Advances in Strong Gravitational Lensing of Galaxies with (Sub)millimeter Interferometric Arrays: A Postprint

Authors: Liu Xiaohui[1,2], Cao Xiaoyue[1,2], Li Ran[1,2]

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Abstract

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Full Text

Preamble

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Research Progress of Galaxy-Galaxy Strong Gravitational Lensing Observed by (Sub)millimeter Interferometer Arrays

LIU Xiao-Hui[1];[2], CAO Xiao-yue[1];[2], LI Ran[1];[2]

(1. National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China;

2. School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China)

Abstract

In recent years, the selection and study of strong gravitational lens samples have expanded from being primarily confined to optical-infrared bands to the millimeter/submillimeter bands. Currently, approximately 200 strong lensing systems have been discovered in wide-field extragalactic surveys at millimeter/submillimeter wavelengths. High-sensitivity, high-resolution observations from radio interferometer arrays such as ALMA, NOEMA, and SMA, combined with the magnification effect of strong gravitational lensing, have ushered in a new era for studying high-redshift galaxies. This paper reviews the current research status of galaxy-galaxy strong lensing at (sub)millimeter wavelengths from three perspectives: observational samples, modeling methods, and scientific applications.

Keywords: strong gravitational lensing; high-redshift galaxies; dark matter; interferometer arrays
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1 Introduction

Light from a background source is deflected by an intervening mass structure during its propagation toward the observer, creating a magnification effect analogous to an optical lens. This phenomenon is known as gravitational lensing. The mass structures causing the deflection are generally referred to as “lenses,” which can be galaxies, galaxy clusters, or even large-scale cosmic structures. When the deflection effect of the foreground lens is sufficiently strong to produce multiple images of the background luminous source, it forms a “strong gravitational lensing” phenomenon.

Current astronomical instruments only allow us to study structures on sub-kpc scales in the nearby universe (z 0.1). To investigate the high-redshift universe, we must rely on the magnification effect of gravitational lensing to observe fainter objects and resolve smaller structural details. For instance, Meštrić et al. [1] utilized the strong lensing effect of galaxy cluster MACS J0416 to study 166 clump structures in source galaxies at redshifts 2–6, finding typical effective radii of 2–500 pc and stellar mass ranges of 10^5–10^9 M . Shu et al. [2] and Ritondale et al. [3] discovered star-forming clumps of about 100 pc in Lyman-alpha emitting galaxies at redshifts 2–3 through the magnification of early-type galaxies at z 0.5. In contrast, Förster Schreiber et al. [4] studied 35 unlensed star-forming galaxies at z 2 using the Very Large Telescope (VLT) in 2018. Limited by instrumental resolution, the clump structures they could study had effective radii of only 0.8–9.5 kpc and stellar masses of approximately 2×10^9–3×10^{11} M , making smaller clumps difficult to resolve. Strong lensing also enables kinematic studies of high-redshift galaxies; for example, Dye et al. [5], Cheng et al. [6], and Rizzo et al. [7] reconstructed the intrinsic velocity fields of galaxies at z 4 using integral field spectroscopy data from the Atacama Large Millimeter/submillimeter Array (ALMA).

Because high-redshift galaxies are extremely distant, there is a higher probability that foreground lensing galaxies or clusters will intersect the line of sight, creating strong lensing systems. The millimeter/submillimeter band is particularly suitable for studying high-redshift galaxies for two main reasons. First, this radio band lies in atmospheric transmission windows, enabling ground-based observations. More importantly, the intense UV photons from young stars in high-redshift galaxies are absorbed by interstellar dust and re-radiated at far-infrared to millimeter wavelengths. Due to negative K-correction, the flux density of high-redshift galaxies in the millimeter/submillimeter band remains nearly constant across redshifts 1–10. As shown in [Figure 1: see original paper], some bands even exhibit the counter-intuitive feature that more distant galaxies appear brighter, contrary to the inverse square law typically observed in visible light.

Observations using strong gravitational lensing in this special band allow us to detect fainter galaxies at the same redshift, facilitating studies of galaxy formation and evolution. With continuously improving sensitivity and resolution of instruments like ALMA, researchers can now conduct in-depth studies of galaxy-galaxy strong lensing systems with high-redshift background sources using millimeter/submillimeter interferometer observations.

The following sections briefly introduce the principles of gravitational lensing and interferometer arrays, lens modeling methods, and recent research progress in radio strong lensing observations.

2.1 Introduction to Gravitational Lensing

For a general gravitational lens, assuming light from a background source passes through a weak gravitational field at position $x = (x_1, x_2)$ on the lens plane (not applicable near black holes), the two-dimensional deflection angle $\hat{\alpha} = (\alpha_1, \alpha_2)$ can be expressed as:

$$\hat{\alpha}(x) = \quad (\quad dz \quad (x\ , z)\ x - x \quad | \ x - x \quad |^2$$

where (x , z) is the density at position (x , z), z is the coordinate along the line of sight, G is the gravitational constant, and c is the speed of light.

In real astronomical lensing systems, the physical size of the lens along the line of sight is much smaller than the angular diameter distances from observer to lens (D_L) and from lens to source (D_{LS}). Therefore, we can approximate the lens mass distribution as confined to a thin plane (the "lens plane"). Similarly, the background source is assumed to lie in a thin plane (the "source plane"). Under this "thin lens" approximation (shown in [Figure 2: see original paper]), Equation (1) becomes:

$\hat{\alpha}$(x) = Σ(x) x − x | x − x $|^2$ d^2x

where the surface mass density Σ(x) is defined as:

Σ(x)   (x, z) dz

From Figure 2, when the deflection angle $\hat{\alpha}$ is small, the position x on the lens plane relates to the position β on the source plane as:

β = x − (D_{LS} / D_S) $\hat{\alpha}$(x)

where D_L, D_S, and D_{LS} represent the angular diameter distances from observer to lens plane, observer to source plane, and lens plane to source plane, respectively. Substituting β = D_S and x = D_L ˆ into Equation (4) yields the mapping between source plane position and image plane position ˆ:

 = ˆ − (D_{LS} / D_S) $\hat{\alpha}$(D_L ˆ) ˆ − α(ˆ)

where the reduced deflection angle α = (D_{LS} / D_S) $\hat{\alpha}$(D_L ˆ). Equation (2) can be rewritten in dimensionless form:

α(ˆ) = (1/π) (ˆ) (ˆ − ˆ) / | ˆ − ˆ $|^2$ d^{2}ˆ

where the normalized surface density (convergence) is:

 (ˆ) Σ(D_L ˆ) / Σ_{crit}

with Σ_{crit} being the "critical surface mass density" of the lens:

Σ_{crit} c^2 D_S / (4πG D_L D_{LS})

The lensing potential (ˆ) is defined as:

 (ˆ) = (1/π) (ˆ) ln | ˆ − ˆ | d^{2}ˆ

Using the identity ln | | = / | $|^2$, the reduced deflection angle relates to the lensing potential as:

α(ˆ) = (ˆ)

Thus Equation (5) becomes:

 = ˆ − (ˆ)

This shows that image positions depend directly on the first derivative of the lensing potential . Furthermore, the normalized surface density relates to the potential through:

 (ˆ) = (1/2) ^{2} (ˆ)

Gravitational lensing magnifies background sources without changing their surface brightness (or intensity). The magnification effect comes from angular size amplification. The distortion of an infinitesimal area element in the source plane under lensing can be described by the Jacobian matrix:

A(ˆ) = / ˆ = δ_{ij} − ^{2} (ˆ) / _i _j = [1 − − γ_1 −γ_2] [−γ_2 1 − + γ_1]

where γ_1 and γ_2 represent the two components of shear $\gamma = \gamma_1 + i\gamma_2$, defined as:

$\gamma_1 = (1/2)(\ ^2 /\ _1{}^2 - \ ^2 /\ _2{}^2)$, $\gamma_2 = \ ^2 /\ _1\ _2$

Here is the "normalized surface density" defined in Equation (7). The effects of and γ on images are: causes isotropic scaling of image size without changing shape, while γ distorts image shape without changing size (as shown in [Figure 3: see original paper]).

The magnification tensor is defined as the inverse of the Jacobian matrix, M(ˆ) = A^{-1}. Its determinant (det M) represents the differential magnification (ˆ) at position ˆ. The closed curve formed by points on the lens plane with infinite magnification (det A = 0) is called the critical curve. Critical curves are typically divided into radial and tangential types; for tangential critical curves, the average normalized surface density inside is unity (¯ = 1). Projecting critical curves back to the source plane via the lens equation yields caustic curves, also divided into radial and tangential caustics. Caustics help qualitatively understand lensing image characteristics—when a background source crosses a caustic, the number of images increases or decreases in pairs (depending on crossing direction). [Figure 4: see original paper] shows imaging features for source positions at different locations relative to caustics.

For extended sources, the total magnification can be considered as a weighted integral of ( ) over the source with surface brightness I_s( ):

_{tot} = I_s( ) d^2 / I_s( ) ( ) d^2

Although the local magnification ( ) is wavelength-independent, the total magnification _{tot} depends on wavelength. Since the calculation involves weighting by source plane surface brightness I_s( ), the total magnification is identical at two wavelengths only if their surface brightness distributions I_s(λ_1) and I_s(λ_2) are exactly the same. For example, in B1938+666, the radio jet of the source galaxy happens to cross a caustic, producing extremely high magnification with _{radio} 173 [12], while in the optical band its magnification is _{opt} 13 [13]. Since millimeter/submillimeter studies of strong lensing systems utilize multiple spectral lines (e.g., dust continuum, various CO transitions, H_2O, HCN, HCO^+, etc.), and different lines correspond to different interstellar medium components, observations at different wavelengths likely have different magnifications [14].

For an ideal axisymmetric lens, when the background source, foreground lens, and observer are perfectly aligned, the source's image forms a ring. The radius of this ring is called the Einstein radius, and the mass enclosed within it is the Einstein mass. For more complex mass distributions, one can define an "effective Einstein radius" using critical curves (see Section 4.1 of Meneghetti et al. [15]). If the area enclosed by the critical curve is S_{crit}, the effective Einstein radius is defined as:

_E = √(S_{crit} / π)

2.2.1 Visibility Function

A radio interferometer array is a network of interconnected antennas (dishes or dipoles). The relative distance between a pair of antennas is called a baseline. Interferometer arrays do not measure sky surface brightness distribution directly; instead, they measure the so-called visibility function V(u, v, w), which is essentially a discretely sampled two-dimensional Fourier transform of the sky brightness distribution I(l, m):

$$V(u,\ v,\ w) = \quad A(l,\ m)\ I(l,\ m)\ e^{-2}\pi i[ul + vm + w(\surd(1-m^2-l^2)-1)]\ /\ \surd(1-m^2-l^2)\ dl\ dm$$

where l and m are coordinates on the celestial tangent plane (using standard radio astronomy notation, l points west and m points north), A(l, m) is the primary beam pattern, and u, v, w are baseline coordinates in the Fourier plane [16]. The w-term, representing array depth, becomes important only at large angular distances from the phase center (wide-field observations). For typical galaxy-galaxy strong lensing observations, the pointing is accurate within a few arcseconds, so the w-term can be safely neglected. Equation (18) then simplifies to:

$$V(u, v) = \quad A(l, m)\ I(l, m)\ e^{-2}\pi i(ul + vm)\ dl\ dm$$

However, sampling of the uv-plane is incomplete; we only obtain visibility functions V(u, v) at specific positions sampled by the interferometer baselines. Direct Fourier transformation of the observed data does not yield the true sky image I(l, m) but rather a “dirty image” with brightness distribution:

$$I\_D(l, m) = A(l, m)^{-1} \quad S(u, v)\ V(u, v)\ e^{+2}\pi i(ul + vm)\ du\ dv$$

where S(u, v) is the uv-plane sampling function (equal to 1 at sampled (u, v) positions and 0 elsewhere). The dirty image can be viewed as the convolution of the true sky brightness distribution I(l, m) with the dirty beam B, which is the inverse Fourier transform of the sampling function:

$$B(l\ ,\ m\ ) = \quad S(u, v)\ e^{+2}\pi i(ul\ + vm\ )\ du\ dv$$

Incomplete sampling in the uv-plane produces sidelobe structures in the image plane that depend heavily on the interferometer baseline configuration and visibility weighting scheme (e.g., natural, uniform, or Briggs weighting [16]). Consequently, noise in the image plane becomes highly correlated, and signals contain sidelobe structures that must be removed through deconvolution to recover the true image. [Figure 5: see original paper] illustrates different stages of aperture synthesis imaging.

2.2.2 Aperture Synthesis Imaging

The most direct method to reconstruct sky brightness distribution assumes the dirty image is formed by convolution of the dirty beam with a collection of point sources. This concept underlies the CLEAN algorithm [17, 18], the most widely used deconvolution algorithm in radio astronomy. CLEAN iteratively decomposes the sky into a series of point source components and subtracts their corresponding responses from the observed visibility function until reaching a specified threshold. This algorithm works well for point source data but becomes problematic for high-resolution extended source observations, as it decomposes all extended emission into point source collections, yielding suboptimal imaging results. This issue is particularly severe when studying strong lensing systems.

To improve imaging of extended sources, several enhanced CLEAN algorithms have been developed, including W-CLEAN [19] and MS-CLEAN [20]. For instance, MS-CLEAN decomposes the true sky brightness into both point-like and extended structures of various sizes, modeling sources accordingly. By selecting components at different scales, this algorithm provides more accurate fitting of extended structures compared to the original CLEAN algorithm.

3 Current Status of Observational Samples

Most current radio strong lens samples come from wide-field extragalactic surveys at millimeter/submillimeter wavelengths [21–26]. The South Pole Telescope (SPT) [27], Herschel Space Observatory (HSO) [28], and Planck satellite [29] have provided substantial data. Major associated survey projects—including the Herschel Astrophysical Terahertz Large Area Survey (H-ATLAS) [30], Herschel Multi-Tiered Extragalactic Survey (HerMES) [31], SPT survey [27], and Planck All-Sky Survey to Analyze Gravitationally-lensed Extreme Starburst (PASSAGES) [32, 33]—have been dedicated to detecting lensing candidates at submillimeter wavelengths.

3.1 Survey Project Introductions

The Herschel Space Observatory is a 3.5-meter space telescope operated by the European Space Agency (ESA) that observed in the far-infrared to submillimeter range (55–672 m). Launched in 2009 to the Sun-Earth L2 point, it ceased operations in April 2013 after exhausting its liquid helium coolant. The H-ATLAS and HerMES surveys covered 500 and 100 square degrees at three bands (approximately 250, 350, and 500 m), respectively.

The SPT is a 10-meter single-dish telescope operating at approximately 1.4 mm, 1.0 mm, and 3.0 mm. The SPT survey covers 2,500 square degrees with resolutions of 0.7 and 1.6 at 1.4 mm and 2 mm, respectively. Due to different operating wavelengths (far-infrared for Herschel versus millimeter for SPT), these telescopes observe sources with different selection effects. Generally, SPT detects sources at higher redshifts with greater far-infrared luminosities than Herschel sources [8].

3.2 Selection Methods for Lensing Candidates

Searching for strong lensing systems in optical bands is extremely time-consuming [34–36]. In contrast, submillimeter surveys can relatively easily detect strong lensing systems, with background sources typically at high redshifts.

First, due to negative K-correction, we can detect very distant dusty star-forming galaxies (DSFGs) [37, 38]. Negative K-correction means that at a fixed observed wavelength, the brightness of the dust continuum increases with redshift. Consequently, DSFGs can be detected relatively uniformly across a wide redshift range [8, 39]. For a galaxy with intrinsic spectral energy distribution f_ and luminosity L, the observed flux density S_ at frequency is:

S_ = L_ / (4π D_L^2)

where D_L is the luminosity distance and L_ is the source luminosity at the given frequency. Under the Rayleigh-Jeans approximation, L_ ^(2+β) T_{dust}, with = $_0$/(1+z). The luminosity distance evolves as D_L (1+z)^2. Thus Equation (22) can be rewritten as:

S_ (z) (1+z)^(β+2) / (1+z)^4 = (1+z)^(β−2)

For typical dust spectral energy distributions, the spectral index β = 1.5–2.0 [8], making S_ (z) nearly constant across a large redshift range. Particularly at lower frequencies, the observed surface brightness of high-redshift DSFGs shows little decline with redshift, enabling relatively uniform detection of these galaxies at high redshifts.

At high redshifts, the number density of unlensed bright submillimeter sources drops sharply, while numerous galaxies serve as background sources for strong lensing systems. Therefore, in submillimeter/far-infrared observations, one can identify strong lensing systems by simply applying a flux threshold to observed targets. Above this threshold, the number of unlensed sources becomes negligible, as shown in [Figure 6: see original paper].

This selection method yields samples affected by nearby galaxies and bright active galactic nuclei (AGNs). Researchers remove these contaminants through cross-comparison with relevant catalogs [23, 40]. For example, Negrello et al. [22] used similar criteria in 2017 to identify 80 lens candidates from H-ATLAS after selecting sources with flux densities >100 mJy at 500 m and removing contaminants.

After obtaining lens candidates, researchers use higher-resolution imaging and spectroscopic observations from ALMA, NOEMA, and SMA to confirm these systems. For strong lensing systems, high-resolution observations reveal multiple images or Einstein rings, and spectroscopic observations can determine the redshifts of both lens and source galaxies [41].

4.1 Background

Two decades ago, researchers began attempting to reconstruct the surface brightness distribution of source galaxies on pixelated grids using optical data—i.e., gravitational lens modeling [42–44]. These methods later evolved to include adaptive source-plane grids [45], Bayesian evidence for model comparison [46], and pixelated perturbations to the lensing potential [47, 48]. For high-resolution interferometer data, existing image modeling codes must be adapted to perform lens modeling directly on visibility data in the uv-plane.

In 1992, Kochanek and Narayan [49] introduced LensClean, a lens modeling method for interferometer data based on the CLEAN algorithm, later improved by Ellithorpe et al. [43] and Wucknitz et al. [50]. Bussmann et al. [51, 52] and Hezaveh et al. [53] developed alternative lens modeling approaches fitting uv data directly. The former used Markov Chain Monte Carlo (MCMC) to generate numerous paired parametric lens and source models, mapping them to uv-space and determining optimal solutions based on ^{2} calculations. The latter [53] proposed a visibility fitting technique with parametric lens and source models including internal self-calibration, applying it to several lensed systems from the SPT sample with ALMA data. Although these methods significantly improved upon direct modeling of radio CLEANed images, their source models remained parametric (e.g., Sérsic or Gaussian profiles). In 2016, Hezaveh et al. [54] extended their 2013 work [53] by incorporating pixelated source models.

4.2 Traditional Modeling Methods for Images

We first introduce gravitational lens modeling methods for optical images. Galaxy-galaxy strong lens modeling typically involves four components: the lens galaxy's mass distribution model, the lens galaxy's brightness distribution model, the background source's brightness distribution model, and the point spread function (PSF). The modeling process can be summarized as: for any given set of model parameters for the lens galaxy (mass and brightness distributions) and background source (brightness distribution), one generates an ideal model image through the lens equation; this ideal image is then convolved with the telescope PSF and compared with the observed image to find the optimal model parameters that produce the closest match.

In practice, researchers typically use parametric models to describe the source galaxy's brightness distribution (e.g., Sérsic profiles). These simple parametric macro models can reproduce many lens system features, such as lens galaxy mass, ellipticity, position angle, and source galaxy effective radius and luminosity. However, for applications requiring more detailed source brightness information, non-parametric models are needed, treating the source brightness distribution on the source plane as discrete pixels where each pixel is a free parameter [55–57]. Similarly, both parametric and non-parametric mass models exist for lens galaxies.

4.3 Modeling Methods for Visibility Functions

Unlike optical images, interferometer arrays directly measure the Fourier-space signal of the target's brightness distribution, which must be "inverted" and "CLEANed" to obtain conventional brightness images. Since CLEANed images contain irregular correlated structures and the CLEANing process itself introduces unestimable brightness errors, proper assessment of model errors becomes difficult during lens modeling, potentially leading to biased solutions.

Although modeling in the image plane is more efficient than processing large visibility datasets directly, researchers often use CLEANed images for lens modeling of ALMA high-resolution data [5]. To fully exploit interferometer observations, more sophisticated lens modeling requires direct use of visibility data. Several groups have developed techniques for lens modeling visibility data directly in the uv-plane [41, 53, 58, 59]. The advantage of direct visibility modeling is that it incorporates self-calibration (e.g., antenna phase correction) as part of model optimization, thereby including all measurement uncertainties. Current implementations differ in source reconstruction: Hezaveh et al. [53] and Bussmann et al. [41] use parametric source models, while Rybak et al. [58, 59] employ a pixelated reconstruction technique within a Bayesian framework, extending the method of Vegetti and Koopmans [48] to interferometer data. Rybak's approach enables detailed study of background source galaxies' complex brightness distributions using ALMA's high-resolution data.

The basic procedure for lens modeling using visibility functions is: (1) Guess a set of model parameters (including lens galaxy mass model and source brightness distribution) to generate an ideal lensed image; (2) Fourier transform this image to obtain "visibilities" on a regular uv grid; (3) Perform linear interpolation at arbitrary uv coordinates to obtain corresponding model "visibility" values; (4) Compute differences between observed and model visibilities (^{2}) to iteratively guess new parameters until the entire parameter space is sampled to obtain the optimal solution.

4.4 Kinematic Modeling with Interferometer Data

Studying galaxy kinematics and morphology requires high-resolution observations of the interstellar medium (ISM), typically resolving molecular gas at scales of tens to ~100 pc. As natural telescopes, strong lensing effects enable spatially resolved studies of background source galaxies' velocity fields [5, 60–64]. For example, Stark et al. [60] in 2008 used strong lensing to study the kinematics of a galaxy at z = 3.07 with ~120 pc resolution.

Optical studies of background source kinematics typically feature: (1) Lens galaxy mass models derived from high-resolution imaging data [60–62, 65, 66]; (2) Kinematic modeling of de-lensed three-dimensional integral field unit (IFU) data [62, 65], obtaining velocity and dispersion information by Gaussian-fitting emission lines in the source plane or by de-lensing moment maps and mapping them to the source plane [61, 66], with lens mass models held fixed; (3) Fitting de-lensed velocity fields with analytic functions (e.g., arctangent) to derive rotation curves (the “kinemetry” method).

Similar approaches are used for molecular line data from interferometers [5, 59, 67]. First, lens galaxy mass models are obtained by modeling radio continuum data observed in the same band as the spectral lines. Then this mass model is used for lens modeling of three-dimensional spectral line data to reconstruct the source brightness distribution and compute source-plane moment maps. Finally, kinematic parameters are derived by applying kinemetry methods [68] to first- and second-moment maps [59] or by applying dynamical models to first-moment maps [5, 67].

Most strong lensing studies to date analyze two-dimensional surface brightness data. For three-dimensional data cubes, the common approach applies two-dimensional modeling techniques [60] by treating data cube channels as independent sources during reconstruction. However, the correlations between reconstructed channels are unknown beforehand. To increase signal-to-noise ratios and reduce visibility data sizes, researchers typically preprocess data by averaging in time and combining channels to achieve sufficient width for high signal-to-noise while maintaining adequate narrowness for kinematic studies. As shown in [Figure 7: see original paper], each channel slice has a width of 50 $\mathrm{km \cdot s^{-1}}$. If the background source galaxy has a rotating disk structure, an analytic disk model can fit each channel’s reconstructed image to derive kinematic parameters [69].

These methods are suboptimal for two main reasons: First, keeping the lens galaxy mass model fixed prevents quantifying degeneracies with source galaxy kinematic properties; second, kinematic fitting is performed on reconstructed sources rather than directly on the data. Pixels in the source plane are correlated, and resolution varies with lensing magnification, introducing difficult-to-quantify systematic errors when deriving kinematic parameters. In 2018, Patrício et al. [70] partially addressed these issues using forward modeling that directly fits data in the image plane by Gaussian-fitting emission lines to obtain velocity field information. However, like previous methods, this approach is imperfect because it relies on a fixed lens mass model from Hubble Space Telescope (HST) data and models two-dimensional velocity fields rather than complete three-dimensional data cubes.

In 2018, Rizzo et al. [71] proposed a new method modeling complete three-dimensional data by directly fitting simulated emission line IFU observations (or radio interferometer data) with parametric rotating disk models, enabling simultaneous determination of lens mass models and source kinematic information. Both Patrício et al. [70] and Rizzo et al. [71] solved the problem of non-uniform source-plane resolution. Rizzo et al.’s method allows simultaneous optimization of lens and source parameters, enabling study of parameter degeneracies.

While parametric kinematic models are effective in many cases, simple parametric models poorly represent disturbed kinematic structures with complex morphologies or multiple components.

Below we briefly introduce the kinemetry method [68] and the dynamical modeling approach proposed by Rizzo et al. [71] in 2018.

The kinemetry method is based on the idea that reconstructed source velocity/dispersion fields can be described by a series of concentric ellipses with increasing major axis lengths. Along each ellipse, the first moment (velocity field) and second moment (velocity dispersion) can be decomposed into Fourier series with few harmonic terms. The first moment M_1(x, y) and second moment M_2(x, y) are:

M_1(x, y) = (I__ (x, y))$^{-1}$ I__ (x, y) v__
M_2(x, y) = (I__ (x, y))$^{-1}$ I__ (x, y) (v__ − M_1(x, y))^2

where (x, y) are source-plane coordinates,   and v__ are frequency and rest-frame velocity for a given channel, and I__ (x, y) is surface brightness at frequency   and position (x, y).

In kinemetry, the i-th moment decomposes as:

M_i(a,) = A_0,i(a) + Σ_n [A_n,i(a) sin(n) + B_n,i(a) cos(n)]

where a and   are the semi-major axis and position angle of the ellipse. This can be rewritten as:

M_i(a,) = A_0,i(a) + Σ_n k_n,i(a) cos[n(− ϕ_n,i(a))]

with k_n,i and ϕ_n,i defined as:

k_n,i(a) = √(A_n,i^2(a) + B_n,i^2(a))
ϕ_n,i(a) = arctan(A_n,i(a) / B_n,i(a))

For an ideal rotating disk, the only non-zero terms are B_1,v and A_0,σ, corresponding to circular velocity and constant velocity dispersion.

In 2018, Rizzo et al. [71] used a modified version of 3DBAROLO's building-model module [72] to construct kinematic models. Rotating galaxies are simulated as a series of concentric rings using the tilted-ring model [73]. Gas positions are randomly selected on each ring such that, on average, gas is uniformly distributed across the surface. Each ring is described by: (1) central coordinates x_s, y_s; (2) inclination i (i = 0° for face-on, i = 90° for edge-on); (3) position angle PA; (4) face-on gas column density Σ; (5) systemic velocity V_{sys}; (6) rotation velocity V_{rot}; (7) velocity dispersion σ_{gas}.

The line-of-sight projected velocity V_{los} at radius R is:

V_{los}(R) = V_{sys} + V_{rot}(R) cos ϕ sin i

where ϕ is the azimuthal angle in the galaxy plane. Three empirical functions describe rotation curves: arctangent, hyperbolic tangent, and multi-parameter functions:

$$V_{rot}(R) = V_t \arctan(R / R_t)$$
$$V_{rot}(R) = V_t \tanh(R / R_t)$$
$$V_{rot}(R) = V_t [1 + (R / R_t)^\alpha]^{(-1/\alpha)}$$

where R_t is the turn-over radius separating inner and outer rotation curve regions, V_t is the asymptotic velocity for arctangent and hyperbolic tangent functions, and α is a parameter for the multi-parameter function. The arctangent function is commonly used for high-redshift galaxy kinematic modeling [66, 74] but lacks flexibility in inner regions where bulges may exist, making the more flexible multi-parameter function preferable.

Velocity dispersion profiles can be described by power-law, linear, or exponential functions:

$$\sigma\{gas\}(R) = \sigma_0 \ (R / R_0)^\beta$$
$$\sigma\{\mathrm{gas}\}(\mathrm{R}) = \sigma_0 + \ \mathrm{R}$$
$$\sigma_{\mathrm{gas}}(\mathrm{R}) = \sigma_0 \ \mathrm{e}^{(-\mathrm{R}/\mathrm{R}_0)} + \sigma_1$$

5.1 Studies of High-Redshift Galaxies

Millimeter/submillimeter strong lensing systems enable detailed studies of star formation and interstellar medium in distant galaxies.

5.1.1 High-Redshift Spectral Lines

Thanks to strong lensing magnification and ALMA's extreme sensitivity, studies of high-redshift galaxies can utilize not only traditional molecular and atomic lines (e.g., CO, [C II]) but also weaker emission and absorption lines in the submillimeter band. In 2014, Spilker et al. [75] first demonstrated ALMA's capability to detect rich spectral features in high-redshift sources using stacked spectra of SPT sources, identifying 16 lines with signal-to-noise >3 and providing the first constraints on many molecular species at high redshift. These "non-traditional" lines open new windows for studying distant star-forming galaxies. Below we discuss several detectable lines in lensed sources:

(1) Dense Gas Tracers
While CO is typically the most easily detected molecule in high-redshift star-forming galaxies, its relatively low critical density (n_H_2 10^2–10^3 cm^{-3}) makes it a poor tracer of typical star-forming regions (dense molecular cloud cores). Molecules with higher critical densities ($n > 10^4$ cm^{-3}, such as HCN, HNC, HCO^+, CN) are considered more reliable tracers of star formation. Recently, astronomers [75, 76] used ALMA observations of strongly lensed star-forming galaxies to constrain the density, temperature, and excitation conditions of interstellar medium in star-forming regions using line intensity ratios.

(2) H_2O
Water is also a dense gas tracer and considered one of the most abundant molecules in molecular clouds, making it crucial for studying interstellar medium in dust-obscured galaxies [77]. Highly excited water lines (above 500 K from

ground state) have brightness comparable to CO in the same frequency range. Early work by Omont et al. [78, 79] and Yang et al. [80] used PdBI/NOEMA to observe water lines in the brightest lensed submillimeter galaxies (SMGs). ALMA now enables high-resolution water observations; for example, in 2014 researchers used ALMA to observe water lines in the strongly lensed source SDP.81 (z = 3.042) at 0.9 resolution [81]. In 2019, Yang et al. [64] achieved 0.4 resolution when observing water lines in the lensed system G09v1.97 (z = 3.63).

(3) CO Isotopologues
^{13}CO and C^{18}O are typically optically thinner than ^{12}CO, making them tracers of total molecular column density. Additionally, carbon and oxygen isotopes have different formation pathways, allowing study of high-redshift nucleosynthesis through their ratios with ^{12}CO. In 2014, Spilker et al. [75] detected multiple ^{13}CO transitions in stacked spectra, calculating that ALMA could detect (and even spatially resolve) these faint lines with only 30 minutes of observation per line for a galaxy with L_{IR} = 5 × 10^{13} L . This opens a new window on cosmic isotope enrichment history. In 2018, Zhang et al. [82] found low ^{13}CO/C^{18}O abundance ratios in four strongly lensed SMGs at z 2–3. Based on models by Romano et al. [83, 84], they suggested this low ratio might indicate a top-heavy initial mass function (IMF) with a higher proportion of massive stars in high-redshift SMGs.

(4) Atomic Fine-Structure Lines
These include some of the brightest far-infrared emission lines in star-forming galaxy spectra, many of which have been detected even in non-lensed galaxies. Particularly [C II] (158 m) is typically the strongest line in the long-wavelength spectra of star-forming galaxies. ALMA now routinely detects and resolves [C II] in both lensed [85] and non-lensed galaxies [86–88]. Strong lensing enables initial statistical studies of important atomic fine-structure lines. For instance, Bothwell et al. [89] in 2017 studied [C I] in 13 SPT lensed sources at z 2–5. Since [C I] is considered a good tracer of cold molecular gas in the interstellar medium [90, 91], it can be used to estimate H_2 masses. Bothwell et al. derived gas masses from [C I] and found significant deviations from CO-based estimates, suggesting these sources have denser, more carbon-rich media than nearby starburst galaxies.

(5) Molecular Absorption Lines
Since absorption line strength does not diminish with distance but depends only on background source brightness, they are sensitive to small amounts of molecular gas along the line of sight. Molecular absorption lines serve as important tracers of interstellar molecules and markers of molecular outflows. A decade ago, only five sources beyond the local universe showed detected absorption lines [92]. Later, Herschel's SPIRE instrument detected some OH absorption signals in the brightest lensed SMGs [93, 94]. ALMA now enables studies of high-redshift molecular absorption lines [95]. For example, Spilker et al. [96] in 2018 demonstrated spatially resolved molecular absorption via OH 119 m

ground-state doublet transitions in a strongly lensed starburst galaxy at z = 5.3. OH observations indicate that rapid molecular outflows can remove most gas required for star formation, providing new observational evidence for self-regulating feedback mechanisms. ALMA observations of lensed sources have also discovered new molecules at high redshift; Falgarone et al. [97] detected CH^+ ground-state transition absorption and emission in six lensed starburst galaxies at z 2.5 ([Figure 8: see original paper]). Recent ALMA observations of two lensed SMGs at z 2.3 have also detected OH^+ and H_2O^+ ground-state absorption lines [98], which can measure cosmic ray ionization rates in extended gas halos.

5.1.2 High-Redshift Star Formation

Far-infrared continuum and CO emission lines enable studies of star formation and molecular gas in high-redshift galaxies. An important area is using resolved (sub-galactic scale) observations to study the relative efficiency of gas conversion into stars within individual galaxies (e.g., the Kennicutt-Schmidt relation). Many groups have conducted studies on very luminous or strongly lensed galaxies using (sub)millimeter data [99–105]. Recent ALMA observations of lensed galaxies have further advanced this field [106–111]. Using ALMA data from some samples, researchers can even study star-forming clumps in high-redshift galaxies. For example, Sharda et al. [109] used high-resolution ALMA data to study a resolved star-forming region in SDP.81 discovered by Swinbank et al. [67], testing various star formation models and finding that the multi-freefall (turbulent) model [112] best matched observations. Subsequent analysis of two star-forming clumps in the non-lensed SMG AzTEC-1 (z 4.3) yielded similar conclusions [110], suggesting that high star formation rates in high-redshift starburst galaxies may be sustained by interactions between gravity and turbulence.

5.1.3 [C II]/FIR Deficit

When a galaxy's total far-infrared luminosity L_{FIR} exceeds 10^{11} L , the L_[C II]/L_{FIR} ratio declines significantly with increasing L_{FIR}, a phenomenon known as the [C II]/FIR deficit [113–115]. ALMA's high-resolution observations of both [C II] line and far-infrared continuum in high-redshift galaxies have yielded substantial progress in this area. In 2016, Spilker et al. [116] analyzed strongly lensed SPT sources and found that the L_[C II]/L_{FIR} ratio characterizes far-infrared surface mass density well, extending the results of Díaz-Santos et al. [117] from low-redshift galaxies by two orders of magnitude ([Figure 9: see original paper]). Subsequent work extended this to galaxies at z > 5 [118].

Lamarche et al. [119] and Litke et al. [63] found through sub-galactic scale observations of lensed galaxies that if this deficit occurs at a physical scale, it must be at sub-kpc scales, indicating the [C II]/FIR deficit is generated locally, consistent with Smith et al.'s [120] 2017 results from nearby galaxies.

5.1.4 Kinematics and Morphology of High-Redshift Galaxies

When a spectral line such as CO (or [C II]) has sufficient signal-to-noise and spatial resolution, dynamical models can be fitted to study the kinematic properties of high-redshift galaxies. Most such studies have found rotating disk characteristics [5–7, 67, 121, 122]. Various dynamical modeling tools have been used to quantify rotating disk properties, including DYSMAL, GalPak3D, and 3DBAROLO [72, 123, 124]. Analysis of high-resolution data for SDP.81 revealed non-uniform dust distributions with ~200 pc clumps located within a larger, more extended cold gas disk [5, 58, 59]. While Dye et al. [5] and Swinbank et al. [67] argued SDP.81's disk is rotationally supported, Rybak et al. [58, 59] found significant asymmetric kinematic features at larger radii, indicating a perturbed disk with multiple velocity components. SDP.81's Toomre parameter Q 0.3 suggests its disk is unstable [5, 67]. In 2019, Litke et al. [63] used ALMA to observe [C II] in the lensed galaxy SPT0346-52 at z = 5.7, finding two components separated spatially (~1 kpc) and kinematically (~500 $\mathrm{km \cdot s^{-1}}$), connected by a gas "bridge," indicating SPT0346-52 is a major merger system [63].

In 2020, Rizzo et al. [7] used high-resolution ALMA data to study the dynamics of SPT0418-47, a lensed dusty star-forming galaxy at z = 4.2, finding kinematic characteristics similar to nearby spiral galaxies with a dynamically cold disk structure. This suggests that the system's high star formation rate and gas fraction do not induce large turbulent motions, and the galaxy's stability remains unaffected. In 2021, Rizzo et al. [122] further studied five lensed dusty star-forming galaxies at z 4.5 using [C II], obtaining similar results: these systems are dynamically cold with V/σ ratios of 7–15 ([Figure 10: see original paper]). Fraternali et al. [125] reached similar conclusions in 2021 studying two non-lensed dusty star-forming galaxies. Since these galaxies' star formation rates and ISM properties are typical of dusty star-forming galaxies [99, 126, 127], dynamically cold disk structures may be common in high-redshift star-forming galaxies. However, reproducing such high-redshift galaxies with rapid rotation, high star formation rates, and cold gas content remains challenging for most numerical simulations and semi-analytic models [128–131].

5.2 Supermassive Black Holes in Lens Galaxies

Galaxies with bulges typically host supermassive black holes (SMBHs) at their centers. Despite bulges extending beyond the SMBH's dynamical influence, observed correlations exist between black hole mass and bulge properties (luminosity, velocity dispersion, stellar mass) [138–141], suggesting co-evolution of SMBHs and their host galaxies [142]. Determining the origin of these correlations is crucial for understanding galaxy formation and evolution. Current SMBH mass measurements using stellar, gas, or maser dynamics are limited to galaxies within ~150 Mpc [142]. Reverberation mapping [143] extends this to ~1 Gpc for AGN, but only works for galaxies with bright AGN emission, complicating measurements of host bulge properties. Beyond z 0.4, SMBH masses must rely on assumed scaling relations with AGN observables like luminosity. Strong gravitational lensing provides an independent method for measuring SMBH masses at cosmological distances.

Modeling lens galaxies to fit images of background sources allows inference of foreground lens mass distribution properties. Lens theory predicts that nonsingular mass distributions should produce a very faint central image near the lens center [144, 145]. This central image's brightness is extremely sensitive to the mass distribution on small scales (~100 pc) near the lens center; more concentrated mass distributions make the central image fainter ([Figure 11: see original paper]). Current studies use central image brightness to investigate lens galaxy SMBHs [146–150]. Detecting these faint central images is very difficult because they are dim and blend with lens galaxy light. Since lens galaxies are often early-type galaxies with minimal submillimeter emission, detection is most likely in radio bands. Indeed, the only lens system with a detected central image comes from radio observations [146].

Recent submillimeter surveys show numerous strongly lensed dusty star-forming galaxies at z 2–7 [21, 23, 41, 53]. These are among the brightest objects in wide-field submillimeter surveys [152], making them ideal targets for searching for central images. Hezaveh et al. [151] noted in 2015 that ALMA's excellent sensitivity and resolution would enable detection of these central images and constrain lens galaxy nuclear sizes, mass profile slopes, and central SMBH masses.

Even without detecting central images, constraints can be placed on lens galaxy inner mass distributions [153]. In 2015, Wong et al. [148] analyzed high-resolution data for SDP.81 and found that while the lens galaxy's central AGN (z 0.3) showed continuum emission, no central image predicted by strong lens theory was detected in any molecular line observations ([Figure 12: see original paper]). They constrained the lens galaxy's central mass distribution, concluding the central SMBH mass must exceed $10^{8 \cdot 5}$ M . Tamura et al. [147] reached similar conclusions, finding that an SMBH $>3 \times 10^8$ M was needed to explain the missing central image. SMBH mass can also be estimated using the M–σ relation [139, 140]; Tamura et al. [147] applied the Kormendy & Ho [142] relation to estimate ~10^9 M for SDP.81's SMBH, consistent with strong lensing constraints.

5.3 Studies of Dark Matter Substructure

Up to 85% of matter in the universe consists of dark matter, whose nature is one of physics' longest-standing questions [154–156]. In the standard cold dark matter (CDM) cosmological model, dark matter comprises weakly interacting particles with negligible thermal velocities in the early universe, behaving collisionlessly on scales 1 kpc [157, 158]. Cosmic microwave background observations show the early universe was smooth and homogeneous except for small density perturbations [159, 160]. With negligible thermal velocities, CDM particles become trapped in these perturbations, evolving under gravity to form small-scale structures [161, 162]. The distribution and evolution of these perturbations determine the statistical properties of current dark matter distribution, with baryonic matter forming observed galaxies and clusters on the dark matter skeleton [163, 164]. The ΛCDM framework agrees well with observations on large scales, but high-resolution simulations show significant discrepancies with observations on small scales ($<$ few kpc) [165–167]. To alleviate these tensions, alternative dark matter models have been considered, including self-interacting dark matter, warm dark matter (WDM), and fuzzy dark matter [168–171].

A key difference between WDM and CDM is structure evolution on galactic and sub-galactic scales. WDM particles' non-negligible velocities allow them to free-stream from early small-mass density perturbations, suppressing the number density of low-mass dark matter halos and subhalos compared to CDM [172, 173]. Since this suppression scale depends on the dark matter particle's momentum distribution and production mechanism, quantifying the relative abundance of small-mass halos is crucial for dark matter property studies.

Strong gravitational lensing is a powerful tool for probing small-mass halos in distant galaxy dark matter halos [54, 174–181] and along the line of sight [182–184]. It provides an independent method to distinguish dark matter theories [173, 186–188] such as CDM [189], WDM [168, 190], and fuzzy dark matter [191, 192], as different models predict different numbers of small-mass halos [173]. Two mainstream methods currently exist for detecting these small-mass halos with strong lensing.

The first method studies flux ratio anomalies in lensed quasars. Small-mass dark halos alter the relative fluxes of multiple quasar images compared to predictions from smooth lens potentials. By analyzing flux ratios, researchers can probe the total amount of substructure without inferring individual masses or positions. Flux ratios depend non-linearly on combinations of the lens potential's second derivatives near images, serving as a local probe of small-scale structure capable of detecting masses as small as 10^7 M . Mao & Schneider [193] and Metcalf & Madau [194] first proposed that flux ratio anomalies might relate to dark matter substructure in foreground lens galaxy halos, making these systems useful for constraining substructure properties. Subsequent observational and simulation studies confirmed this, establishing flux ratio anomalies as a viable probe [179, 195–197]. In 2020, Hsueh et al. [198] studied flux ratios in seven lensed quasars, finding results consistent with CDM hydrodynamic simulations at 1σ when accounting for stellar disks, bright satellite galaxies, and line-of-sight halos ([Figure 13: see original paper]).

The second method is gravitational imaging. Small-mass dark halos in lens galaxies perturb the surface brightness distribution of extended lensed arcs, allowing researchers to analyze these perturbations to constrain individual subhalo masses and projected positions [47, 48]. To date, this technique has detected halos in the mass range 10^8–10^9 M using optical and infrared images [176–178]. For example, Vegetti et al. [199, 201] and Ritondale et al. [202] used

11–17 lens samples from the Sloan Lens ACS Survey [34] (SLACS) and BOSS Emission-Line Lens Survey [203] (BELLS) to constrain halo mass functions in the 10^8–10^{10} M range, finding results consistent with CDM predictions [200, 204]. In the millimeter/submillimeter band, studies of lens galaxy substructure have begun using a new class of lensed sources—dusty star-forming galaxies [54, 206].

ALMA observations of lensed dusty star-forming galaxies may revolutionize studies of dark matter halo substructure. These sources are advantageous due to their large numbers (compared to lensed quasars) and high redshifts, allowing study of lens galaxies across a wide redshift range to constrain redshift evolution of dark matter substructure. In 2013, Hezaveh et al. [206] constructed lens models for four dusty star-forming galaxies observed during ALMA Cycle 0. The sources' extreme brightness and ALMA's high sensitivity allowed strict constraints on foreground lens mass distributions even with very short total exposure times (~20 s) and only ~15 antennas, demonstrating that future full ALMA observations could enable detailed studies of lens galaxy mass distributions.

In 2016, Hezaveh et al. [54] used ALMA high-resolution data with pixelated source models to directly model visibility data and study substructure in SDP.81. They first tested the method on simulated data. As shown in [Figure 14: see original paper], Δ characterizes the difference in log-posterior probability between two models:

Δ (p_{sub})   _m(p_{sub}) − _m$_0$

where _m$_0$ corresponds to models without subhalos. For simulated data without substructure ([Figure 14a: see original paper]), the method correctly excludes subhalo presence. When a subhalo of mass 4×10^8 M is added at the blue marker position, it is detectable in simulations without antenna phase errors without false signals ([Figure 14b: see original paper]). With antenna phase errors present and uncorrected ([Figure 14c: see original paper]), numerous false signals appear. When phase errors are included in the model ([Figure 14d: see original paper]), these false substructure signals disappear. This test demonstrates that using visibility data rather than CLEANed images is crucial when searching for dark matter substructure with interferometer data, as CLEANed images fix antenna phases and prevent their optimization during model comparison, leading to spurious signals. In [FIGURE:14b,d], only one region shows significant signal, while more massive subhalos produce signals in multiple regions ([Figure 15: see original paper]), with the true subhalo corresponding to the position of minimum Δ . When fixing the first subhalo's parameters to search for a second ([Figure 15b: see original paper]), other substructure signals from [Figure 15a: see original paper] disappear.

Detected substructure signals may be affected by unknown errors in interferometer data (e.g., rapidly varying antenna phase errors). Since these errors are time-variable, analyzing multiple observations from different times can reveal whether results are affected. Hezaveh et al. [54] applied this method to search for dark matter substructure in SDP.81, analyzing Band 6 and Band 7 data separately (observed on different dates). During initial subhalo searches, subhalo mass was fixed ($M = 10^{8 \cdot 6}$ M) while considering Δ at different positions. Results from Band 6 and Band 7 were consistent ([Figure 16: see original paper]).

Based on these results, Hezaveh et al. [54] further added subhalo parameters (M_{sub} and position coordinates x_{sub}, y_{sub}) to fit the combined Band 6 and Band 7 data, re-optimizing all lens model parameters. The results revealed a subhalo with $M = 10^{8 \cdot 96} \pm^{0 \cdot 12}$ M at 6.9σ significance. Inoue et al. [207] independently analyzed SDP.81's substructure, imaging Band 7 continuum and Band 6 CO (8–7) line data, then searching for subhalos using CLEANed images. They detected substructure signals at similar positions to Hezaveh et al. [54] but found a compact low-density region near one arc. This discrepancy may arise because Inoue et al. [207] analyzed CLEANed images where antenna phase errors generate false substructure signals ([Figure 14: see original paper]). Thus, direct analysis of visibility data is necessary for dark matter substructure studies with interferometer observations.

Simulations show that Hezaveh et al.'s method can successfully detect lensing effects from subhalos with masses $>10^7$ M [54]. Li et al. [173] note that theoretical predictions for CDM and WDM models differ completely in this mass range. With numerous galaxy-galaxy strong lensing systems already known from millimeter/submillimeter observations, future high-quality data will help distinguish between different dark matter models.

6 Summary and Outlook

This paper reviewed current research on galaxy-galaxy strong lensing systems observed by (sub)millimeter interferometer arrays, covering observational samples, lens modeling methods for interferometer data, and research status. With the advent of wide-field extragalactic surveys at submillimeter wavelengths by facilities like Herschel [28], numerous galaxy-galaxy strong lensing systems have been discovered. ALMA's excellent sensitivity and spatial resolution enable detailed studies of star formation, interstellar medium properties, and kinematic characteristics of high-redshift galaxies at kpc and even sub-kpc scales. These observations constrain central supermassive black hole masses in lens galaxies like SDP.81 and advance our understanding of dark matter substructure in lens galaxies using extremely bright, high-redshift lensed dusty star-forming galaxies at (sub)millimeter wavelengths.

The current sample of radio galaxy-galaxy strong lenses contains ~200 systems, with the Square Kilometre Array expected to increase this to ~10^5 [208]. McKean et al. [208] predicted in 2015 that future larger-baseline radio interferometers (such as SKA) could potentially detect dark halos with masses $<10^7$ M . High-resolution observations of these strong lensing systems by ALMA and future

SKA will provide deeper insights into galaxy formation and evolution theories and dark matter models.

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