Inversion Methods for Lunar Regolith Dielectric Constant and Applications (Postprint)

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Abstract

Lunar exploration constitutes humanity's first step in space exploration. The dielectric constant is a parameter that describes a material's capacity to store and release energy under an external electric field; measurements of the dielectric constant of lunar regolith facilitate investigations into regolith thickness and subsurface structure, thereby enhancing understanding of the Moon's origin and evolutionary history. Humanity primarily employs three approaches to measure the dielectric constant of the lunar surface and interior: examination of lunar regolith samples, microwave remote sensing, and in-situ radar detection. This article principally introduces the inversion methodologies, results, and primary applications of lunar regolith dielectric constant measurements from various nations. Concurrently, the article provides a brief comparative analysis of the advantages and disadvantages of different measurement techniques based on empirical results, summarizes the distinctions among these methods, and, building upon prior research, prospects the progress of China's endeavors in detecting the dielectric constant of lunar regolith collected by Chang'E-5 and inverting the dielectric constant of Mars.

Full Text

The Methods of Calculating Lunar Regolith Dielectric Constants and Their Applications

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Abstract: Exploring the Moon represents humanity's first step into space. The dielectric constant is a parameter that describes a material's ability to store and release energy in an applied electric field, and measuring the dielectric constant of lunar regolith helps researchers investigate regolith thickness and subsurface structure, thereby understanding the Moon's origin and evolutionary history. The dielectric constant of the lunar surface and interior is primarily measured through three approaches: direct testing of lunar soil samples, microwave remote sensing detection, and radar in-situ 探测. This paper mainly introduces various methods for inverting lunar regolith dielectric constants, their results, and primary applications. Additionally, it briefly compares the advantages and disadvantages of different measurement methods, summarizes their differences, and based on previous research, prospects for China's progress in testing the dielectric constant of Chang'e-5 collected samples and inverting the dielectric constant of Mars.

Keywords: lunar; dielectric constant; microwave remote sensing detection; radar in-situ detection

1 Introduction

As Earth's only natural satellite, the Moon is the closest celestial body to our planet. It records a unique history of early planetary formation and development, as well as changes in the space environment and solar radiation over billions of years, making it scientifically significant for human cosmic exploration. Studying the Moon's characteristics, particularly its potential resources, is crucial for planning humanity's future in space. Consequently, the Moon has remained a primary target for space exploration. On January 2, 1959, the Soviet Union successfully launched Luna 1, the world's first lunar probe, and on July 21, 1969, American astronaut Neil Armstrong took humanity's first step on the lunar surface. Since the mid-20th century, advances in aerospace technology and space science have accelerated the pace of lunar and deep space exploration.

During lunar exploration, lunar regolith and subsurface structure have been key research focuses, helping us understand the Moon's origin and evolutionary history. Researchers worldwide have employed various methods to probe the Moon, obtaining distributions of subsurface structure and regolith thickness, which have deepened our understanding of lunar topography, structure, and mineral resources. As a critical parameter for studying regolith dielectric properties and thickness, inverting the dielectric constant of lunar regolith has become a focal point for researchers. Numerous techniques have been used to measure regolith dielectric constants, including direct laboratory measurement of lunar soil samples, microwave remote sensing through instruments like the Lunar Radar Sounder (LRS), Miniature Synthetic Aperture Radar (Mini-SAR), and Miniature Radio Frequency (MiniRF), and radar in-situ detection.

In China's lunar exploration program, both Chang'e-3 and Chang'e-4 carried Lunar Penetrating Radar (LPR), whose data can be used for dielectric constant inversion through various methods such as the reflection method and hyperbolic fitting method. Additionally, Chang'e-5 successfully completed a drilling sampling mission, returning regolith samples that can be directly measured for dielectric constants.

This review synthesizes different research methods and results for inverting lunar regolith dielectric parameters from domestic and international researchers, which is significant for future lunar research and for inverting dielectric constants of Mars and other planets.

2 Dielectric Constant and Loss Tangent

The dielectric constant describes a material's ability to store and release energy in an applied electric field, reflecting the degree of polarization under electromagnetic field excitation. In a given electromagnetic field, greater polarization in a dielectric material corresponds to a larger dielectric constant. Typically, the dielectric constant is expressed as the dimensionless relative permittivity:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where ε is the material's dielectric constant and ε_0 is the permittivity of free space or vacuum. In some cases, the relative permittivity is expressed in complex form:

$$\varepsilon_r = \varepsilon_r' + j\varepsilon_r''$$

where ε_r' is the real part of the complex permittivity, representing energy storage and constituting the conventional dielectric constant, and ε_r'' is the imaginary part, related to energy loss in the medium and used to calculate the loss tangent described below. The dielectric constant and loss tangent are two important parameters for studying lunar regolith dielectric properties.

The loss tangent describes the energy loss in a dielectric material after applying an electric field and can be expressed as the ratio of the imaginary to real parts of the complex permittivity:

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

Olhoeft et al. found that when temperature is below 200°C, the dielectric constant of dry lunar soil is independent of detection frequency above several kilohertz. Therefore, within the operating frequency band of Ground Penetrating

Radar (GPR), the real and imaginary parts of the dielectric constant show similar trends with frequency, and dielectric loss variation with frequency can be neglected in the GPR band.

Studying the dielectric constant and loss tangent of lunar regolith enables understanding of its dielectric properties, thereby inverting regolith thickness and subsurface structure. Additionally, these parameters help explore the presence of water ice, which is crucial for lunar resource exploration, development, and utilization.

3 Dielectric Constant Inversion Methods

Relative permittivity is closely related to regolith bulk density, electromagnetic wave velocity, and loss tangent, making accurate measurement of relative permittivity essential for lunar observation. Currently, lunar material dielectric constant measurements primarily fall into three categories: direct lunar sample testing, microwave remote sensing detection, and radar in-situ detection. [Figure 1: see original paper] marks the exploration regions for lunar regolith dielectric constants discussed in this paper. On the lunar nearside, yellow text indicates areas explored by Japan's Kaguya spacecraft LRS, blue text shows Lunar Reconnaissance Orbiter detection sites, and green marks denote Chang'e-3 and Apollo 15 landing sites. The lunar farside shows the Chang'e-4 landing site.

3.1 Lunar Sample Laboratory Testing

Direct measurement of dielectric constants on lunar soil samples is the most direct and accurate method. In the latter half of the 20th century, the U.S. Apollo series sampling missions returned large quantities of lunar soil samples. Using methods such as coaxial resonant cavities, short-circuited waveguide methods, and transmission line methods, researchers measured relative permittivity values between 2.3 and 6.5 for lunar soil samples.

Olhoeft and Strangway analyzed nearly 92 Apollo lunar soil samples to study dielectric properties of materials within 100 m of the lunar surface. By examining several samples collected from 1–3 m depth under controlled laboratory conditions, they developed empirical formulas relating regolith dielectric constant and loss tangent to density and $FeO+TiO_2$ content, estimating relative permittivity for the Mare Imbrium region as 2.5–3.5:

$$\varepsilon_r' = 1.919\rho^{0.179}$$

$$\tan\delta = 100.038S + 0.312\rho - 3.26$$

where ρ and S represent bulk density and weight percentage of $FeO+TiO_2$, respectively. However, since their data came only from fine grains on the lunar surface, not representative of larger rock mixtures or solid rock, they suggested their 100 m depth estimates might only apply to 7–10 m below the surface.

Based on Carrier et al.'s empirical formulas, Fa and Jin measured mineral content in lunar soil through other methods, assuming a density of $\rho = 1.8\,\mathrm{g/cm}^3$ to calculate dielectric constant and loss tangent. A common approach involves using spectrometers or Gamma Ray Spectrometers (GRS) to invert FeO and TiO_2 content and distribution, then estimating dielectric constants through empirical formulas.

3.2 Microwave Remote Sensing Detection

3.2.1 Reflection Amplitude Method On November 20, 2007, Japan launched the Kaguya spacecraft (SELENE) equipped with a Lunar Radar Sounder (LRS) operating at 5 MHz, capable of probing several kilometers below the lunar surface. Ono et al. used reflected signals to detect subsurface structures in nearside mare regions, observing multilayer structures in Imbrium, Crisium, and Oceanus Procellarum. To verify whether detected echoes represented subsurface waves, they needed to calculate subsurface dielectric constants. Using three independent observational parameters—surface echo amplitude, subsurface echo amplitude, and time delay of subsurface echoes relative to surface echoes—they assumed normal incidence and a moderate top-layer dielectric constant value of 4, then inversely solved the Fresnel model for vacuum and two subsurface layers to calculate loss tangent of lunar surface materials and dielectric constant of the second layer material. Results are shown in .

Table 1 Dielectric constants and loss tangents for Procellarum, Imbrium, and Crisium regions

Region	Top layer tan δ	Second layer ε_r	Second layer tan δ
Procellarum (44.5°N, 69.3°W)	0.001–0.019	5.86–8.08	0.001–0.010
Imbrium (36.0°N, 15.3°W)	0.001–0.016	5.77–9.04	0.001–0.008
Crisium (57.0°N, 15.5°W)	0.002–0.005	6.45–6.76	9.25–9.71

The results show that dielectric constants generally increase with depth, consistent with Apollo sample measurements.

3.2.2 Radar Depth to Regolith Thickness Ratio Method Ishiyama et al. also used LRS data from Japan's Kaguya spacecraft, combined with Multiband Imager (MI) and Terrain Camera data, to determine dielectric constants and porosity of the uppermost basalt layer to depths of several hundred meters in Mare Humorum and Mare Serenitatis, studying geology beneath lava flow units. They proposed a method using the ratio of apparent radar depth (the penetration depth when medium permittivity is 1) to the uppermost basalt layer thickness to calculate the dielectric constant:

$$\varepsilon_r = \left(\frac{c \cdot \Delta t}{2d_{\text{radar}}}\right)^2 \cdot \frac{1}{d}$$

where c is light speed, Δt is the time delay between surface and subsurface echoes near craters measured by LRS, d_{radar} is apparent radar depth, and d is the uppermost basalt layer thickness. Their estimated results showed dielectric constants of 2.8–5.5 for Mare Humorum lava flow units and 4.2–18.0 for Mare Serenitatis. However, these values are much smaller than the previously assumed dielectric constant of 8–9 based on lunar radar observations, which the authors attributed to porosity in the uppermost basalt layer.

3.2.3 Particle Anisotropy Method Verma et al. presented a model using SAR data to estimate lunar surface dielectric constants based on MiniRF SAR data from the Lunar Reconnaissance Orbiter (LRO), applying it to Apollo landing site Taurus-Littrow valley and Sinus Iridium region. First, they derived a coherence matrix from Stokes parameter SAR images and expressed it in terms of particle anisotropy. Then, comparing matrix elements with those provided by Cloude, they established relationships between particle anisotropy and coherence matrix elements. Finally, they determined dielectric constants from particle anisotropy:

$$\varepsilon_r = \frac{(1 + A_p)^2}{(1 - A_p)^2}$$

where A_p represents particle anisotropy. Applying this model, Verma et al. calculated surface dielectric constants for Taurus-Littrow valley and Sinus Iridium region, shown in [Figure 2: see original paper]. In Taurus-Littrow valley, the average dielectric constant was 2.56 ± 0.25, with 97.45% of sampling points below 3. Similarly, Sinus Iridum region showed an average of 2.54 ± 0.43, with most sampling points below 3. The particle anisotropy method yielded results for Taurus-Littrow valley closer to previous findings. Compared with laboratory measurements, this method is more practical as it doesn't require surface material composition, content, or bulk density data—only microwave scattering information to estimate dielectric constants. The authors believe this model can be applied not only to the Moon but also to estimate dielectric constants of other celestial bodies.

3.2.4 Brightness Temperature Spectral Model Inversion Method

Previous studies estimated microwave-frequency dielectric constants based primarily on FeO and TiO_2 content and spectral albedo from Apollo samples, which significantly limited calibration accuracy. Gong et al. combined Diviner IR-measured surface temperatures and regolith density with Chang'e-1 and Chang'e-2 microwave radiometer observations to estimate effective dielectric constants at multiple frequencies for Apollo 15 site and lunar equatorial highlands. They established a model predicting lunar regolith brightness temperature spectra in microwaves using Diviner-measured surface temperatures and thermal properties from Apollo heat flow probes:

$$T_{Bv} = (1 - r_v) \int_0^{\infty} \rho(z) k_v(z) T(z) e^{-\int_0^z \rho(z') k_v(z') dz'} dz$$

where r_v is surface reflectivity at frequency v, $\rho(z)$ represents density, $k_v(z)$ is the absorption coefficient primarily related to the imaginary part of dielectric constant ε_r, and $T(z)$ represents the physical temperature profile at the measurement site. In this expression, r_v and k_v are unknowns that can be obtained using Chang'e-1 and Chang'e-2 microwave radiometer data combined with least mean square methods to derive surface effective reflectivity r_v and absorption coefficient k_v, thereby deriving the effective complex permittivity expression at different frequencies. The real part of effective permittivity can be calculated from $r_v = \frac{\sqrt{\varepsilon_r'} - 1}{\sqrt{\varepsilon_r'} + 1}$, and the imaginary part from $\varepsilon_r'' = \frac{k_v \bar{\rho} c}{2\pi f}$. They obtained complex permittivity at different frequency channels, shown in [Figure 3: see original paper] and [Figure 4: see original paper] for Apollo 15 site and lunar equatorial highlands regolith, respectively.

The results indicate that dielectric constants inverted from brightness temperature data are frequency-dependent. Compared with Apollo 15 site results, high-frequency channel dielectric constants for equatorial highlands are similar, while low-frequency channel values are much lower. The loss tangent is also lower than at Apollo 15 site. The authors attribute this to reduced FeO and TiO_2 content in equatorial highlands, potentially leading to lower reflectivity and higher emissivity.

3.3 Radar In-situ Detection

3.3.1 Real Part of Dielectric Constant Inversion 1) Reflection Method

On December 14, 2013, China's Chang'e-3 (CE-3) landed in a previously unexplored northwestern region of Mare Imbrium, carrying the first Lunar Penetrating Radar (LPR) to explore regolith thickness and subsurface structure. Before exploring regolith thickness, the authors calculated surface dielectric constants using the reflection method, modeling lunar subsurface structure as three uniform planar layers: vacuum, surface layer, and subsurface layer. The surface layer dielectric constant can be calculated from the relationship between vacuum permittivity and reflected signal amplitude:

$$\varepsilon_r = \varepsilon_0 \frac{1 - A_0}{1 + A_0}$$

where ε_0 is vacuum permittivity and A_0 is relative reflection amplitude—the ratio between surface reflection amplitude and incident signal amplitude. They obtained regolith dielectric constants between different navigation points, shown in [Figure 5: see original paper]. Dielectric constants for three traversed terrains were 2.95 ± 0.36, 2.83 ± 0.33, and 2.94 ± 0.34, respectively. The overall relative permittivity at Chang'e-3 landing site was 2.9 ± 0.4, consistent with Apollo sample measurements.

2) Hyperbolic Fitting Method

Ding Chunyu also used Chang'e-3 LPR data to invert internal regolith dielectric constants using hyperbolic fitting. When LPR encounters rocks or other reflectors within regolith, radar echoes produce hyperbolic curves on echograms. Based on geometric relationships between electromagnetic wave propagation and target reflectors, the author derived a hyperbolic expression to invert internal regolith dielectric constants.

As shown in [Figure 6: see original paper], based on right triangle geometry, the time t for electromagnetic wave transmission and reception is:

$$t = t_0 + \frac{2\sqrt{(x - x_0)^2 + h^2}}{v}$$

where t_0 is wave propagation time in air (negligible in this model), x_0 is the position directly above the target, h_0 is target depth, x is radar position, and v is electromagnetic wave velocity in regolith. This equation transforms into a hyperbolic expression:

$$\frac{(t - t_0)^2}{a^2} - \frac{(x - x_0)^2}{b^2} = 1$$

where $a = 2h_0/v$ and $b = h_0$. By inputting point coordinates (t, x) on the hyperbola, remaining parameters can be fitted to obtain target depth h_0 and wave velocity v. Finally, the dielectric constant is obtained from $\varepsilon_r = (c/v)^2$. The manually selected hyperbolas from LPR data and inverted dielectric constant distribution results are shown in [Figure 7: see original paper] and [Figure 8: see original paper].

The author selected 75 hyperbolas. Inversion results indicate that internal dielectric constants in the northwestern Mare Imbrium landing region range from

1.5 to 16.4, mostly distributed between 2 and 8, consistent with Apollo sample test results for internal regolith layers.

3) Optimized Hyperbolic Fitting Method

On January 3, 2019, China's Chang'e-4 (CE-4) successfully landed in Von Kármán crater within the South Pole-Aitken basin, achieving the first soft landing on the lunar farside. Similar to Chang'e-3, Chang'e-4 also carried LPR to explore subsurface structure. Wang et al. proposed an improved dielectric constant inversion method better suited for LPR data based on hyperbolic fitting. The traditional method assumes antennas are close to the lunar surface without considering LPR antenna height and spacing, yielding only an equivalent relative permittivity from one hyperbola. The authors incorporated antenna height and spacing effects, proposing a new electromagnetic propagation model and relative permittivity calculation method that still uses geometric relationships between wave propagation and target reflectors but adds antenna height and spacing parameters.

In this new method, once the hyperbola peak position is determined, any point on the hyperbola can yield a relative permittivity based on geometric relationships, obtaining the average dielectric constant for all points on the curve. The schematic diagram is shown in [Figure 9: see original paper]. According to Snell's law, the dielectric constant can be expressed as:

$$\varepsilon_r = \frac{\sin\theta_{d1}}{\sin\theta_{u1}} = \frac{\sin\theta_{d2}}{\sin\theta_{u2}}$$

where $\theta_{u1}, \theta_{u2}, \theta_{d1}, \theta_{d2}$ are incident and reflection angles for upward and downward waves, respectively. Using geometric relationships with parameters including antenna height h, antenna spacing L, target depth H, incident and reflection point positions x_1, x_2, position directly above target x_0, and radar position x, the authors expressed the sine terms and derived dielectric constant ε. Wang et al. compared traditional and optimized hyperbolic fitting results, shown in [Figure 10: see original paper]. The comparison indicates that the new method generally yields higher dielectric constants than the traditional method, with more accurate results, particularly improved for shallow layers.

4) 3D Velocity Spectrum Analysis Method

Dong et al. applied 3D velocity spectrum analysis to Chang'e-4 LPR data combined with hyperbolic fitting for dielectric constant inversion. The 3D velocity spectrum can automatically search for hyperbolas in radar images and use velocity axis maxima with a soft threshold function to obtain horizontal position, two-way reflection time, and velocity for each hyperbola. To obtain the 3D velocity spectrum, they applied a normalized expression for stacked amplitude:

$$C_{i,j,k} = \frac{\sum_{j=1}^{L+i-1} S_{i,j,k}^2(t_{i,j,k}; x_j)}{\sum_{j=1}^{L+i-1} f^2(t_{i,j,k}; x_j)}$$

where $i = 1, 2, 3, \dots, n_t$, $j = 1, 2, 3, \dots, n_x$, $k = 1, 2, 3, \dots, n_v$, with n_t, n_x, n_v representing sample points per trace, number of traces, and number of calculated velocities, respectively; N_i represents horizontal calculation region size for the i-th selection; L is time gate; x_j is horizontal distance between the j-th hyperbola point and extremum point; $t_{i,j,k}$ is two-way travel time for the j-th point; and $S_{i,j,k}$ is stacked amplitude before normalization. $C_{i,j,k}$ is the 3D data whose local maxima represent hyperbola vertex time, horizontal position, and velocity. Finally, $\varepsilon_r = (c/v)^2$ yields the dielectric constant.

Automatically selected hyperbolas and dielectric constant results from 3D velocity spectrum are shown in [Figure 11: see original paper] and [Figure 12: see original paper]. Thirty hyperbolas were automatically selected, each vertex being a local maximum along the velocity axis. Results show that most dielectric constants in the studied region range between 3 and 5.5. Previous research indicates lunar basalt relative permittivity is approximately 8. Therefore, the authors conclude their studied region represents lunar regolith based on obtained velocities and relative permittivity structure.

3.3.2 Loss Tangent Inversion 1) Energy Attenuation Method

The methods above invert the real part of complex permittivity, while obtaining the imaginary part requires inverting the loss tangent. In 2019, Lai et al. used signal energy attenuation to calculate the loss tangent of fine-grained regolith at Chang'e-4 landing site. They employed Skolnik's radar equation:

$$\frac{P_r}{P_t} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^{3R^4}} e^{-4\alpha R}$$

where P_r is received power, P_t is transmitted power, G is system gain, λ is wavelength in the medium, σ is scattering cross-section, R is propagation distance, and $e^{-4\alpha R}$ represents medium loss. For backscatter correction, three reflection target models must be considered: smooth plane reflection ($P_r \propto 1/R^2$), discrete scattering ($P_r \propto 1/R^4$), and intermediate cases ($P_r \propto 1/R^3$). Attenuation β (dB/m) was obtained through linear least squares fitting of two-way distance versus dB power, and loss tangent derived from:

$$\tan\delta = \frac{\beta}{9.1 \times 10^{-8} \cdot f}$$

The authors obtained fine-grained regolith loss tangent results shown in [Figure 13: see original paper]. Results show the average loss tangent for fine-grained regolith along Yutu-2 rover's 0–105 m traverse path is 0.0039 ± 0.0002, roughly consistent with Apollo lunar soil sample laboratory measurements at the lower limit. In 2021, Lai et al. again studied Chang'e-4 landing site regolith loss tangent, but focused on coarse-grained material in the 12–18 m depth range. Using similar signal energy attenuation methods, they validated feasibility with Stochastic Media model and three reflection target simulation models. Simulation results showed that when propagation medium is uniform with minimal scattering, obtained dielectric constants are closer to true values with 15% error. This method yielded an average loss tangent of 0.0104 ± 0.0027 for coarse-grained material at Chang'e-4 landing site, significantly higher than the 2019 fine-grained regolith value, indicating different compositions between the two layers.

2) Frequency Shift Method

Li et al. used frequency shift method to calculate loss tangent of fine-grained regolith at Chang'e-4 landing site. The ground acts as a low-pass filter, and as electromagnetic waves propagate underground, the center frequency shifts—a phenomenon related to loss tangent. By obtaining instantaneous frequency through time-frequency analysis and combining with slope, loss tangent can be calculated:

$$\tan\delta = -\frac{8\pi f_0^2}{c \cdot k_f}$$

where k_f represents the slope of center frequency change over time and f_0 is original center frequency. The authors calculated frequency attenuation for 0–450 ns two-way travel time, obtaining an average loss tangent of $(5 \pm 2) \times 10^{-3}$ for the entire radar section, very close to Lai et al.'s result of 0.0039 ± 0.0002.

4 Applications of Dielectric Constant

Inverting dielectric constants is fundamental for in-depth lunar regolith research. Dielectric constants enable time-depth conversion, transforming radar wave travel time in regolith to depth for calculating regolith thickness. Additionally, since raw radar signals have low signal-to-noise ratios, direct radar profiles cannot clearly reveal geological structures like subsurface rock distribution, requiring migration processing. Dielectric constant is a critical parameter in migration processing. LPR on Chang'e-3 and Chang'e-4 used inverted dielectric constants in F-K migration and Kirchhoff migration methods, improving post-migration profile signal-to-noise ratios and revealing clear rock distribution and layered structures. Therefore, dielectric constants play a key role in studying regolith physical properties, inverting regolith thickness, and exploring regolith layering.

4.1 Study of Regolith Physical Properties

The most basic application of dielectric constants is studying regolith physical properties such as density and iron-titanium content. With known regolith dielectric constants, Olhoeft and Strangway's empirical formulas can calculate regolith density. Dielectric constants also reflect iron-titanium content, enabling researchers to calculate regolith composition more clearly.

4.2 Regolith Thickness Inversion

A primary application of dielectric constant inversion is regolith thickness estimation. Some researchers used hyperbolic inversion of Chang'e-3 landing zone dielectric constants to estimate regolith density and thickness, treating lunar rock or silicate dielectric constants as density functions to obtain regolith density. Through regression analysis of density and depth, they derived optimal fitting curves to obtain regolith depth. They found regolith thickness near the landing zone relatively small, with nearly half of fresh impact craters in 2–3 m thick regolith, and maximum average thickness not exceeding 8 m.

4.3 Study of Regolith Layered Structure

Dielectric constant inversion helps explore internal regolith layering. For example, Iraklis et al. used Chang'e-4 data to identify four layered structures within the top 10 m of lunar regolith. They found relatively low dielectric constants in the first and third layers, while the second and fourth layers showed values up to 10, enabling differentiation of four layer structures in the top 10 m at Chang'e-4 landing site based on varying dielectric constants.

5 Summary and Outlook

Since the mid-20th century, human lunar exploration has never ceased. As a crucial parameter for lunar research, dielectric constants play a vital role in understanding regolith dielectric properties, exploring regolith thickness, and subsurface structure, with important implications for lunar resource exploration, development, and utilization. This paper reviewed different inversion methods and results for lunar surface and interior dielectric constants since the 20th century, including sample measurement, microwave remote sensing, and radar in-situ detection.

Direct measurement of Apollo samples is the most accurate method, but limited samples restrict study to Apollo landing sites. Moreover, laboratory environments differ from lunar conditions, affecting accuracy. Microwave remote sensing overcomes geographic limitations, enabling large-scale or even global measurements, though with lower spatial resolution than radar in-situ detection. For Chang'e-3 and Chang'e-4, the reflection method measures surface dielectric constants, while hyperbolic fitting inverts subsurface values. Optimized hyperbolic fitting and 3D velocity spectrum analysis improve upon classical hyperbolic fitting: the former considers antenna spacing and height for greater accuracy, while the latter automatically selects hyperbolas via 3D velocity spectrum, improving efficiency over manual selection. Inverted dielectric constants enable regolith thickness inversion, migration imaging, and layered structure exploration.

Based on previous research, we hope to provide references for China's subsequent lunar exploration and dielectric constant studies of Mars and other solar system bodies: (1) On November 24, 2020, China's Chang'e-5 successfully returned 2 kg of regolith samples, making China the third nation to collect lunar samples. Previous dielectric constant inversions can provide references for testing Chang'e-5 samples and comparing results. (2) Lunar regolith dielectric constant inversion methods can assist Chinese researchers in calculating dielectric constants for Mars and other solar system bodies. On July 23, 2020, China's Tianwen-1 successfully launched, with its rover beginning surface morphology and soil property detection in May 2021. Methods used for lunar regolith can inform Mars dielectric constant estimation, helping China understand Martian soil dielectric properties and subsurface structure. (3) Based on dielectric constant inversion research, Chinese scientists can improve existing methods and explore new approaches for more accurate measurements.

As technology advances, China has accelerated exploration of the Moon, Mars, and other solar system bodies. Domestic and international dielectric constant research methods can help avoid detours. While learning from these methods, we must also develop new approaches to achieve more precise dielectric constant inversion results.

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Note: Figure translations are in progress. See original paper for figures.