Pulsed intense electron beam emittance measurement Postprint

Authors: ZHANG Zhuo, JIANG Xiao-Guo

Date: 2023-06-18T00:00:00+00:00

Abstract

Recently we measured with the Modified Three Gradient Method (MTGM) [1, 2] the beam emittance of an injector constructed in 2012, which was designed to provide a 2.4 kA, 2.6 MeV electron beam. The MTGM is a non-intercept indirect method, which is based on the three gradient type measurements of beam profiles and subsequent data processing which helps to get the least square solution to the beam emittance. Beam profiles under different currents of guiding coil are measured using Cerenkov radiation given off by a piece of quartz glass in the beam tube, which is recorded with a CCD camera. MTGM Code is developed to realize the data fitting as well as beam transport simulation, in which both the σ matrix method and the numerical solution of root-mean-square beam envelope equation are used. The error is also analyzed.

Full Text

Preamble

Pulsed Intense Electron Beam Emittance Measurement

ZHANG Zhuo (张卓)[1],* and JIANG Xiao-Guo (江孝国)[1]

[1]Institute of Fluid Physics, China Academy of Engineering Physics, Mianyang 621900, China

(Received January 22, 2014; accepted in revised form May 19, 2014; published online December 15, 2014)

Abstract. We recently measured the beam emittance of an injector constructed in 2012 using the Modified Three Gradient Method (MTGM). This injector was designed to provide a 2.4 kA, 2.6 MeV electron beam. The MTGM is a non-intercepting, indirect method based on three gradient-type measurements of beam profiles and subsequent data processing to obtain the least-square solution for beam emittance. Beam profiles under different guiding coil currents are measured using Cerenkov radiation emitted from a quartz glass plate in the beam tube, which is recorded with a CCD camera. MTGM code was developed to perform data fitting and beam transport simulation, employing both the σ matrix method and numerical solution of the root-mean-square beam envelope equation. Error analysis is also presented.

Keywords: Emittance measurements, High current beam, Modified three gradient method
DOI: 10.13538/j.1001-8042/nst.25.060201

Introduction

In most applications, emittance and brightness are the primary figures of merit for particle beams. The horizontal and vertical emittances relate to beam brightness, defined as $B = I/(\pi^2 \varepsilon_x \varepsilon_y)$, where the horizontal (or vertical) emittance is typically defined as the area (optionally divided by π) of the ellipse in x-x' phase space that contains 95% of all particles. Low beam emittance is essential for achieving the required spot size at the beam line's output focus.

Liouville's theorem [1] states that phase space volumes are invariant for Hamiltonian systems. However, in linear accelerators where particle energy varies, emittance is not invariant. Instead, we define the normalized emittance $\varepsilon_n = \beta\gamma\varepsilon$, which is conserved during acceleration.

Conventional emittance measurement techniques include the pepper-pot method [2], the three gradient method [3], and the Modified Three Gradient Method (MTGM) [4,5]. We employ MTGM due to its advantages, particularly its online feasibility as a non-destructive procedure. The main concept of MTGM is illustrated in [Figure 1: see original paper]. The method relies on three gradient-type measurements of beam radius and data fitting to the measured crossover curve.

Under a given guiding coil current, assuming ε and initial conditions (R_0, R_0') are known, the root-mean-square beam envelope of an axially symmetric beam is described by the following equation [6]:

$$R''_{rms} + k^2 R_{rms} - \frac{2K}{R_{rms}} - \frac{\varepsilon^2}{R^3_{rms}} = 0 \tag{1}$$

where $K = \frac{I_b}{17045\beta^3\gamma^3}$. Here, R_{rms} is the beam radius and ε_{rms} is the beam emittance normalized to beam energy. It is found that ε_{rms} remains constant during non-accelerating transport. The subscript "rms" denotes root-mean-square, which will be omitted hereafter for brevity.

Note that all ε values discussed below refer to edge emittance normalized to energy. The parameter k represents half the cyclotron wavenumber, with B_z being the axial magnetic field determined by the guiding coil current. K is the generalized perveance coefficient, with I_b being the beam current measured by current viewing resistors (CVRs). The relativistic factors β and γ are calculated from beam energy, which is measured using a capacitive probe mounted between accelerator cells.

Equation (1) can be solved numerically by converting it to a first-order differential equation system, which is then integrated using the Runge-Kutta method [7]. Solving the equation under different magnetic fields generates a set of beam radius data, from which a crossover curve revealing the relationship between beam radius at the analysis plane and magnetic field can be constructed. Conversely, if the crossover curve can be obtained experimentally, (ε, R_0, R_0') can be deduced through data fitting. In our work, the σ matrix method is used as an aid to find the least-square fit to the crossover curve.

Measurements

Beam profiles are measured using Cerenkov Radiation (CR) [8], which is emitted when charged particles traverse a transparent dielectric medium at velocities exceeding the speed of light in that medium. This technique is preferred in experiments due to its well-defined emission direction, rapid response time, and the proportional relationship between photon yield and electron number.

The CR light is emitted at the analysis plane, located 574.5 mm downstream from the center of the guiding coil. It is reflected by a mirror and captured by a CCD camera, then transmitted via optical fiber to an online computer. The experimental setup is shown schematically in [Figure 2: see original paper].

The parameters of the solenoid coils in the experimental beam line are listed in . Images of beam profiles under different focusing currents are shown in [Figure 3: see original paper]. Scanning these images radially produces grayscale curves from which beam radii are extracted (see in Section III). At 11 A, the beam profile reaches its minimum size.

Simulation Results and Discussion

MTGM has been applied to measure the emittance of a space-charge-dominated electron beam and has proven effective. The magnetic field distributions under different current supplies were calculated. [Figure 4: see original paper] plots the magnetic field curve from $z_0 = 3$ m to $z_0 = 4.642$ m, revealing the proportional relationship between magnetic field and supply current.

The calculation starting point is selected at $z = 3$ m, where the magnetic field of MC01 is negligible and initial conditions remain relatively constant across different measurements.

MTGM code was designed to determine $(\varepsilon, R_0, \tilde{R}_0')$ by performing least-square fitting to the measured (R_i, I_i) data. The beam envelope equation is a second-order differential function with varying coefficients, whose explicit solution is difficult to obtain through direct integration [9]. However, the σ matrix method provides a concise way to establish the direct relationship between R and initial conditions, enabling the use of least-square data fitting [10]. In fact, the σ matrix method serves as an independent beam transport solution technique, though it complements the beam envelope method in this application.

After extensive manipulation, we obtain the following functions. By solving the equation set, we can determine the solution $(R_0^*, \varepsilon^*, \tilde{R}_0'^*)$ that provides the least-square approximation to the measured data:

$$R = \sqrt{M_{11}^2 R_0^2 + 2M_{11}M_{12}R_0\tilde{R}_0' + M_{12}^2\tilde{R}_0'^2} \tag{2}$$

$$\tilde{R}' = \sqrt{\frac{R^2 - M_{11}^2 R_0^2 - 2M_{11}M_{12}R_0\tilde{R}_0' - M_{12}^2\tilde{R}_0'^2}{M_{11}^2}} \tag{3}$$

The least-square solution for $(\varepsilon, R_0, \tilde{R}_0')$ is listed in .

TABLE 2. The least-square solution of $(\varepsilon, R_0, \tilde{R}_0')$

Parameters	Values
R_0 (mm)	[value]
$\tilde{R}_0'$ (mrad)	[value]
ε^* $(\pi \cdot mm \cdot mrad)$	[value]

Here, M is the transport matrix of the entire beam line [11]. Special attention must be paid to $\tilde{R}_0'$ associated with the σ matrix, as it is not the derivative of R with respect to z, but rather the beam angular envelope—the maximum r_0' in phase space [12]—which can be calculated by solving the ellipse function in phase space. In fact, $(\varepsilon, R_0, \tilde{R}_0')$ represents a set of initial conditions for the σ matrix method, equivalent to (ε, R_0, R_0') in the beam envelope equation. Both sets can be derived from one another by differentiating Eq. (3) with respect to z at $z = z_0$. The sign of the second term is determined by the ellipse orientation in phase space: negative for converging beams and positive for diverging beams.

The least-square method requires minimizing $\|\delta\|^2 = \sum_i [R_{(ii)} - R_i]^2$. This demands zero derivatives of $\|\delta\|^2$ with respect to ε, R_0, and R_0':

$$\frac{\partial\|\delta\|^2}{\partial\varepsilon} = 0, \quad \frac{\partial\|\delta\|^2}{\partial R_0} = 0, \quad \frac{\partial\|\delta\|^2}{\partial R_0'} = 0$$

A three-dimensional equation set for R_0, R_0', and ε can be obtained by substituting the expressions for R and $\|\delta\|^2$.

Using the $(\varepsilon^*, R_0^*, \tilde{R}_0'^*)$ values above, the numerical solution of the beam envelope equation is obtained with the fourth-order Runge-Kutta method. The measured R-I data and calculated values are listed in .

TABLE 3. Radii versus focusing current

I (A)	R_i (mm)	$R(I_i)$ (mm)	δ_i (mm)
[value]	[value]	[value]	-0.15
[value]	[value]	[value]	-0.68
[value]	[value]	[value]	-3.41

The crossover curve corresponding to $(\varepsilon^*, R_0^*, \tilde{R}_0'^*)$ is plotted in [Figure 5: see original paper], and the beam envelopes are shown in [Figure 6: see original paper].

The method's precision requires sufficient data acquisition across a wide range, which is achievable by carefully selecting the distance between the guiding coil center and analysis plane, as well as their positions within the beam envelope evolution.

We can rewrite Eq. (1) as:

$$R''_{rms} = -k^2 R_{rms} + \frac{2K}{R_{rms}} + \frac{\varepsilon^2}{R_{rms}^3}$$

Here, the left term represents a force on the beam envelope, while the right-side terms represent contributions from different forces: the focusing force from axial magnetic field B_z and defocusing forces from space charge effects and beam emittance. This becomes clearer when examining the three functions individually.

The focusing effect of the axial magnetic field over a small step can be described by:

$$R = \sqrt{(kR_0)^2 + R_0'^2} \sin\left[k(z - z_0) + \arctan\left(\frac{R_0'}{kR_0}\right)\right]$$

The effective emittance force describes beam transport in drift space:

$$R = \sqrt{(R_0 R_0')^2 + \varepsilon^2 (z - z_0)^2 + R_0^2 R_0'^2}$$

This equation defines a hyperbolic evolution of the beam envelope under emittance effects, with the minimum beam radius given by:

$$R_{\varepsilon,min} = \frac{\varepsilon}{R'_0}$$

The space charge effect cannot be expressed explicitly. However, integrating the equation over a small step yields:

$$R' = \pm\sqrt{R_0'^2 + K\ln\left(\frac{R^2}{R_0^2}\right)}$$

or equivalently:

$$R = R_0 e^{-\frac{R'^2 - R_0'^2}{2K}}$$

This equation represents the expanding effect of space charge force, with the minimum beam radius restricted to:

$$R_{sc,min} = R_0 e^{-\frac{R_0'^2}{2K}}$$

When R is large, or at positions far from the crossover curve minimum where magnetic focusing dominates, R is sensitive to magnetic field variations. However, near the crossover minimum where R is small, the radius is primarily determined by emittance and space charge forces. Therefore, accurately locating the crossover curve's minimum is crucial for determining beam emittance. A practical approach is to acquire as much data as possible near the crossover minimum by reducing the guiding current step size. Repeated measurements at the same current also help eliminate intrinsic beam size uncertainties.

The main error sources are: (1) beam radius measurement error; (2) beam energy and current errors; (3) MTGMCODE errors. Detailed error analysis has been performed elsewhere [13], revealing that source (1) significantly affects precision while (2) and (3) are relatively minor.

We simulated beam transport for various emittances increasing arithmetically from 120 to 1720 π mm mrad, as shown in [Figure 7: see original paper]. Each step between adjacent curves is 20 π mm mrad. By taking partial derivatives of Eq. (3) with respect to ε, R_0, and $\tilde{R}'_0$, we obtain the error formula:

$$\Delta R = \alpha_{R_0}\Delta R_0 + \alpha_{\tilde{R}'_0}\Delta\tilde{R}'_0 + \alpha_\varepsilon\Delta\varepsilon$$

Here, R represents the least-square fit to measured data. For each point in [Figure 7: see original paper], the emittance error is roughly proportional to the radius error:

$$\Delta\varepsilon(\pi \cdot mm \cdot mrad) \approx (200 \sim 1600)\Delta R(mm)$$

This indicates that near the crossover minimum, small radius measurement uncertainties can produce large emittance calculation errors—approximately 8 times greater than those far from the minimum. For example, at the crossover bottom, a 1 mm radius measurement uncertainty causes about 160 π mm mrad emittance error, while at the curve's upper portion, the same uncertainty yields only about 20 π mm mrad error—one-eighth of the former.

The relative error is:

$$\eta = \frac{\Delta\varepsilon}{\varepsilon}$$

Under our measurement conditions, η ranges from 6 to 10, with 10 at the plot's lower portion and 6 at the upper portion. The parameters for each curve are listed in , including emittance and initial values of radius and divergence angle.

TABLE 4. Corresponding Parameters

ε (π mm mrad)	R_0 (mm)	R_0' (mm)
[value]	[value]	[value]

Conclusion

Results from applying the Modified Three Gradient Method (MTGM) to a pulsed high-intensity electron source are presented. The method, experimental setup, and results for non-destructive beam emittance measurements are described. MTGM enables non-destructive emittance determination in the presence of space charge effects. The approach integrates beam cross-section measurements in a three-gradient configuration with the beam envelope equation for axially symmetric beams. Experimental data is processed using a numerical matching program to determine emittance and beam diameter. The program employs the σ matrix method for least-square fitting to the measured crossover curve, while the envelope equation is solved using the fourth-order Runge-Kutta method to obtain beam radius. Error analysis is also presented. For the experimental beam, the normalized edge emittance is approximately 112 π mm mrad.

Acknowledgments

The authors thank the members of the testing and operating team for their contributions to this work.

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Note: Figure translations are in progress. See original paper for figures.