. These questions require understanding the distribution characteristics of step ramp signals in both amplitude and time. Step ramp signals represent detections of characteristic X-rays emitted during atomic deexcitation. According to atomic transition laws, these signals follow an exponential distribution in time [9], while their amplitudes follow the probability density distribution function of the reference spectrum provided by the host computer.

Amplitude Control Algorithm

The output step ramp signal must satisfy the probability density distribution function of the reference spectrum in amplitude. Since the host computer provides different reference spectra, the amplitude distribution varies accordingly, necessitating a random sampling control scheme for amplitude.

Assuming a signal conversion gain of 10 mV/keV, maximum X-ray energy of 50 keV in the measurement spectral line, and 1024-channel resolution, the reference spectrum data[1024] from the host computer defines the occurrence probability for the nth channel (n = 0, 1, 2, ···, 1023) as data[n] divided by the total count data_{sum}[1023]. The amplitude corresponding to the nth channel is 500(n+1)/1024 mV.

The system implements amplitude and time sampling using a randomizer. Random numbers from the randomizer are uniformly distributed, meaning all values have equal probability. To achieve different probabilities, the system assigns different data intervals. As shown in Fig. 3 [Figure 3: see original paper], two arrays are used: the reference spectrum data[1024] (amplitude probability density distribution function) from the host computer, and the cumulative spectrum data_{sum}[1024]. The cumulative value data_{sum}[n] represents the sum of reference spectral data from the first n channels. The difference between consecutive cumulative values (data_{sum}[n] - data_{sum}[n-1]) equals the reference spectrum value data[n]. Consequently, the probability of a random number in the range 0 to data_{sum}[1023] falling between data_{sum}[n-1] and data_{sum}[n] is (data_{sum}[n] - data_{sum}[n-1])/data_{sum}[1023], which matches the probability of outputting the nth channel step ramp signal (data[n]/data_{sum}[1023]). The system determines output signal amplitude by searching for the range containing the random number within the cumulative spectrum data.

The amplitude sampling process works as follows: A 32-bit pseudo-random number p is generated by the FPGA randomizer. Taking p modulo data_{sum}[1023] yields random number q in the range 0 to data_{sum}[1023]. A successive approximation approach then searches for q' s position in the cumulative spectrum data. If q falls between data_{sum}[n-1] and data_{sum}[n], the output signal amplitude is set to 500n/1024 mV, thereby implementing the probability density function distribution in amplitude.

This analysis assumes q is uniformly distributed in 0 to data_{sum}[1023]. However, if the maximum random number M generated by the randomizer is not divisible by data_{sum}[1023], smaller values of q become slightly more probable than larger ones, creating a non-uniform distribution. For a 32-bit randomizer, M = 2^{32} - 1. When the total reference spectrum count data_{sum}[1023] does not exceed 1,000,000, the probability error (1/R) is less than: 1000000/(M - M%1000000) 0.00023288, where % denotes remainder. This error probability of <0.023% is acceptable. Equation (4) shows that increasing the randomizer bit width reduces this error probability (1/R). Additionally, the FPGA can adjust by discarding random numbers p outside the range 0 to (M - M%data_{sum}[1023]) and generating new ones to eliminate this error.

Time Control Algorithm

Step ramp signals follow a Poisson distribution in count rate. When the average count rate is m, the probability of observing n step signals within time t is given by [9]:

P(t, n) = $(mt)^n$e⁻ /n!

The condition for two adjacent step ramp signals separated by time interval t requires: no signals appearing within time t after the first signal (probability P(t, 0)), and one signal appearing in the subsequent infinitesimal time dt (probability P(dt, 1)). Since these events are independent, the probability dP(t) of adjacent signals separated by interval t is:

dP(t) = P(t, 0)P(dt, 1) = e⁻ × $(mdt)^1$e⁻ me⁻ dt

Therefore, the probability of step ramp signals appearing in time interval (c, d) is:

P(d > t > c) = _c^d dP(t) = e⁻ - e⁻

The average time for interval (c, d) is:

t̄(d > t > c) = c + _c^d t dP(t) = c + (c + 1/m)e⁻ - (d + 1/m)e⁻

After outputting the first step ramp signal, the system divides the possible appearance time of the second signal into x intervals, calculating each interval' s average time and appearance probability using Equations (8) and (7) (see Table 1). Random sampling based on these probabilities determines the next output time. For instance, if sampling yields 1.5–1.6 μs, the next signal should appear at the average time of this interval.

Table 1 lists probabilities and average times for different intervals when average count rate m = 52.194 k/s and time division x = 1024. Theoretically, 1024 data points exist, but space limitations show only 19 intervals.

The last two columns of Table 1 are used for FPGA time sampling. The process divides the signal appearance time into 1024 intervals based on average count rate m. Using Equations (7) and (8), it calculates the probability (Column 3) and average time data_{time}[1024] (Column 5) for each interval. The probability is multiplied by base number K = 10,000,000 and stored in array data_{pro}[1024] (time probability distribution array, Column 4). A cumulative time probability distribution array data_{{{pro}}{{sum}}}[1024] is then created, where data{{{pro}}{{sum}}}[n] equals the cumulative sum of the first n *elements of data*{pro}[1024]. The base number K theoretically equals the total count data_{{{pro}}*{{sum}}}[1023] of array data*{pro}[1024].

A random number p from the randomizer is taken modulo data_{{{pro}}*{{sum}}}[1023] to obtain random number q in range 0 to data*{{{pro}}*{{sum}}}[1023]. Successive approximation searches for q' s position in data*{{{pro}}*{{sum}}}[1024]. If q falls between data*{{{pro}}*{{sum}}}[n] and data*{{{pro}}*{{sum}}}[n+1], the next step ramp signal outputs at time data*{time}[n+1], achieving exponential time distribution.

Like amplitude sampling, time sampling must address non-uniform probability distribution when maximum random number M is not divisible by data_{{{pro}}_{{sum}}}[1023]. The base number K can be selected to ensure divisibility, such as 1,114,129; 3,342,387; or 5,570,645, rather than 10,000,000.

Testing shows that larger time division intervals x yield output signals whose count rate distribution more closely approximates Poisson distribution. However, excessively large x increases FPGA storage requirements and random number search time, consuming more FPGA resources and reducing output signal count rate. Considering these trade-offs, x = 1024 provides an appropriate balance.

Randomizer Design in FPGA [20]

The system employs a Linear Feedback Shift Register (LFSR) to generate uniformly distributed random numbers. The LFSR design is shown in Fig. 4 [Figure 4: see original paper], where feedback factors C_i (1 ≤ i ≤ f-1) are binary ("0" or "1"). Addition ignores carry, and multiplication is a selection operation.

The shift register equation in Fig. 4 can be expressed as:

X(t + D) = T X(t)

where D is one clock delay. Generally, an e-bit randomizer requires e clock cycles per random number. To accelerate generation, we iterate Equation (10) e times (e < f):

X(t + eD) = T X(t)

Thus, e-bit random numbers can be obtained in one clock cycle using T as the feedback network. This system uses e = 32, with f determined by feedback network complexity. Table 2 shows complexity for f ranging 45-50.

When f = 49, T has the fewest "1" s and shortest feedback chain, making it the optimal choice. Therefore, this system uses 49 shift registers.

Logical Design of Amplitude and Time Sampling in FPGA

Based on the theoretical calculations and methods described above, a hardware logic control block diagram within the FPGA is designed as shown in Fig. 5

[Figure 5: see original paper].

The diagram comprises storage, randomizer, amplitude sampling block, time sampling block, and output control. The PC transmits reference spectral data and average count rate to FPGA storage via serial port. The time sampling block divides time into 1024 intervals, calculating appearance probability and average time for each interval based on the average count rate, then samples to determine the next signal' s appearance time. The amplitude sampling block obtains cumulative spectrum data from the reference spectrum and samples amplitudes to determine the next signal' s amplitude. The output control block combines sampled time and amplitude to generate the step ramp signal.

The system uses EP2C8T144C8N as the FPGA chip. At a 100 MHz system clock, the FPGA logic resource utilization is shown in Table 3 .

The randomizer, time sampling block, amplitude sampling block, and output control can be implemented in parallel using a pipeline structure, significantly improving output signal speed.

Test Results

System testing focuses on waveform, noise, amplitude distribution, and time distribution of output step ramp signals, based on their characteristic properties.

Waveform Testing

Figures 6(a) and 6(c) show output signals from a silicon drift detector (SDD), while Figs. 6(b) and 6(d) show signals from the spectrum generator, both at approximately 2.5 k/s count rate. The signals from the SDD and spectrum generator are nearly identical, confirming the generator meets practical requirements.

System Noise Testing

Nuclear spectrum generator noise is evaluated through signal amplitude spectrum broadening. The generator outputs a series of constant-amplitude step signals into an MCA to obtain an amplitude spectrum. The full width at half maximum (FWHM) of this spectrum represents signal noise. Before testing, the MCA must be calibrated: the generator outputs two constant-amplitude signals (V_{P1} and V_{P2}), and the MCA measures their peak positions (x_1 and x_2). The voltage per channel is then (V_{P1} - V_{P2})/(x_1 - x_2). Noise voltage is calculated as:

FWHM_{NV} = (V_{P1} - V_{P2}) × FWHM / (x_1 - x_2)

The physical test system is shown in Fig. 7 [Figure 7: see original paper]. Module (a) is the designed nuclear spectrum generator, and module (b) is the digital multichannel analyzer (DMCA). In the DMCA, the CR differential shaping time constant is 3.2 μs and RC integral shaping time is 25 μs. Using this method at approximately 10 k/s output rate, system noise calculated by Equation (12) is 2.43 mV, demonstrating high performance.

Amplitude Distribution Testing

Output step ramp signals must satisfy the probability density distribution function of the given reference spectrum in amplitude. Using an X-ray fluorescence spectrum from gold jewelry as the reference, with output signals at 2 k/s, the system' s output was tested with a digital X-ray fluorometer. Spectra obtained at different measurement times were compared with the reference spectrum. In Fig. 8 [Figure 8: see original paper], the black solid line represents the reference spectrum, while dotted lines show Spectrum No. 1 and No. 2 obtained at total counts of 1,000,000 and 400,000, respectively. The measured spectra closely match the reference waveform. Though resolution covers 1024 channels, Fig. 8 displays only channels 300–700 for clarity.

Linear fitting between reference and measured spectra yields linearity of 0.9995, as shown in Fig. 9 [Figure 9: see original paper]. The reference spectrum contains 689,336 total counts. Table 4 provides total counts, fitting coefficients, and relative errors for Spectra No. 1 and No. 2. Relative fitting coefficient errors are within $±$0.076%, confirming that output step ramp signals satisfy the reference spectrum' s probability density distribution function in amplitude.

Time Distribution Testing

Equation (5) describes Poisson distribution. The step ramp signal count rate distribution was compared against theoretical Poisson distribution, with results in Table 5 . Data were collected at 52.194 k/s output rate, measuring 100,000 times for 1 ms each. Under these conditions, the probability peaks at 52 counts.

Table 5 shows that with extensive repeated measurements, the measured probabilities for different counts essentially match theoretical Poisson probabilities, with maximum relative error of $±$1.65%. This error decreases with longer measurement times. Therefore, the step ramp signal generator output satisfies Poisson distribution in count rate. Due to space constraints, Table 5 shows only probabilities and relative errors for 43–65 step signals at 1 ms measurement time.

Conclusion and Discussion

An arbitrary X-ray fluorescence spectrum generator has been developed. Through random sampling, its output step ramp signals satisfy Poisson distribution in count rate and the reference spectrum' s probability density function in amplitude, conforming to the distribution laws of semiconductor X-ray detector outputs. The generator connects to a PC for real-time modification of reference spectral data and signal output speed, with control software designed in C++. Additionally, reference spectral data for Ag, Fe, Cu, and Zn characteristic X-rays are stored in FPGA memory, enabling offline operation while maintaining the specified amplitude distribution.

This X-ray spectrum generator can measure the system response function of X-ray fluorometers under various count rates. Combined with inverse convolution methods, it can eliminate adverse effects of ballistic deficit and electronic noise on energy resolution, thereby improving system performance. The generator can also measure maximum pulse throughput rate, energy linearity, spectral line energy range, and dead time of X-ray fluorometers.

References

[1] Wang N X and Zheng H W. Nucl Sci Tech, 2006, 17: 11-15.
[2] Ge L Q, Zhou S C, Lai W C. In-Situ X Radiation Sampling Technique. Chengdu (China): Sichuan Science and Technology Press, 1997, 33-66, 150-212. (in Chinese)
[3] Fernández Timón A, Jurado Vargas M, Martín Sánchez A. Appl Radiat Isotop, 2010, 68: 941-945.
[4] Wang J J, Fan T M, Qian Y G. Nuclear Electronics. Beijing (China): Atomic Energy Press, 1983, 158-170. (in Chinese)
[5] Gal J, Hegyesi G, Kalink G, et al. Nucl Instrum Meth A, 1997, 399: 407-413.
[6] Zhou C Y, Su H, Kong J, et al. Nucl Sci Tech, 2012, 23: 239-
[7] He Z S. Signals and systems. Higher Education Press, 2008, 48-86. (in Chinese)
[8] Cheng X, Han C, Mats P, et al. Nucl Instrum Meth A, 2013, 715: 11-17.
[9] Fudan University, et al. Experimental Nuclear Physics. Beijing (China): Atomic Energy Press, 1985, 1-26. (in Chinese)
[10] Attwenger W, Gruber G, Patzelt R. Nucl Instrum Methods, 1969, 70: 103-105.
[11] Byung yang Ahn. in Proc. IEEE Asia Pacific Conference on Circuits and Systems, 1996, 11:18-21.
[12] Pan D J. Atom Energy Sci Technol, 1986, 4: 431-434.(in Chinese)
[13] International Electrotechnical Commission, IEC: standard No. 659, Edition, 1980.
[14] Ziegler F, Beck D, Brand H. Nucl Instrum Meth A, 2012, 679:
[15] Wiernik M. Nucl Instrum Methods, 1971, 96: 325-329.
[16] Mattingly J K and Mihalczo J T. Tennessee: Nuclear Materials Management and Storage Program Office March 13, 1998,1.
[17] Hamers H C and Marseille A. Physica, 1956, 22: 563-568.
[18] Analog Devices. 16-Bit 30MSPS D/A Converter AD768 Data Sheet.
[19] Texas Instruments. Wideband, Current Feedback Operational Amplifier with Disable OPA 691 Data sheet.
[20] Du X F and Wu J. J Univer Sci and Technol China, 2006, 9: 990-994. (in Chinese)

Note: Figure translations are in progress. See original paper for figures.