

## Mass thickness measurements for dual-component samples utilizing equivalent energy of X-rays postprint

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### Abstract

In this paper, equivalent energy method is introduced for measuring mass thickness of dual-component samples using dual-energy X-rays. Approximately, the method adopts equivalent mass attenuation coefficients of the two components in mass thickness measurements for dual-component samples, in a certain range of thicknesses. Feasibility of the method is proven by numerical calculations and Monte Carlo simulations (EGSnrc package). The results of absorption experiments using an X-ray machine at tube voltages of 30 and 45 kV, the relative errors are less than 5% between the nominal and detected values. Also, optical low energy is discussed at given high voltages.

### Full Text

### Preamble

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**Mass Thickness Measurements for Dual-Component Samples Utilizing Equivalent Energy of X-rays**

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This paper introduces the equivalent energy method for measuring mass thickness of dual-component samples using dual-energy X-rays. The method approximates the mass attenuation coefficients of the two components as constants over a certain thickness range. Feasibility is demonstrated through numerical calculations and Monte Carlo simulations (EGSnrc package). Absorption experiments using an X-ray machine at tube voltages of 30 and 45 kV show relative

errors of less than 5% between nominal and detected values. Optimal low energy selection is also discussed for given high voltages.

**Keywords:** X-rays, Optimal dual-energy, Dual-sample, Mass thickness

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## Introduction

X-ray sources are widely employed in non-destructive testing, where beam-hardening artifacts limit quantitative analysis capabilities and must be eliminated [1, 2]. In medical applications, researchers have studied optimal energies for dual-energy computed tomography and optimal tube voltage selection for chest imaging, addressing dual-energy CT challenges across various anatomical regions [3, 4]. This paper investigates mass thickness measurements for dual-component samples, using aluminum and plexiglass (PMMA, poly(methyl methacrylate)) as phantom materials for bone density measurements.

In previous work, we introduced the equivalent energy method for single-sample measurements [5], demonstrating a log-linear absorption rate across a range of material thicknesses when using an additional filter, validated through numerical calculations and experimental measurements. This study extends the method to mass thickness measurements of dual-component samples (Al and PMMA) through numerical calculations and absorption experiments.

## II. Materials and Methods

### A. Equivalent Energy Method

Electron beam-induced X-rays consist of continuous and characteristic radiation. In the equivalent energy method, a suitable filter absorbs characteristic photons and pre-hardens the X-ray spectrum at a specific voltage. The hardened spectra exhibit a peaked energy distribution, making the beam-hardening effect negligible for a certain range of sample thicknesses.

For current-mode detection, the current signals from dual materials with mass thicknesses  $M_1$  and  $M_2$  are proportional to the photon energy and number of photons deposited in the detector. For dual-component samples, Eq. (1) can be obtained:

$$i(M_1, M_2) = ki'(M_1, M_2) = k \int_{E_0} \int_{E'} qE(E_0 - E)e^{-\mu_1(E)M_1 - \mu_2(E)M_2} dE, \\ qE(E'_0 - E)e^{-\mu'_1(E)M_1 - \mu'_2(E)M_2} dE,$$

where  $k$  is the detector transmission factor;  $E_0$  and  $E'$  are maximum photon energies corresponding to low and high voltages ( $V_L$  and  $V_H$ ), respectively;

$\mu_1(E)$ ,  $\mu'_1(E)$ ,  $\mu_2(E)$  and  $\mu'_2(E)$  are mass attenuation coefficients of the two components at  $V_L$  and  $V_H$ , respectively; and  $q$  is the charge activated per unit photon energy. The components  $k \int_{E_0}^{\infty} qE(E_0 - E)dE$  and  $k \int_{E'}^{\infty} qE(E'_0 - E)dE$  represent the original signals before X-ray attenuation in the sample, assuming 100% detection efficiency. Solving Eq. (1) requires expressing  $M_1$  and  $M_2$  as functions of  $i$  and  $i'$ , which becomes computationally intensive when processing large datasets.

Using the equivalent energy method, the coefficients  $\mu_1$ ,  $\mu'_1$ ,  $\mu_2$  and  $\mu'_2$  become constants, allowing Eq. (1) to be written as:

$$\begin{cases} \ln(i_0/i) = \mu_1(E_{eq})M_1 + \mu_2(E_{eq})M_2, \\ \ln(i'_0/i') = \mu'_1(E'_{eq})M_1 + \mu'_2(E'_{eq})M_2, \end{cases}$$

where  $i_0 = k \int_{E_0}^{\infty} qE(E_0 - E)e^{-\mu_0(E)M_0}dE$  and  $i'_0 = k \int_{E'}^{\infty} qE(E'_0 - E)e^{-\mu'_0(E)M_0}dE$  are the initial X-ray intensities at  $V_L$  and  $V_H$ , respectively;  $i$  and  $i'$  are the transmitted X-ray intensities at  $V_L$  and  $V_H$ , respectively;  $\mu_0(E)$  and  $\mu'_0(E)$  are the mass attenuation coefficients of the filter at  $V_L$  and  $V_H$ , respectively;  $M_0$  is the filter mass thickness;  $\mu_1(E_{eq})$ ,  $\mu'_1(E'_{eq})$ ,  $\mu_2(E_{eq})$  and  $\mu'_2(E'_{eq})$  are the equivalent mass attenuation coefficients of the two materials at equivalent energies  $E_{eq}$  and  $E'_{eq}$ , respectively. These equivalent coefficients are determined by voltage, filter material, and sample thickness factors. Eq. (2) is linear in the two unknown materials ( $M_1$  and  $M_2$ ) when other parameters are known, reducing computation time and improving accuracy. In this paper, dual-component samples consist of different pieces of 0.35-mm thick aluminum and 2-mm thick plexiglass.

## B. Numerical Calculations

The photon absorption process can be well computed considering the effects of photoemission, Compton scattering, Rayleigh scattering, and electron pair effect ( $E > 1.02$  MeV). Based on Kramer's continuous spectra formula and detection principles, mass thickness measurement results can be calculated through numerical calculations.

## C. Monte Carlo Simulation

Monte Carlo simulations using EGSnrc packages are widely employed in absorption simulation [6, 7], capable of simulating electron and photon transport in arbitrary geometries for particle energies from a few keV to several hundred GeV [8]. The geometry configuration used in our simulation is shown in Fig. 1 [Figure 1: see original paper]. A total of  $10^8$  particle tracks were simulated, with photons passing through samples scored while excluding scattered ones.

## D. Experiments

The experimental setup is shown in Fig. 1. The X-ray generator with a Cu target is from Dandong High-voltage Device Factory, Liaoning, China. A 0.2-mm thick Ni filter is used at low tube voltages, while 1-cm thick Cu is used at high tube voltages. Collimated X-rays pass through the dual-component sample (0.35-mm aluminum and 2-mm plexiglass). The detector uses a GOS scintillator and silicon photodiode. An HPGc detector with a 0.127-cm thick Al window is also employed.

## E. Error Analysis

This section discusses errors of the equivalent energy method at different tube voltages. From Eq. (2),  $M_1$  and  $M_2$  depend closely on the mass attenuation coefficients of the two components. According to NIST data [9], mass attenuation coefficients of both aluminum and plexiglass decrease with increasing photon energy in the 10–100 keV range, but become insensitive above 100 keV. Therefore, low-energy X-rays ( $E < 100$  keV) play a relatively important role in determining these coefficients.

In this study, high-energy X-rays generated at 140 kV tube voltage are used, with low tube voltages of 50, 60, 70, 80, and 90 kV selected to study their effects on mass attenuation coefficients. Taking the solution for  $M_1$  as an example, according to Eq. (2),  $M_1$  can be expressed as:

$$M_1 = [\mu_2 \ln(i'_0/i') - \mu'_2 \ln(i_0/i)] / (\mu'_1 \mu_2 - \mu_1 \mu'_2).$$

Assuming detector readings at different voltages are uncorrelated, for a small change in  $M_1$ , Eq. (3) yields the uncertainty:

$$\sigma_{M_1} = \{[(\sigma_{i'}/i')\mu_2]^2 + [(\sigma_{i_0}/i_0)\mu'_2]^2\}^{1/2} / |\mu'_1 \mu_2 - \mu_1 \mu'_2|,$$

where  $\sigma_{i'}/i'$  and  $\sigma_{i_0}/i_0$  are uncertainties in dose at  $V_H$  and  $V_L$ , respectively, expressed as  $1/\sqrt{N}$  where  $N$  is the photon count integrated over the entire energy spectrum. Only detector uncertainties are considered, as the effective dual-energy has been identified.

## III. Results and Discussion

### A. Numerical Calculations and Monte Carlo Simulation

Figure 2 [Figure 2: see original paper] shows  $\ln(i_0/i)$  as a function of mass thickness for aluminum and plexiglass, at 70 kV tube voltage with 0.2-mm Ni filter, and at 140 kV with 1-cm Cu filter. Results were obtained by Monte Carlo simulation using EGSnrc codes. The equivalent attenuation coefficients remain

similar across the investigated mass thickness range for both materials, demonstrating the feasibility of the equivalent energy method for dual-component sample measurements.

According to Eq. (2),  $\mu_1$ ,  $\mu_1'$ ,  $\mu_2$  and  $\mu_2'$  must first be identified to solve for  $M_1$  and  $M_2$ . Approximately, the equivalent mass attenuation coefficients of the single components can be used, though this introduces additional errors since X-rays are hardened more in dual-component samples than in single-component ones. These errors were studied through numerical calculations at tube voltages of 70 and 140 kV with various thickness combinations, with results given in Table 1. Relative errors remain below 2% for both components as X-ray energy increases.

## B. Experimental Errors

Experimental results for mass thickness measurements of dual-component samples (Al and PMMA) are presented in Table 2. The two X-ray energies were 30 and 45 kV, with tube current set to 30 mA to ensure sufficient X-ray intensity. The nominal values are provided by the manufacturer. Detector output exceeding 10,000 counts was ensured to reduce background effects. Table 2 shows errors below 5% within the investigated thickness ranges of the dual-component samples.

For quantitative characterization of spectrum hardening, X-ray spectra with and without a 1.4-cm thick PMMA sample were measured at 35 kV using an HPGe detector (Fig. 3 [Figure 3: see original paper]). A 0.2-mm Ni filter was used. The peak positions of the two spectra are close, indicating that equivalent X-ray energy can be treated as constant in experiments using the HPGe detector with a 0.127-cm Al window. Results show that equivalent energy in experiments is near the peak position.

## C. Optimal Low Voltage

The squared statistical counting error ( $1/N$ , where  $N$  is photon number) as a function of low voltage is shown in Fig. 4 [Figure 4: see original paper]. Results were obtained by EGSnrc Monte Carlo simulation for different Al and PMMA thickness combinations, assuming 100% detection efficiency. Statistical errors are insensitive to thickness combinations within the investigated range; the curves for “2Al + 3PMMA” and “4Al + 1PMMA” overlap in Fig. 4. At low voltages above 60 kV, statistical errors for all thickness combinations become approximately constant.

Equivalent mass attenuation coefficients of Al and PMMA at low voltages of 50–90 kV with 0.2-mm Ni filter were calculated and can be fitted by exponential functions of low voltage ( $x$ ) with  $R^2 > 0.999$ :

$$\begin{aligned}\mu_1 &= 4.36341e^{-x/23.21801} + 0.29753, \\ \mu_2 &= 0.46335e^{-x/27.13392} + 0.18987.\end{aligned}$$

Substituting factors  $\mu_1, \mu_2, \sigma_1/i, \sigma_2/i$  and  $\sigma\{i\}/i$  into Eq. (4) yields errors in  $M_1$  caused by statistical errors. Results at high voltages of 130 and 140 kV are shown in Fig. 5 [Figure 5: see original paper], where each curve corresponds to a thickness combination from Table 1. Minimum error occurs at low voltages of approximately 60 and 65 kV for high voltages of 130 and 140 kV, respectively. At 130 kV high voltage, different low voltage choices produce error variations from 0.05 g/cm<sup>2</sup> to 0.08 g/cm<sup>2</sup>.

This study focuses on aluminum and plexiglass dual-component samples for human bone density measurement applications, where the human body can be approximated as dual-component samples similar to aluminum (bone mineral) and plexiglass (water or soft tissue). However, the method should be effective for other dual-component materials if equivalent energy can be determined, and the numerical calculation model will be helpful when material properties are known.

#### IV. Conclusion

The equivalent energy method is introduced for mass thickness measurements of dual-component samples. This approach solves linear equations rather than integral equations, providing a simple, fast method with low detector requirements. Based on known dual-component composition, it can be applied to DEXA (dual-energy X-ray absorptiometry) and dual-energy CT for diagnostics of human body or printed circuit boards. With material property knowledge, feasibility can be predicted through numerical calculations to guide experiments. Monte Carlo simulation results agree well with numerical calculations in this study. For investigated Al and PMMA thickness combinations, numerical result relative errors are less than 2%. Considering counting statistical errors, an optimal low voltage is identified for a given high voltage. As an effective method for examining dual-component sample mass thicknesses, the equivalent energy method can be well applied in clinical and industrial settings.

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